ECE 2C Mid-Term Exam
May 5, 2011

Do not open exam until instructed to.
Closed book: Crib sheet and 1 page personal notes permitted
There are 2 problems on this exam, and you have 75 minutes.
Use any and all reasonable approximations (5% accuracy is fine), AFTER STATING and approximately Justifying them.

Name: Solution A

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Problem 1. 85 points

You will be working on the circuit below:

\[\text{Q}_1 \text{ is a velocity-limited FET, i.e. } I_d = (v_m c_w) W_c (V_{gs} - V_{th}) (1 + \lambda V_d)\]
where \(v_m c_w W_c \approx 2 \text{ mA/V}\), \(\lambda = 0.1 \text{ V}^{-1}\), and \(V_{th} = 0.3 \text{ V}\).

\(V_{th} = \pm 2.0 \text{ volts.}\)
\(C_m\) and \(C_w\) are very big and have negligible AC impedance.
\(R_L = 5 \text{ kOhm}\)
\(R_{gen} = 100 \text{ kOhm}\)
Part a, 10 points
DC bias.
Q1 is to be biased with 0.5 mA drain current.
The source of Q1 is to be biased at 1.0 Volts
The DC current in Rgl is 2 μA
Ignore α while solving this part.

Find: Rgl = 225 kΩ, Rg2 = 775 kΩ, Rgs = 2 kΩ

\[
\begin{align*}
E_d &= 2 \times 1.6 \text{V} \quad (V_g - V_t) \\
0.5 \text{mA} &= 2 \times 1.6 \text{V} \quad (V_g - 0.5 \text{V}) \\
0.25 \text{V} + 0.5 \text{V} &= V_g = 0.75 \text{V} \\
V_b &= 1 \rightarrow V_g = V_b + V_g = 1.55 \text{V}
\end{align*}
\]

Total bias = Rg1 x CURRENT = Rg1 = 2 kΩ.

\[
2.5 \left[ R_{g2} = \frac{1.85 \text{V}}{2 \text{mA}} = 775 \text{kΩ} \right]
\]

\[
2.5 \left[ R_{gs} = \frac{2.0 - 1.55 \text{V}}{2 \text{mA}} = 225 \text{ kΩ} \right]
\]

\[
2.5 \left[ R_{gs} = 14/0.5 \text{mA} = 28 \text{kΩ} \right]
\]
Part b, 10 points

DC bias

On the circuit diagram above, label the DC voltages at ALL nodes and the DC currents through ALL resistors.

2 pts. each.
Part c, 10 points

Find the small signal parameters of Q1. Use the constant-velocity model.

\[ g_m = 2 \text{ mS} \]
\[ R_d = 10 \Omega \text{ k} \Omega. \]

\[ \begin{align*}
I_d &= \frac{2 \text{ mA}}{1 + \frac{1}{g_m}} = 2.2 \text{ mA} \\
&= 2 \text{ mA} \text{ (in practice)} \text{ for the small model.}
\end{align*} \]

\[ \begin{align*}
R_{ds} &= \left( \frac{V_{ds} + \frac{1}{g_m}}{I_d} \right) \frac{V_{ds} + 10V}{10 \Omega} = 220 \Omega \\
&= \frac{1}{10 \Omega} = 10 \Omega \text{ either answer accepted.}
\end{align*} \]
Part e, 10 points.

Find the small signal voltage gain \((V_{out}/V_{in})\) of Q1

\[
\frac{V_{out}}{V_{in}} = 0.714
\]

\[
5 \left[ V_{out} = \frac{R_{os} + R_{es}}{R_{l}} V_{in} \right]
\]

\[
5 \left[ \frac{V_{out}}{V_{in}} = \frac{R_{os}}{R_{l}} \text{ for } R_{es} \gg R_{l} \right]
\]

\[
\frac{R_{es}}{R_{l}} = \frac{2.5}{1 + 2.5} = 0.71
\]
Part 1. 10 points

Find the amplifier input resistance, \( V_{in/V_{gen}} \), and \( V_{out/V_{gen}} \): 

\[
\text{Rin_{amplifier}} = 1742 \Omega \\
V_{in/V_{gen}} = 0.635 \\
(V_{m/v/V_{gen}}) = 0.45
\]

\[
4 \left\{ \begin{array}{l} 
R_{inc} = R_{g1}/R_{g2} = 220k \Omega / 225k \Omega \\
= 1742 \Omega \\
V_{in} \frac{V_{ga} + R_{inc}}{R_{inc} + R_{gs}} = \frac{1046e-6}{1742} = 0.635 \\
V_{out} = \frac{V_{in}}{V_{gs}} = 0.45
\end{array} \right.
\]
Part g. 15 points:

