• Notes Set 10: Noise in Field-Effect Transistors

- velocity field curves. Ad-hoc electron termperature model of transport.
- gradual-channel model, integrals, derivation of channel noise current
- modification of model for velocity-saturated case
- Gate current noise and correlation with drain current noise
- simplified, pragmatic noise model

ECE ___ Notes Set 10

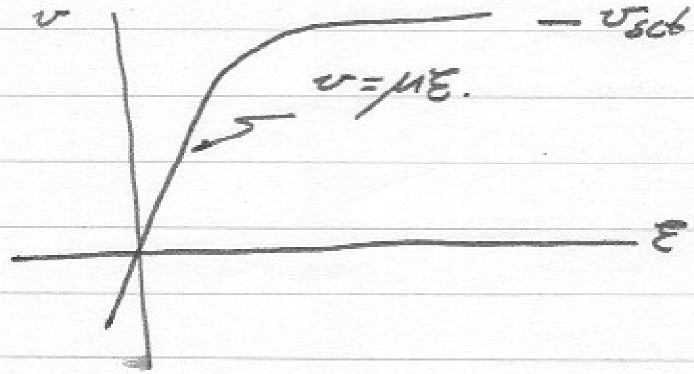
Noise in Field-Effect transistors:

☆ As with bipolar transistors, we must first work the basic device physics in order to later develop a noise model.

☆ Bipolar transistor analysis involves more basic principles, but the solution of the resulting equations is generally straightforward.

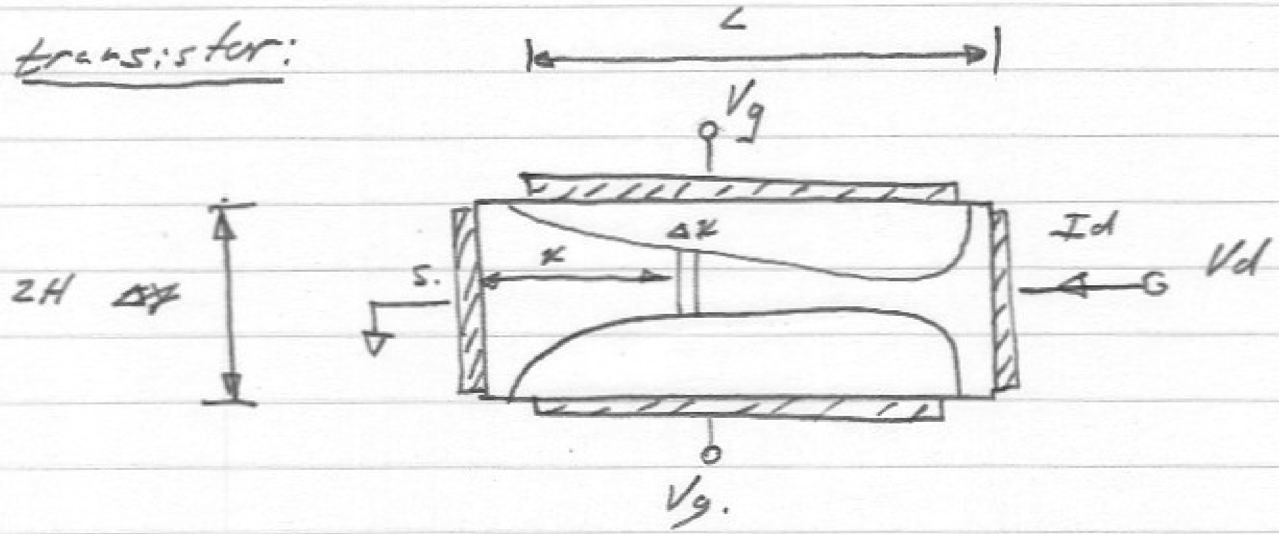
☆ Field-effect transistor analysis involves fewer basic principles, but the resulting equations generally involve solving Poisson's equation in 2 dimensions, so the math is often approximate or lengthy and the answers are generally complicated, or may not be closed-form.

Fet analysis is further complicated by high-field effects, e.g. the intervalley scattering processes which result in a saturation drift velocity:



While velocity saturation is important in all semiconductor devices, the bjt current fluxes, hence noise, are determined by transport in low-field regions, hence we need not model the noise of high-field transport. In fets the control region itself is usually in high fields. Treatment of noise in high field regions is somewhat empirical.

Lets start with the <u>junction field-effect transistor</u>:



~~[illegible]~~ This will give us a picture to work with. First lets develop some general relationships.

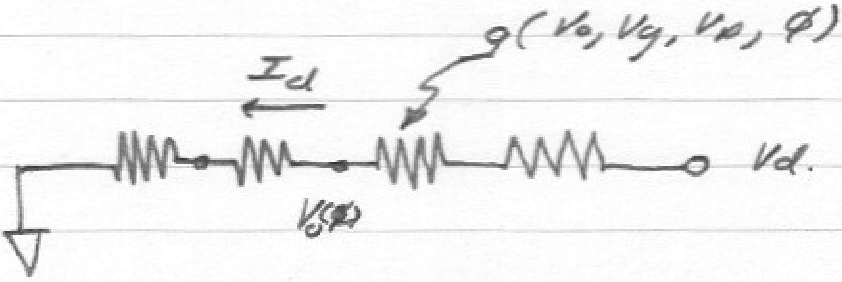
Analysis of FET I-V relationships involves noting that at the position $\underline{x}$, the <u>channel</u> is at a voltage $V_o(x)$. This modulates the width of the depletion region as a function of:

$v_o(x)$ the voltage on the channel

V_g the gate voltage

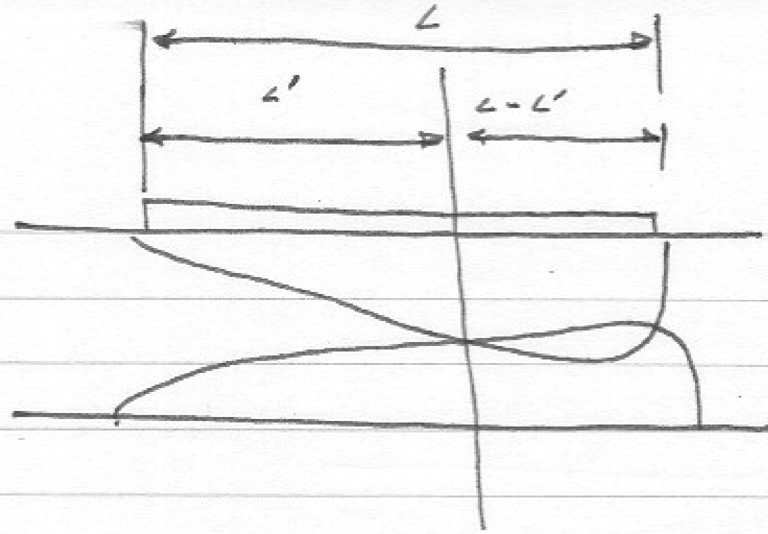
V_p the pinchoff voltage, e.g. the voltage necessary to fully deplete the channel.

