

- Notes Set 12: Physics and Noise of
Resonant tunnel diodes

- this is a very long and very detailed treatment of a very specialized device
- mostly hobby horse interest.

This leaves us with only a few devices left to model:

Optical devices

Lasers, photodiodes, avalanche photodiodes.

Mixers (really a circuit problem)

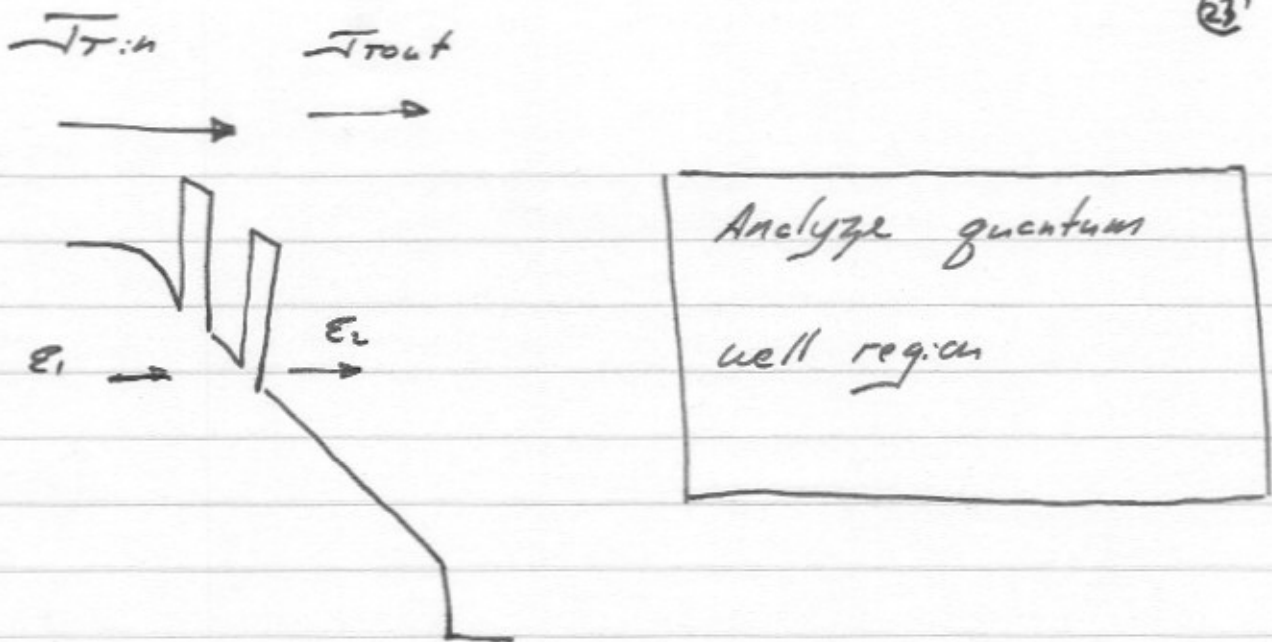
2-terminal negative-resistance devices.

IMPATTs, Gunn's, RTDS.

We will defer the optical devices until we consider optical communication, & the mixers until we consider heterodyne receivers. 2-terminal negative-resistance devices are generally hard to model for noise,

As the processes involve either avalanche multiplication or intervalley scattering with negative differential mobility. We will have to analyze noise in the avalanche photodiode - but later...

Will now analyze RTD (resonant-tunnel-diode) noise for no reason other than it being of hobby interest. Madhakar Reddy is developing very fast MOS that potentially have low noise.



Flux into well

$$J_{T:in} = G_1 E_1$$

stored charge in well

$$P_g \equiv \tau_g \cdot J_{T:out}$$

Flux out of well

$$\begin{aligned} J_{T:out} &= J_{T:in} + j\omega P_g \\ &= J_{T:in} + j\omega \tau_g J_{T:out} \end{aligned}$$

$$J_{T:out} = \frac{J_{T:in}}{1 + j\omega \tau_g} = \frac{G_1 E_1}{1 + j\omega \tau_g}$$

ii we will add noise to the model later by adding shot noise fluctuations to the tunnelling current into the well

iii This assumes that all tunnelling events into the well are statistically independent.