Now you must find the maximum signal swings. Find the output voltage due to the knee voltage and due to cutoff in Q1. Since this is a constant-velocity model, the knee voltage of the transistor must be specified: $V_{knee} = V_{E_{min}} = V_{D_{max}} = 0.30$ Volts, i.e. the FET must have a minimum 0.30 Volts between drain and source for linear operation.

Cutoff of Q1: Maximum $\Delta V_{out}$ resulting $= \boxed{-0.30}$

Knee voltage of Q1: Maximum $\Delta V_{out}$ resulting $= \boxed{+0.7V}$
Problem 2, 20 points
Nodal analysis

Part a, 15 points

Using nodal analysis, determine $V_{out}/I_{in}$.

5. Nodal equation at $V_{in}$:
   \[ V_{in}R_{F} - V_{gs}R_{F} = I_{in} \]

5. Nodal equation at $V_{out}$:
   \[ (V_{out} - V_{gs})R_{L} + V_{out}(1/R_{F} + 1/R_{L}) = 0 \]

5. \[
   \frac{V_{out}}{I_{in}} = \frac{1 - g_{m}R_{F}}{g_{m} + 1/R_{L}}
   \]

6. \[
   (V_{out} - V_{in})R_{L} = I_{in} \rightarrow V_{out}R_{L} + V_{out}(1/R_{F} + 1/R_{L}) = I_{in}
   \]

6. \[
   V_{out}R_{F} + (V_{out} - V_{in})R_{L} + g_{m}V_{in} = 0
   \]

6. \[
   V_{out}(g_{m} - 1/R_{F}) + V_{out}(1/R_{F} + 1/R_{L}) = 0
   \]
\[
\begin{bmatrix}
G_0 & -G_1 \\
G_0 - G_{in} & G_1 + G_{in}
\end{bmatrix} \begin{bmatrix} V_{in} \\ V_{out} \end{bmatrix} = \begin{bmatrix} I_c \\ 0 \end{bmatrix}
\]

\[V_{in} = \frac{N}{5} - 6V_0\]

\[N = \begin{bmatrix} G_0 & I_c \\ G_0 - G_{in} & 0 \end{bmatrix} = I_c (G_0 - G_{in})\]

\[D = \begin{bmatrix} G_0 & -G_1 \\
G_0 - G_{in} & G_1 + G_{in} \end{bmatrix} = G_0 G_{in} + G_1 G_{in} + 0 + G_0 G_{in} - G_1 G_{in}
\]

\[= (G_0 + G_{in}) G_{in}\]

\[\frac{V_{out}}{I_c} = \frac{G_0 - G_{in}}{(G_0 + G_{in}) G_{in}} = \frac{(1000 - 1000) G_{in}}{G_{in}} \]

\[= 1 - \frac{1000 R_f}{1000 + 1000 R_f}\]
Part b. 5 points

$R_i = 1 \text{kOhm}, \quad R_f = 10 \text{kOhm}, \quad g_n = 100 \text{mS}.$

Find the numerical value of $V_{in}/I_m$:

$$\frac{V_{in}}{I_m} = \frac{-9.88 \text{mV}}{}$$

$$\frac{V_{in}}{I_m} = \frac{1 - g_m R_f}{R_i + 100 \text{mS}}$$

$$= \frac{1 - 1000 \times 100 \text{mS}}{1000 \text{Ohm} + 100 \text{mS}}$$

$$= \frac{-999}{101 \text{mS}}$$

$$= -9.88 \text{mV}$$