ϕ the junction built-in potential.

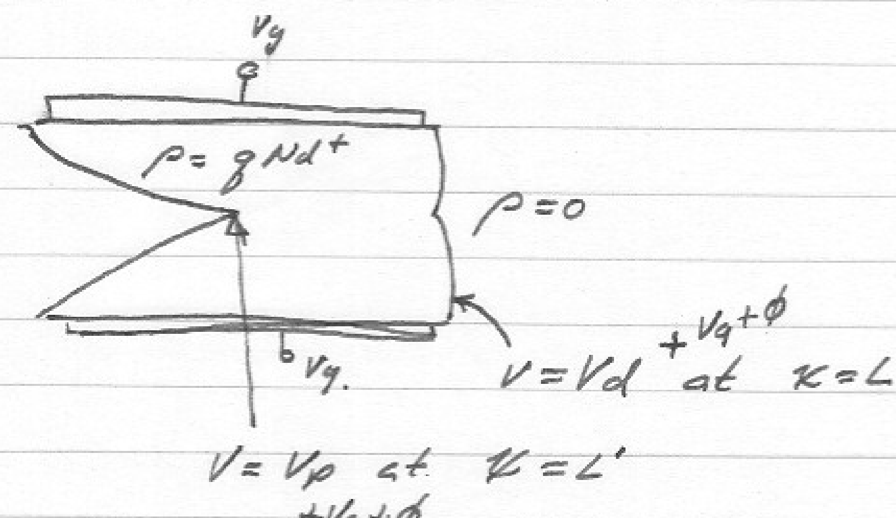


$$\frac{\partial}{\partial x} v_o(x) = \frac{I_d}{g(v_o(x))}$$

This is the model by which a fet is normally analyzed.



If the drain voltage is larger than the pinchoff voltage, then the ~~drain~~ voltage at the channel at the point of full pinch-off is $V_0(L') = V_p$, and the drain current is found for these conditions. Finding L' from $V_d - V_p$ is generally difficult, involving the electrostatic problem below:

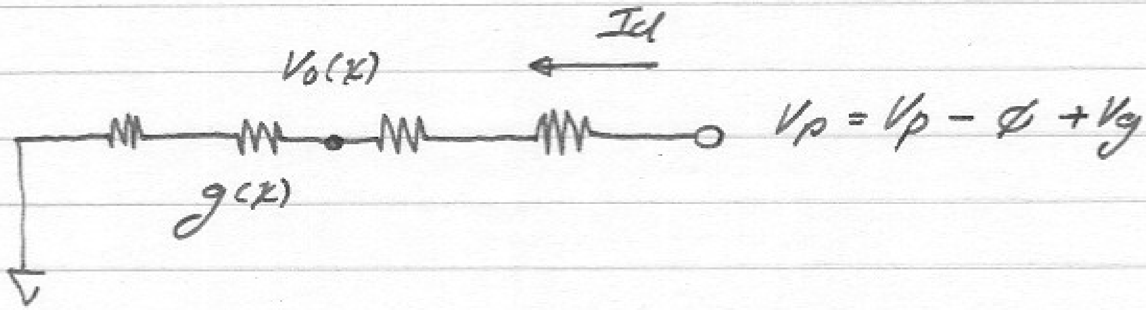


solving this problem would give us the variation of L' with $(V_d - V_p)$ hence the variation of I_d with V_d, e.g. the fet output conductance. <u>This is ~~relatively~~ never done by hand</u> as the electrostatics are not closed-form.

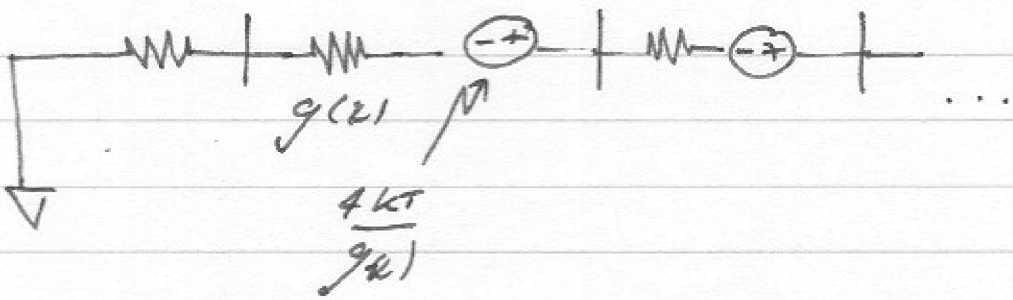
Insted we always take the simplifying approximation that $V_d = V_p + V_g - \phi$ and solve for I_d. Variation of I_d with V_d is taken to be small beyond this point. Here $\underline{L' = L}$

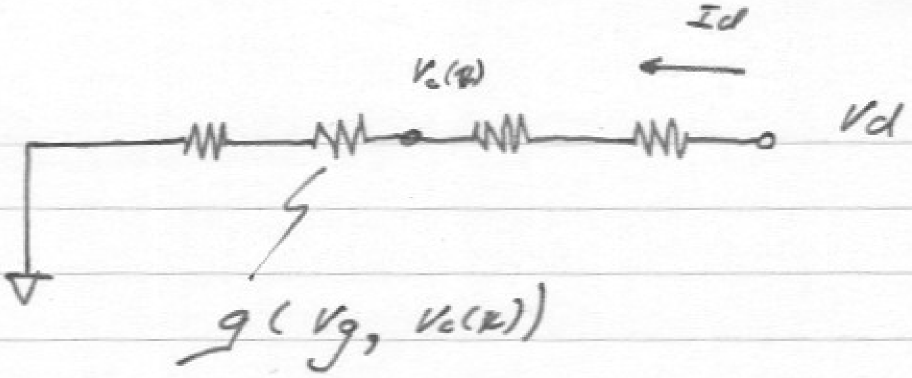
Noise analysis will follow this method.

DC I-V model:



Noise Model obtained by taking each resistor as having thermal noise...