There will be a dc tunnelling current flux into the well, and through the device, of I_{dc} . From this, the ac small signal tunnelling current into the well will be:

$$I_{T:n} = G_1 E_1 + I_n$$

$$I_n = \text{noise current}; \frac{d}{df} \langle I_n I_n^* \rangle = 2q I_n$$

... we will add the noise current later

Note also

$$E_2 = E_1 + \frac{\rho_g}{\epsilon} = E_1 + \frac{\tau_g \cdot J_{\text{total}}}{\epsilon}$$

$$= E_1 + \frac{(\tau_g G_1 / \epsilon) E_1}{1 + j\omega \tau_g}$$

$$= \frac{E_1 + j\omega \tau_g E_1 + (\tau_g G_1 / \epsilon) E_1}{1 + j\omega \tau_g}$$

$$E_2 = E_1 (1 + \tau_g \cdot G_1 / \epsilon) \frac{1 + j\omega \tau_g / (1 + \tau_g G_1 / \epsilon)}{1 + j\omega \tau_g}$$

This is real and significant in high-performance

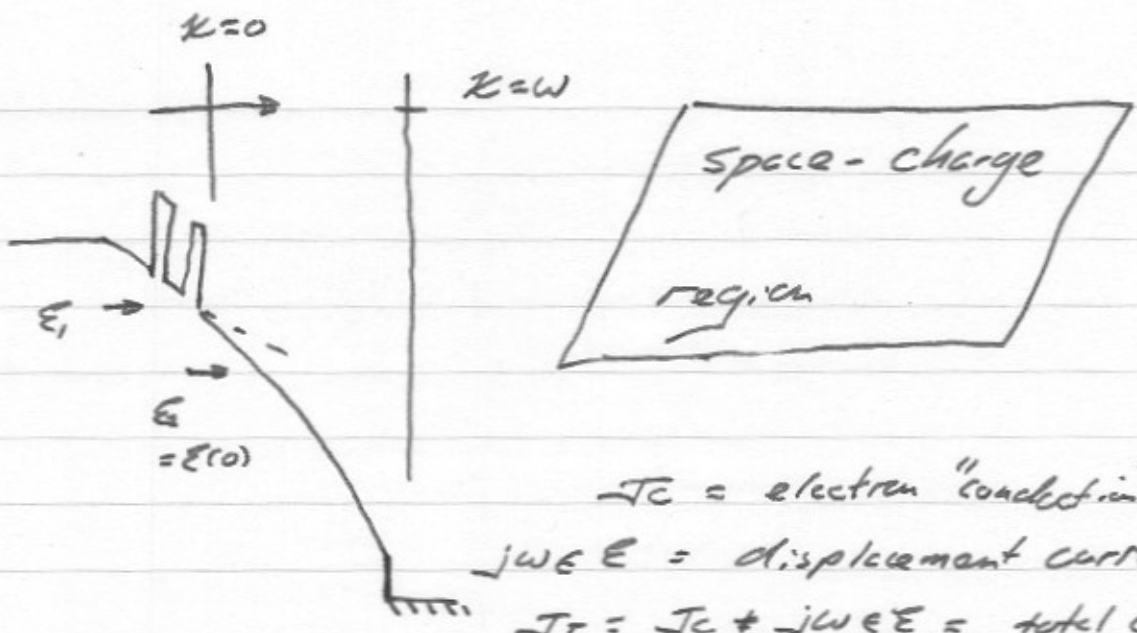
RTD's. $G_1 = \frac{\partial J_g}{\partial E_g}$, and hence $\frac{1}{\epsilon} G_1 = \frac{1}{\epsilon} \frac{\partial J_g}{\partial E}$

is akin to an "RC" time constant of the quantum well,

while τ_g is its charge storage time. In RTD's with

thin barriers, $\tau_g G_1 / \epsilon \sim 0(1)$. The term

$(1 + \tau_g G_1 / \epsilon)$ represents screening of the AC field by the charge in the well.