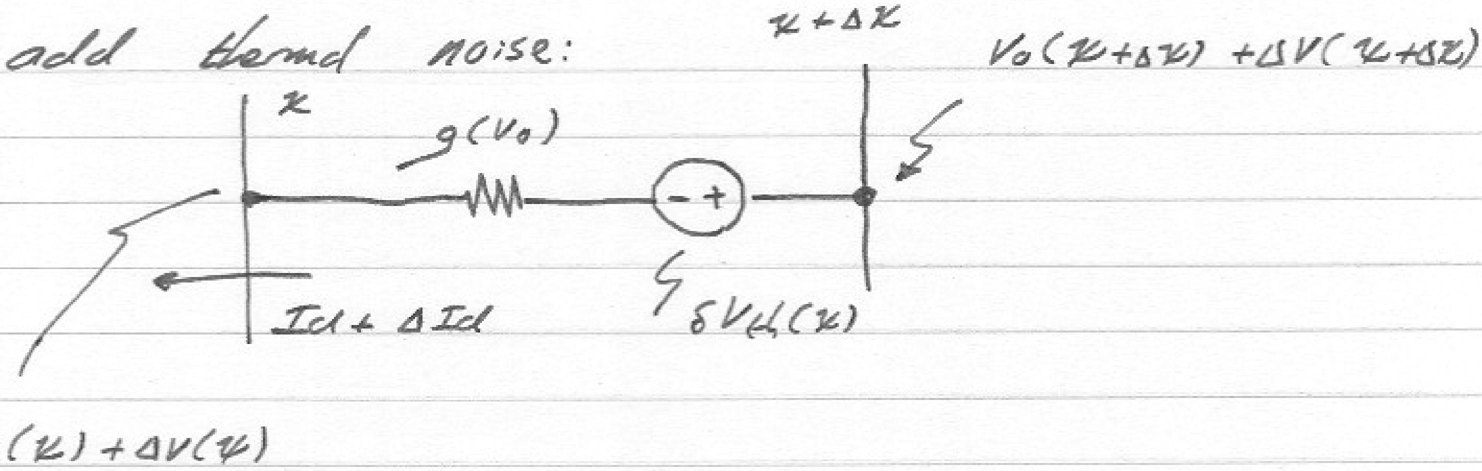
$$I_d = g(V_g, V_o(x)) \cdot \frac{dV_o}{dx}$$

hence
$$\int_0^L I_d \, dx = \int_0^{V_d} g(V_g, V_o(x)) \, dV_o$$

$$\boxed{I_d = \frac{1}{L} \int_0^{V_d} g(V_g, V_o) \, dV_o}$$

by this equation, $I_d(V_g)$ is found.

add thermal noise:



here $V_0(x)$ & I_d are bias quantities.

ΔI_d & ΔV are noise fluctuations:

$$\boxed{\frac{d}{dx}\left[\cancel{g(V_0)} \, \Delta V(x) \right] = \delta V_{th}(x) + \frac{\Delta I_d}{g(V_0)}}$$

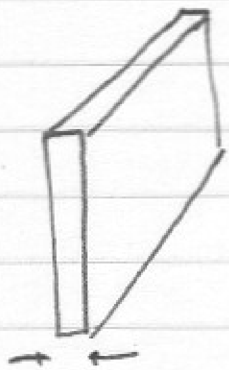
Lets solve this ...

$$\Delta I_d = g(V_0) \cdot \frac{d}{dx}\left[\Delta V(x)\right] - g(V_0)\, \delta V_{th}(x)$$

$$= g(V_0) \cdot \frac{d}{dx}\left[\Delta V(x)\right] + h(x)$$

Here δV_{th} has a power spectral density

$$\frac{d}{df}\langle \delta V_{th}\, \delta V_{th}^{*} \rangle = \frac{4kT}{g(V_0)}$$



δx

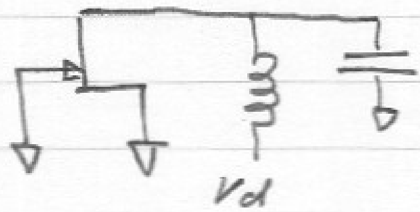
$[g(V_0)]^{-1}$ is a resistance per unit length, $\frac{\text{ohms}}{\text{meter}}$

so $\frac{4kT}{g(V_0)}$ has units of $\frac{V^2}{Hz} \cdot \frac{1}{m}$

The math has to be done carefully to avoid infinities, as $\frac{d}{df}\langle h(x) h(x)^{*}\rangle$ has units of $\left[\frac{A^2}{Hz} \cdot \text{meters}\right]$

$$\frac{d}{df}\langle h(x) h(x)^{*} \rangle = 4kT\, g(V_0)$$

we now find the short-circuit drain noise current:



ac short on drain.

hence $\Delta V = 0$ at $x=0$ and at $\underline{x=L}$.

$$\Delta I_d L = \int_0^L \frac{d}{dx}\left(g(V)\, \Delta V(x)\right) \cdot dx + \int_0^L h(x,t)\, dx.$$

$$\Delta I_d = \frac{1}{L}\int_0^L h(x,t) \cdot dx$$

but $$\frac{d}{df} \langle h(x)\, h(x)^* \rangle = 4kT\, g(V_0)$$

$$\cancel{\frac{d}{df} \langle \Delta I_d\, \Delta I_d^* \rangle =}$$

so:

$$\langle \Delta Id(t) \cdot \Delta Id(t+\tau) \rangle = \frac{1}{L^2} \int_0^L \int_0^L \langle h(x,t)\, h(x',t+\tau) \rangle \, dx\, dx'$$

but the thermal noise is uncorrellated between positions x & x':

$$\langle \Delta Id(t) \cdot \Delta Id(t+\tau) \rangle = \frac{1}{L^2} \int_0^L \langle h(x,t)\, h(x,t+\tau) \rangle \, dx$$

or writing power spectral densities:

$$\frac{d}{df} \langle \Delta Id\, \Delta Id^* \rangle = \frac{1}{L^2} \int_0^L \frac{d}{df} \langle h(x)\, h(x)^* \rangle \, dx$$

$$= \frac{1}{L^2} \cdot \int_0^L 4KT\, g(V)\, dx$$

but $Id = g(V) \cdot dV/dx$

so $dx = \frac{g(V)}{Id} \cdot dV.$

so finally:

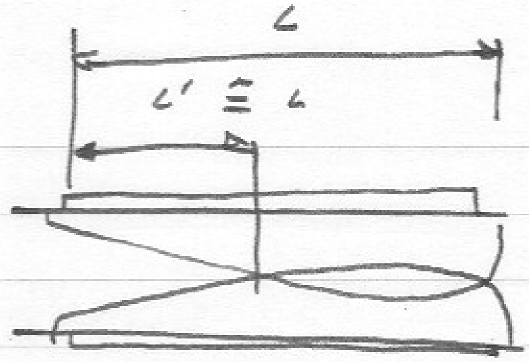
$$\boxed{\frac{d}{df} \langle \Delta I_d \, \Delta I_d^* \rangle = \frac{1}{L^2 I_d} \int_0^{V_d} 4kT \, g^2(V) \, dV}$$

This can now be used with various FET models...

where

$$\boxed{I_d = \frac{1}{L} \int_0^{V_d} g(V_g, V_0) \, dV_0}$$

The JFET



Conductance modulated by depletion depth.

under 2 assumptions:

graded channel: ignore 2 dimensional electrostatics.

constant mobility: $J = \sigma \mathcal{E}$.

$$g(V_0) = g_{open}\left[1 - \left(\frac{\phi - V_g + V_0}{V_{po}}\right)^{1/2}\right]$$

ϕ = built-in potential.

Integrate $I_d = \frac{1}{L}\int_0^{V_g} g(V_g, V_0)\, dV_0$

to find:

$$I_d = \frac{g_{open} V_{po}}{L}\left[1 - Z - \frac{2}{3}(1 - Z^{3/2})\right]$$

where $Z = \dfrac{\phi - V_g}{V_{po}}$

and we integrate the noise equation to find:

$$\frac{d}{df}\langle I_d I_d^*\rangle = 4kT\gamma \cdot g_m$$

where:

$$\gamma = \frac{1}{2}\left[\frac{1 + 3Z^{1/2}}{1 + 2Z^{1/2}}\right]$$

$$g_m = \frac{g_{open} V_{po}}{L}\left[1 - Z^{1/2}\right]\frac{1}{V_{po}}$$

$$g_m = g_{mo}\left[1 - Z^{1/2}\right]$$

So for the JFET under long-channel & constant-mobility assumptions:

$$\frac{\partial}{\partial f} \langle I_d I_d^* \rangle = 4kT\Gamma g_m$$

where $\Gamma = \begin{cases} 1/2 & \text{fully open channel} \\ 2/3 & \text{pinched off} \end{cases}$

$\cong 5/12.$

<u>For the MOSFET,</u>

$$g(V_0) = \mu W \cdot C_{ox}(V_g - V_{g0} - V_0)$$

channel voltage

threshold

gate voltage

μ = mobility, w = channel width,

C_{ox} = oxide capacitance.

From which one can find:

$$I_d = \frac{\mu W C_{ox}}{L} \cdot (V_g - V_{g0})^2$$

and a transconductance:

$$g_m = \frac{\mu W C_{ox}}{L}(V_g - V_{g0})$$

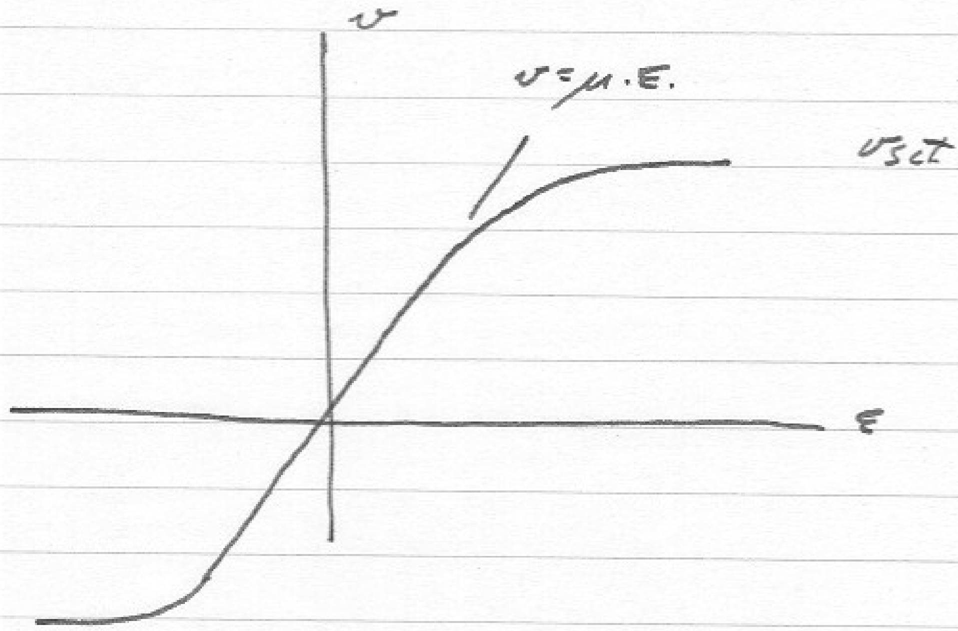
and a drain noise current:

$$\frac{\partial \langle I_d I_d^* \rangle}{\partial f} = 4KT\Gamma g_m$$

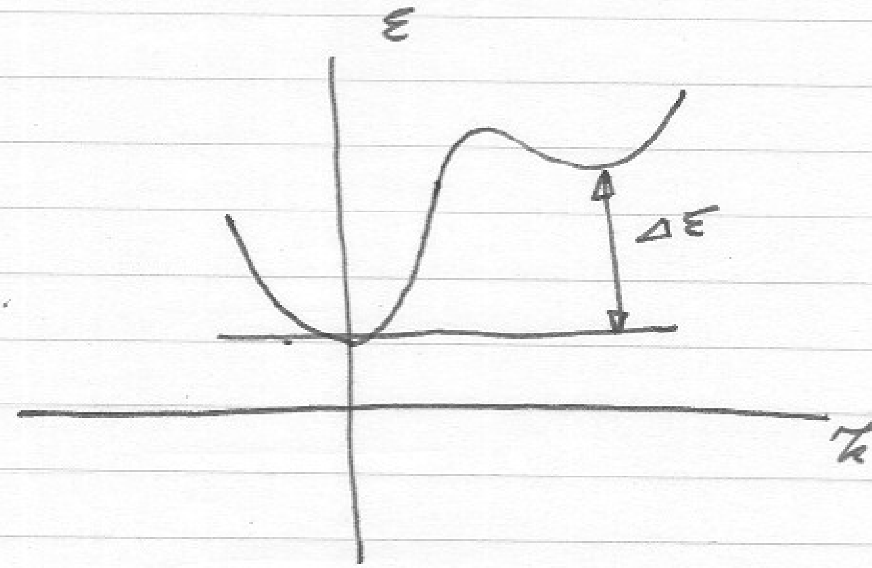
where $\boxed{\Gamma = 2/3}$

MOSFET & HEMT at high fields

Velocity - field characteristics of III-V materials look like so:



This saturation results from intervalley scattering:



Energetic electrons in the lowest valley having kinetic energy above ΔE the intervalley sep' energy, will scatter at some rate into the higher valleys. This scattering is due to phonons & is itself a random process. Once in the higher valley the electrons have much higher effective mass & lower differential mobility. They will subsequently relax to the lower valley, again at some scattering rate.

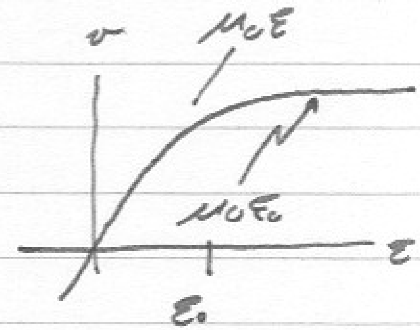
Terminology:

If $\sigma_{v_x}^2$ is the variance of the electron velocity distribution, then define: T_e, the electron temperature: such that:

$$\boxed{\sigma_{v_x}^2 = KT_e/m.}$$

Empirical Model:

$v = \mu(E) \cdot E$, where



$$\mu(E) = \frac{\mu_0}{1 + E/E_0}$$

$v \rightarrow v_{sat} = \mu_0 E_0$ for $E >> E_0$

$$T_e = T\left(1 + \frac{E}{E_0}\right)^n \qquad 0 < n < 2$$

this is simply an empirical fit.

velocity distributions:

zero-field (equilibrium):

gaussian distribution of v_x, v_y, v_z with

$\sigma_{v_x}^2 = kT/m.$

near-zero field (constant mobility regime)

gaussian distribution of v_x with

mean: $\bar{v} = \mu \mathcal{E}.$ and

variance $\sigma_{v_x}^2 = kT/m.$

High-field (nowhere near equilibrium)

some unknown distribution of

mean $\bar{v} = v_{sat}$

and variance $\sigma_{v_x}^2 >> kT/m.$

where T is the crystal temperature.