at $\psi=0$

$$I_C = \frac{G_1 E_1}{1 + j\omega \tau_g}$$

$$= \frac{G_1 E_1}{1 + j\omega \tau_g} \cdot \frac{E_2}{E_1} \cdot \frac{(1 + j\omega \tau_g)}{(1 + \tau_g \cdot G_1/E) (1 + j\omega \tau_g / (1 + \tau_g G_1/E))}$$

$$I_C = \left(\frac{G_1}{1 + \tau_g G_1/E} \right) \cdot \frac{E_2}{1 + j\omega \tau_g / (1 + \tau_g G_1/E)}$$

$$I_C = G_{ex} \cdot \frac{E_2}{1 + j\omega \tau_e}$$

where $G_{ex} = \frac{G_1}{1 + \tau_g G_1/E}$, $\tau_e = \frac{\tau_g}{1 + \tau_g G_1/E}$

Total Current

$$\bar{I}_T = \bar{I}_C(x) + j\omega E \epsilon(x) = \text{constant}$$

$$\bar{I}_T(0) = \left[\left(\frac{G_0 x}{1 + j\omega \tau_0} \right) + j\omega E \right] \epsilon(0)$$

with position!

\uparrow
= E_z

AC small signal quantities throughout...

Charge moves through drift region at velocity v_s .

$$\rightarrow \bar{I}_C(x) = \bar{I}_C(0) e^{-j\omega x / v_s}$$

Voltage drop across device

$$V(\omega) = \int_0^w \bar{E}(x) dx$$

~~$E(x) = \dots$~~

but $\frac{dE}{dx} = \frac{I_C}{\epsilon v_s}$

So:

$$E(x) = E(0) + \int_0^x \frac{J_c(x)}{\epsilon v_s} dx$$

$$= E(0) + \frac{1}{\epsilon v_{set}} \int_0^x J_c(0) e^{-j\omega x / v_{set}} dx$$

$$\approx E(0) + \frac{1}{\epsilon v_{set}} \int_0^x J_c(0) [1 - j\omega x / v_{set}] dx$$

$$= E_0 + \frac{J_c(0)}{\epsilon v_{set}} \left[x - \frac{j\omega x^2}{2v_s} \right]$$

Again, total volt. $V(\omega) = \int_0^W E(x) dx$

$$V(\omega) = E_2 \cdot W + \frac{J_c(0)}{\epsilon v_{set}} \left[\frac{W^2}{2} - \frac{j\omega W^3}{6v_{set}} \right]$$

$$= E_2 \cdot W + \frac{J_c(0)}{\epsilon} \frac{W^2}{2v_{set}} \left[1 - \frac{j\omega W}{3v_{set}} \right]$$

Side note:

We obtained $E(x)$ by integration. This was unnecessary. We could also have used continuity of the total (conduction + displacement) current.

$$\vec{J}_T = \vec{J}_c(x) + j\omega\epsilon E(x); \quad \vec{J}_T(x_1) = \vec{J}_T(x_2) \\ \text{for all } x_1, x_2.$$

$$\text{So: } J_c(0) + j\omega\epsilon E(0) - J_c(x) = j\omega\epsilon E(x)$$

$$J_c(0) \left[1 - e^{-j\omega x/v} \right] + j\omega\epsilon E(0) = j\omega\epsilon E(x)$$

$$E(x) = \frac{J_c(0)}{j\omega\epsilon} \left[1 - e^{-j\omega x/v} \right] + E(0)$$

$$\approx \frac{J_c(0)}{j\omega\epsilon} \left[\frac{j\omega x}{v} - \frac{(j\omega)^2 x^2}{2v^2} \right] + E(0)$$

$$= \frac{J_c(0)}{\epsilon v} \left[x - \frac{j\omega x^2}{2v} \right] + E(0)$$

which is the same answer...