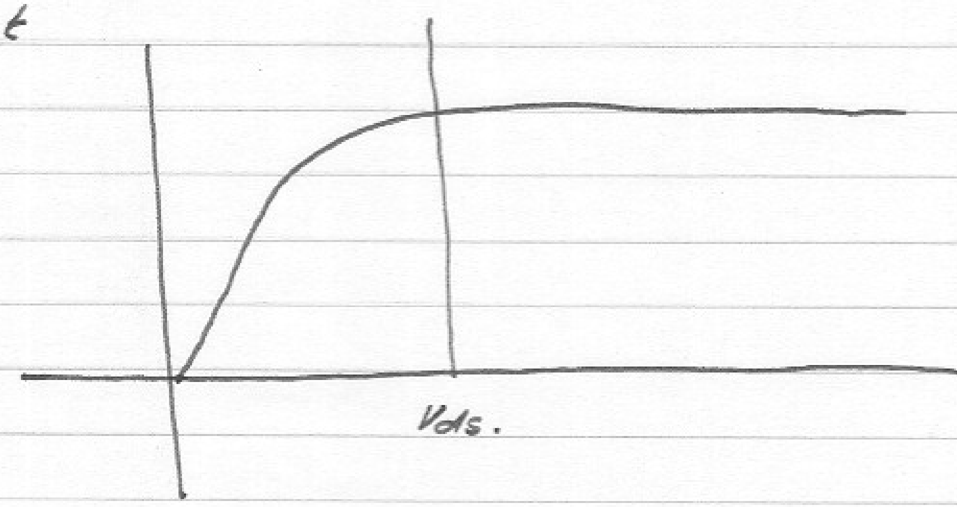
<u>For a MOSFET</u>

$$g(V_0) = \mu(E) \cdot w \cdot C_{ox}(V_g - V_{g0} - V_0)$$

... which is also directly applicable to a HEMT.

writing: $y = \frac{V_g - V_0}{E_0 L}$, the drain characteristics

saturate at



$$\frac{V_{ds}}{E_0 L} = Z = \sqrt{1+2y} - 1, \text{ which is } < V_D$$

with the drain current spectral density

written as $\frac{d}{df}\langle I_d I_d^* \rangle = 4kT\,\Omega\, g_m$,

Van Der Ziel finds that:

$$I = \frac{1}{2}\left[\sqrt{1+2y} + (1+2y)\right] \cdot \frac{1-(Z/y) + 1/3\,(Z/y)^2}{1 - Z/2y.}$$

$$\cdot\ (1+2y)^{1/2}$$

for $T_e = T\left(1+\frac{\varepsilon}{\varepsilon_a}\right)^2$ eg $n=2.$

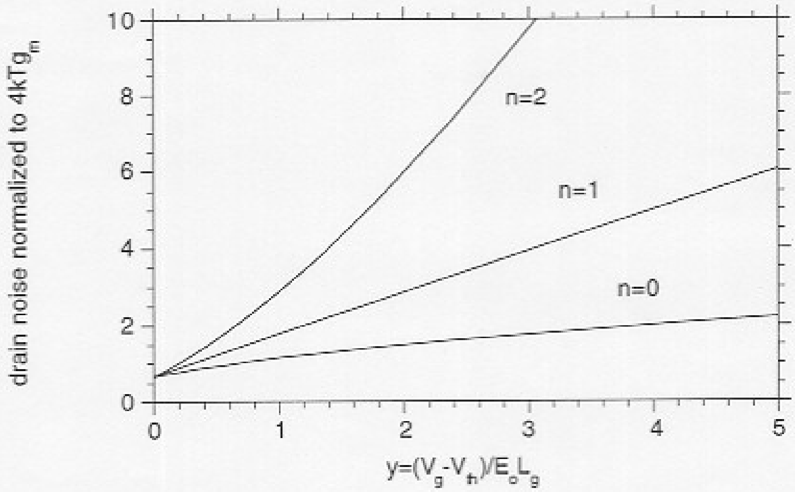
This is not very clear, so I have made a plot.

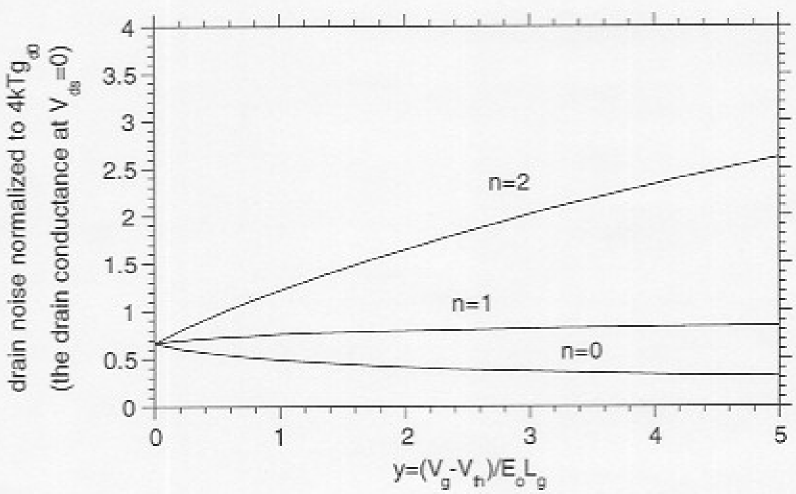
also

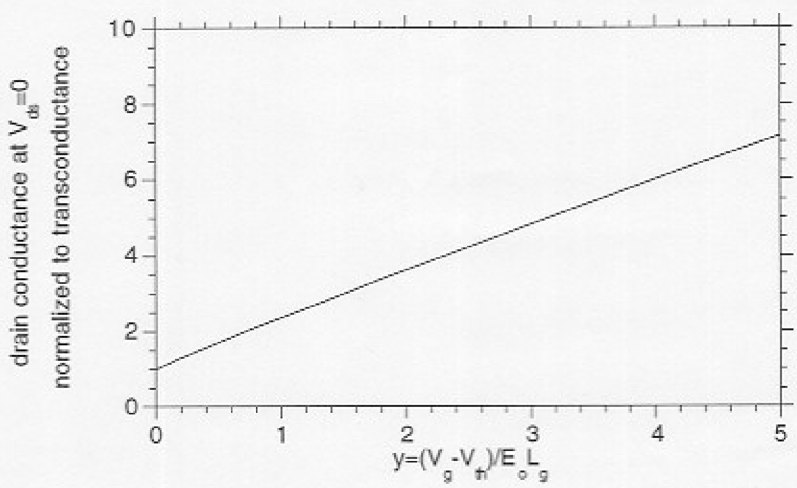
$$\gamma = \left[\frac{1 - Z/y + 1/3\,(Z/y)^2}{1 - Z/2y}(1+2y)^{1/2} + Z\left(\frac{2+Z}{1+Z}\right)\left(1 - \frac{Z}{2y}\right)\right]$$

$$\cdot\ \frac{1}{2}\left[\sqrt{1+2y} + (1+2y)\right]$$

fore $T_e = T.$, e.g. $n=0.$


drain noise normalized to 4kTg_m
n=2
n=1
n=0
0
2
4
6
8
10
0
1
2
3
4
5
y=(V_g-V_th)/E_oL_g


drain noise normalized to 4kTg_d0
(the drain conductance at V_ds=0)
n=2
n=1
n=0
0
0.5
1
1.5
2
2.5
3
3.5
4
0
1
2
3
4
5
y=(V_g-V_th)/E_oL_g


drain conductance at V_ds=0
normalized to transconductance
0
2
4
6
8
10
0
1
2
3
4
5
y=(V_g-V_th)/E_oL_g

Now, in interpreting the above, note:

1) noise analysis is done at saturation point

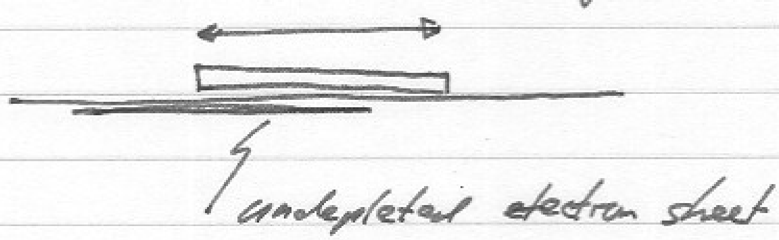
with $y = \frac{V_g - V_{po}}{E_o L}$, the saturation point

is $V_{ds} = E_o \cdot L \cdot Z$, where $Z = \sqrt{1+2y} - 1$

2) Presumption is (?) that g_m & $\frac{d}{dt} \langle I_d I_d^* \rangle$

do not increase markedly for V_{ds} larger than this.

3)



$y = \frac{V_g - V_{po}}{E_o L}$ is $V_g - V_{po}$ normalized to $E_o L$,

e.g. the average field in the undepleted

portion of the channel relative to E_o

This should be kept low.

Note also that if we compare g_m to the zero-V_{ds} output conductance

$$\frac{g_m}{g_{do}} = \frac{2}{(1+2y) + \sqrt{1+2y}} \; ; \; y = \frac{V_g - V_{po}}{E_0 L}$$

$$= 1 \text{ for average fields } \ll E_0$$

$$\ll 1 \text{ for average fields } \gg E_0 .$$

... a large part of Γ lies in the $\frac{g_m}{g_{do}}$ ratio –

I will also point out, without proof or derivation, that for FETS (HEMTS & MOSFETS) in very weak inversion, such that

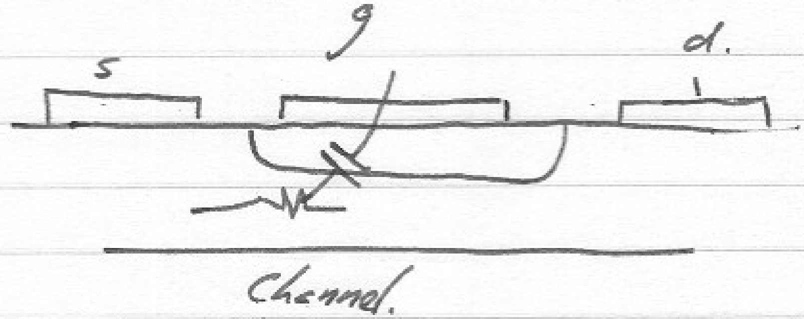
$$I_d \sim I_0 \, e^{q V_{gs}/kT}$$

that the output noise current is $2qI_d$ in spectral density.

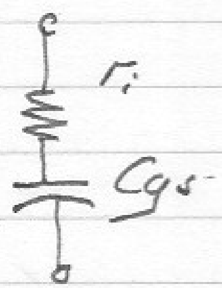
writing $\frac{\partial}{\partial f} \langle I_n I_n^* \rangle = 4kT\Gamma g_m$,

this corresponds to $\Gamma = 1/2$.

Gate / Channel Noise



Input capacitance to device has to be charged through some portion of the channel. This leads to the usual input model of a fet:



Finding R_i involves similar calculations to those shown previously. We will do these in detail later.

One might expect that r_i is an independent physical resistance with available noise power kT, in which case the noise-current into a short-

$$\frac{\partial}{\partial f}\langle I_g I_g^* \rangle = 4kT r_i \cdot \omega^2 C_{gs}^2$$

... but note that r_i noise should be strongly correlated with the drain (channel) noise & that the resistor R_i is not a simple resistor in thermal equilibrium.

Note also in the case of a long-channel

mosfet that

$$C_{gs} = \frac{2}{3} \cdot L_g \cdot W_g \cdot C_{ox}.$$

$$\& \quad r_i = \frac{1}{5 g_m}$$

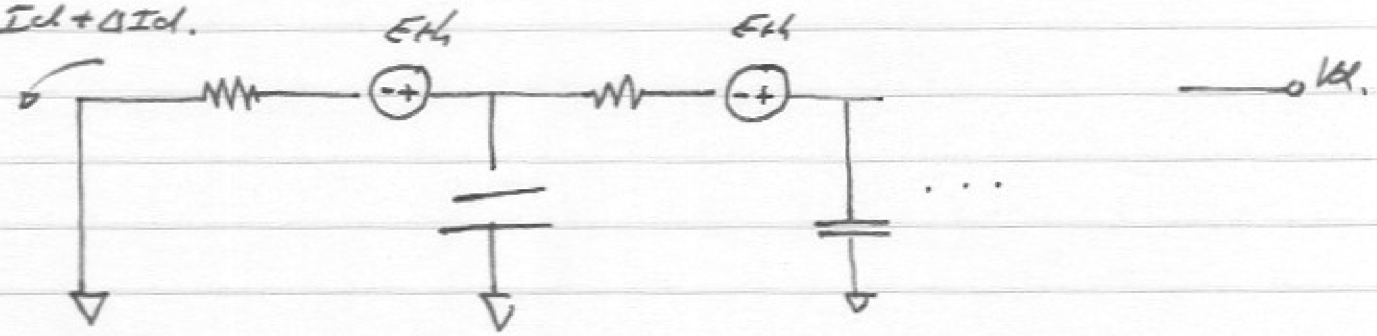
whereas... for short-channel FETs

$$C_{gs} = \frac{2}{3} L_g \cdot W_g \cdot C_{ox}.$$

and typically $r_i \approx \frac{1}{2 g_m}.$

Can we now derive these results?