$$v(\omega) = \epsilon_2 \cdot W + \frac{I_c(0)}{\epsilon} \cdot \frac{W^2}{3v_{scf}} \cdot \frac{3}{2} \left[1 - \frac{j\omega W}{3v_{scf}} \right]$$

$$V(\omega) = \epsilon_2 \cdot W + \frac{I_c(0) \cdot (3/2) W \tau_\pi}{\epsilon} \left[1 - j\omega \tau_\pi \right]$$

$$\text{where } \tau_\pi = W/3v_{scf}$$

$$\text{but } I_c = G_{ex} \epsilon_2 / (1 + j\omega \tau_\pi)$$

S.S.:

$$V(\omega) = \epsilon_2 \cdot W + \frac{G_{ex} \cdot \epsilon_2}{1 + j\omega \tau_\pi} \cdot \frac{3}{2} \frac{\tau_\pi}{\epsilon} \cdot W \left[1 - j\omega \tau_\pi \right]$$

$$\frac{V}{\epsilon_2} = W \left[1 + \frac{3}{2} (G_{ex}/\epsilon) \tau_\pi \frac{1 - j\omega \tau_\pi}{1 + j\omega \tau_\pi} \right]$$

The device admittance is:

$$Y = \frac{I_T}{V} = \frac{E_2}{V} \cdot \frac{I_T}{E_2}$$

$$= \frac{G_{ex} / (1 + j\omega \tau_e) + j\omega E}{W}$$

$$\left[1 + \frac{G_{ex}}{E} \cdot \tau_r \cdot \frac{3}{2} \frac{1 - j\omega \tau_e}{1 + j\omega \tau_e} \right]$$

$$= \frac{(G_{ex}/W) / (1 + j\omega \tau_e) + j\omega E/W}{1 + \frac{G_{ex}}{E} \tau_r \cdot \frac{3}{2} \frac{1 - j\omega \tau_e}{1 + j\omega \tau_e}}$$

$$= \frac{j\omega E/W}{1 + \frac{G_{ex}}{E} \tau_r \cdot \frac{3}{2} \frac{1 - j\omega \tau_e}{1 + j\omega \tau_e}}$$

=

$$Y = \frac{(G_{ex}/\omega) + \frac{j\omega\epsilon}{\omega}(1 + j\omega\tau_e)}{1 + j\omega\tau_e + [3G_{ex}\tau_e/2\epsilon](1 - j\omega\tau_e)}$$

$$= \frac{N}{D}$$

$$\frac{(G_{ex}/\omega) + \frac{j\omega\epsilon}{\omega}(1 + j\omega\tau_e)}{1 + (3G_{ex}\tau_e/2\epsilon) + j\omega[\tau_e - \tau_e(3G_{ex}\tau_e/2\epsilon)]}$$

$$= \frac{(G_{ex}/\omega) + \frac{j\omega\epsilon}{\omega}(1 + j\omega\tau_e)}{1 + (3G_{ex}\tau_e/2\epsilon) + j\omega[\tau_e - \tau_e(3G_{ex}\tau_e/2\epsilon)]}$$

$$= \frac{(G_{ex}/\omega) + \frac{j\omega\epsilon}{\omega}(1 + j\omega\tau_e)}{1 + (3G_{ex}\tau_e/2\epsilon) + j\omega[\tau_e - \tau_e(3G_{ex}\tau_e/2\epsilon)]}$$

$$= \frac{(G_{ex}/\omega) + \frac{j\omega\epsilon}{\omega}(1 + j\omega\tau_e)}{1 + (3G_{ex}\tau_e/2\epsilon) + j\omega[\tau_e - \tau_e(3G_{ex}\tau_e/2\epsilon)]}$$