How FET gate noise is analyzed:



First, approximate that noise current in gate Is relatively small compared to noise current in drain ΔI_d. Then we have:

$$\Delta I_d(t) = \frac{d}{dx}\left[g(V_0)\,\Delta V(x,t)\right] + E_{th}(x)\cdot g(V_0)$$

... as before

the gate noise current is then found from:

$$\Delta I_g = j\omega W_g \int_0^{L_g} C\,\Delta v(x)\,dx$$

This becomes, after a very long calculation, an expression for I_g's spectral density and its cross spectral density with I_d.

The resulting analysis is not simple, and clear analytic answers do not appear in the literature.

General conclusion seems to be:

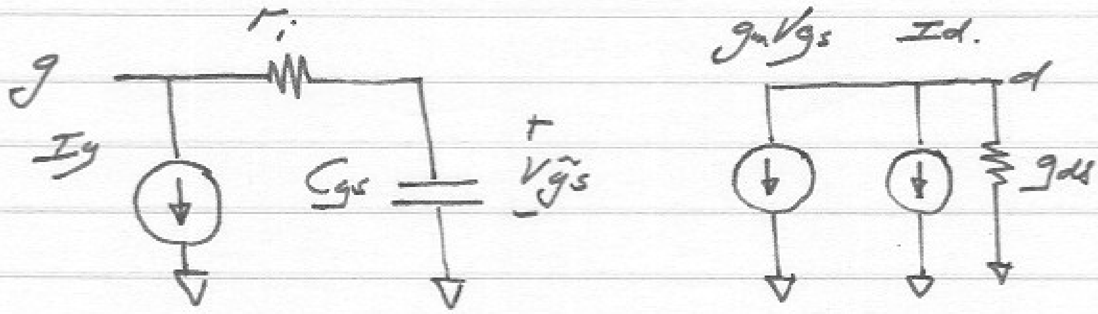
R_i is typically $\sim \frac{1}{g_m}$ to $\frac{1}{2g_m}$

R_i has a noise voltage spectrum of $\sim 4kTR_i$

Correlation of R_i noise with I_d is $\sim 0.3 - 0.4$.

in ~~magnitude~~ magnitude for a constant-mobility model, increasing to $\sim 0.8 - 0.9$ for high velocity saturation.

This is summarized below:



$$\frac{d\langle I_d \cdot I_d^*\rangle}{df} = 4kT\,\Gamma\, g_m$$

$$\frac{d\langle I_g\, I_g^*\rangle}{df} \cong 4kT\, r_i\,(\omega^2 C_{gs}^2)$$

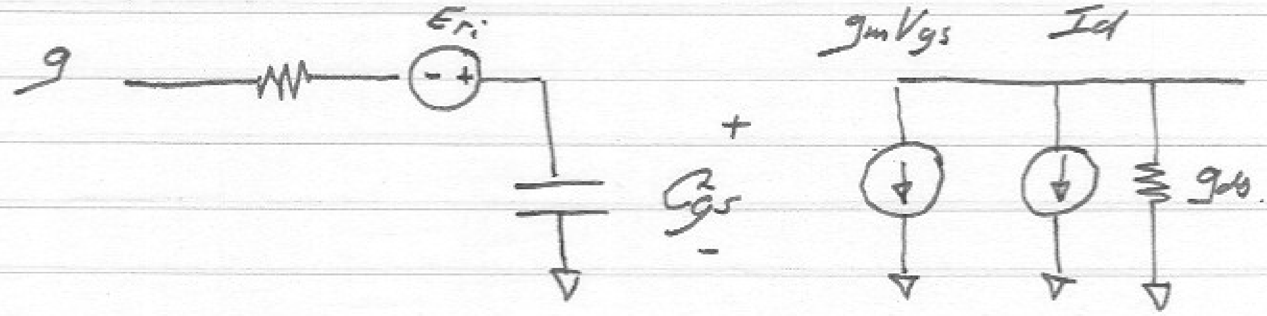
$$\frac{d}{df}\langle I_g\, I_d^*\rangle \cong j c \sqrt{4kT\Gamma g_m}\,\sqrt{4kT r_i (\omega^2 C_{gs}^2)}$$

c: 0.3 - 0.4 constant mobility

0.8 - 0.9 high field.

note that the sign of c depends on the directions assumed for I_g & I_d, and whether we have written $\langle I_g\, I_d^*\rangle$ or $\langle I_g^*\, I_d\rangle$.

Note that this is equivalent to:



$$\frac{d}{df}\langle I_d I_d^*\rangle = 4kT\,\Gamma\, g_m$$

$$\cancel{\frac{d}{df}\langle E_{ri} I_d^*\rangle}$$

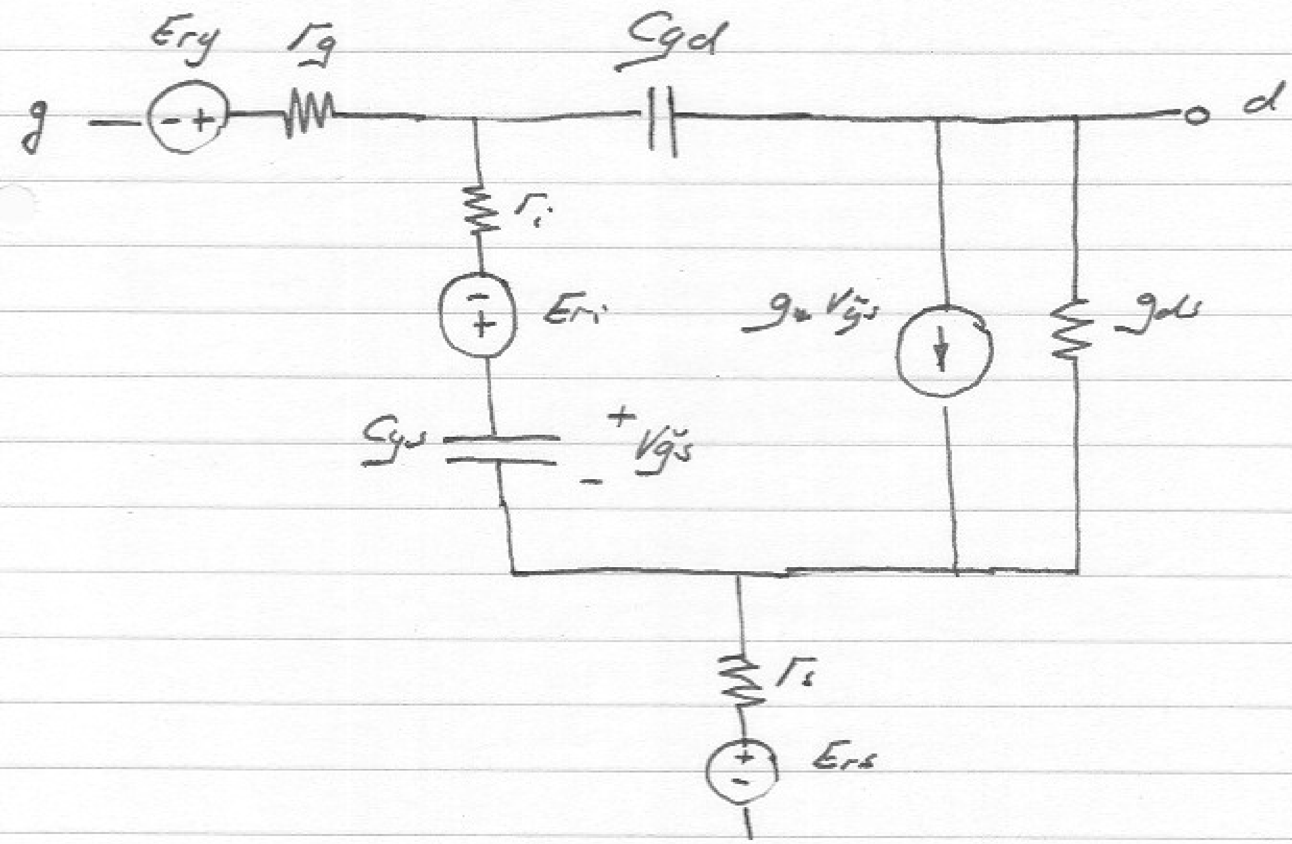
$$\frac{d}{df}\langle E_{ri} E_{ri}^*\rangle \cong 4kT\, r_i$$

$$\frac{d}{df}\langle E_{ri} I_d^*\rangle \cong jC\,\sqrt{4kT\Gamma g_m}\,\sqrt{4kT r_i}$$

0.3 - 0.4 constant mobility

0.8 - 0.9 high field.

In addition to the intrinsic noise sources above, FETS have parasitic series resistance in the gate & ~~drain~~ source circuits, from which the model becomes:



E_{rs} & E_{rg} are the thermal noise of R_s & R_g..

<u>Overall, over-riding comment</u>: re gate noise.

1) I am not a fet expert

2) Literature does not give convincing and consistent answer for $\langle I_g I_g^* \rangle$ and $\langle I_g I_d^* \rangle$

3) In modern FETs typically r_g and r_s are each 1-2 times larger than r_i, so their noise dominates over R_i

4) Consequently, experimental papers show that correlation of r_i noise with I_d can be neglected. Specifically the correlation of r_i noise with I_d noise has too small an effect on transistor noise figure to experimentally verify the theories

Other observations:

Modern very-high-performance HEMTs are made with

1) very small gate-channel separations ~ 100 Å.

2) Advanced semiconductor materials of sufficient novelty that passivation techniques are immature.

... Consequently, significant gate leakage currents are observed due to tunnelling and/or surface leakage.

⇒ This leads to a gate shot noise spectral density $2q\overline{I_{gate}}$.

Example: 1995 InAlAs / InGaAs / InP HEMT with $0.15\mu m \cdot 50\mu m$ gate: $1\mu A$ gate leakage at room temperature, $10\mu A$ @ 85°C.

Other observations:

Modern very-high-performance HEMTs are made with

1) very small gate-channel separations ~ 100 Å.

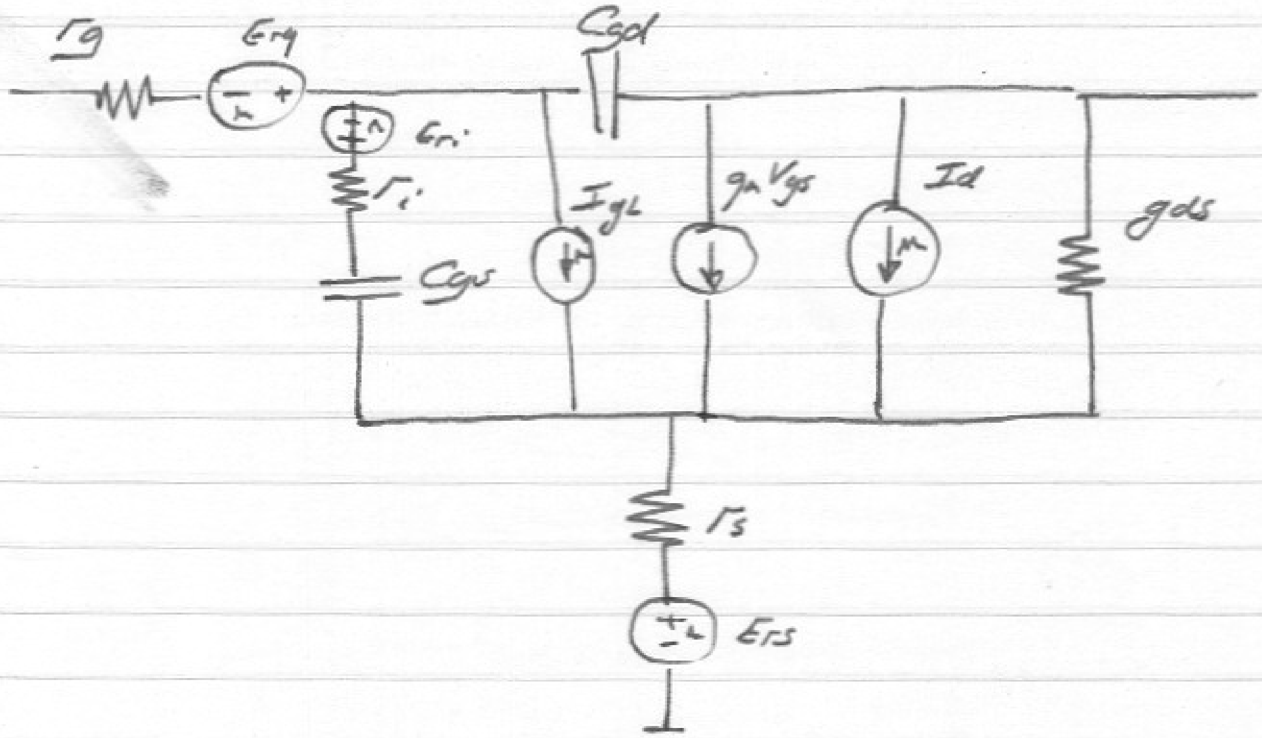
☆ 2) Advanced semiconductor materials of sufficient novelty that passivation techniques are immature.

... Consequently, significant gate leakage currents are observed due to tunnelling and/or surface leakage.

⟹ This leads to a gate shot noise spectral density $2q\overline{I_{gate}}$.

Example: 1995 InAlAs/InGaAs/InP HEMT with 0.15μm · 50μm gate: 1μA gate leakage at room temperature, 10μA @ 85°C.

So the full noise model we will use:



Source	spectral density:
I_d:	$4kT\gamma g_m$
r_g	$4kT r_g$
r_i	$4kT r_i$
r_s	$4kT r_s$
I_{gL}	$2q\overline{I_g}$.

Ignoring all correllations. This is the model generally used by the circuits / IC literature.