At this point I have to simplify

$$\text{use } (1 + j\omega\tau_1)(1 + j\omega\tau_2) \approx 1 + j\omega(\tau_1 + \tau_2)$$

$$\frac{(1 + j\omega\tau_1)}{1 + j\omega\tau_2} \approx 1 + j\omega(\tau_1 - \tau_2), \text{ etc.}$$

$$Y = \frac{G_{ex}}{\omega} \frac{1}{1 + 3G_{ex}\tau_c/2\epsilon} \frac{1}{1 + j\omega \left[\frac{\tau_c - \tau_c(3G_{ex}\tau_0/2\epsilon)}{1 + 3G_{ex}\tau_c/2\epsilon} \right]}$$

$$+ j\omega \frac{(\epsilon/\omega)}{1 + 3G_{ex}\tau_c/2\epsilon} \frac{1}{1 + j\omega \cdot \frac{3G_{ex}\tau_c/2\epsilon}{1 + 3G_{ex}\tau_c/2\epsilon} (-\tau_c + \tau_0)}$$

$$\left[\text{now write } \tau_{sc} = \tau_c \cdot 3/2 = \omega/20\text{scf} \right]$$

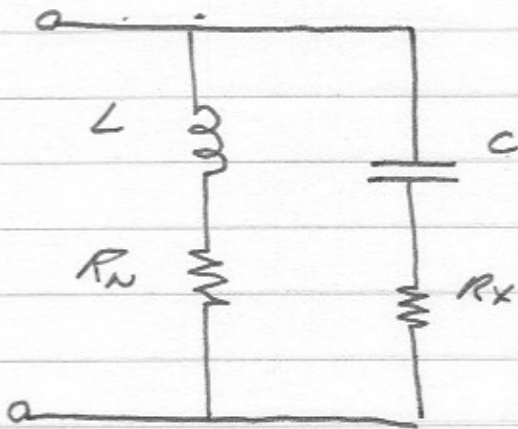
$$Y = \frac{G_{ex}}{\omega} \frac{1}{1 + G_{ex} T_{sc} / \epsilon} \frac{1}{1 + j\omega} \left[\frac{T_{\epsilon} - (2/3) T_{sc} (T_{sc} G_{ex} / \epsilon)}{1 + G_{ex} T_{sc} / \epsilon} \right]$$

$$+ j\omega \cdot \left(\frac{\epsilon}{\omega} \right) \frac{1}{1 + G_{ex} T_{sc} / \epsilon} \frac{1}{1 - j\omega (T_{\epsilon} + (2/3) T_{sc})} \left(G_{ex} T_{sc} / \epsilon \right)$$

where $G_{ex} = \frac{G_1}{1 + G_1 T_g / \epsilon}$ & $T_{\epsilon} = \frac{T_g}{1 + T_g G_1 / \epsilon}$

This allows us to construct an equivalent circuit model of the RTD...

note that G_{ex} & G_1 are negative in the negative differential resistance region.



$$R_w^{-1} = \frac{1}{w} \cdot \frac{G_{ex}}{1 + G_{ex} \tau_{sc} l \epsilon} \quad \text{where} \quad G_{ex} = \frac{G_1}{1 + G_1 \tau_g l \epsilon}$$

L = quantum well inductance

$$\therefore L/R_w = \frac{\tau_e - (2/3) \tau_{sc} (\tau_{sc} G_{ex} l \epsilon)}{1 + G_{ex} \tau_{sc} l \epsilon}$$

negative in NOR region

C = depl. layer capacitance

R_x probably negl. y. bh..

Noise:

$$\overline{I_{cin}} = G_1 E_1 + \delta I_c \quad \text{shot noise}$$

$$\overline{I_{cut}} = \frac{G_1 E_1 + \delta I_c}{1 + j\omega \tau_g}$$

$$E_2 = E_1 + (\tau_g / \epsilon) \overline{I_{cut}}$$

$$= E_1 + (\tau_g / \epsilon) \overline{I_c(0)}$$

In the s.c. region:

$$\frac{dE}{dx} = \frac{J_c}{\epsilon v_{sc}} \cdot e^{-j\omega x/v_{sc}}$$

so:

$$E(x) = E_2 + \frac{J_c(0)}{\epsilon v_{sc}} \left[x - j\omega x^2 / 2v_{sc} \right]$$

and

$$V = E_2 \cdot W + \frac{J_c(0)}{\epsilon} W \frac{W}{2v_{sc}} \left[1 - j\omega \frac{W}{3v_{sc}} \right]$$

$$V = E_2 \cdot W + \frac{J_c(0)}{\epsilon} W \tau_{sc} \left[1 - j\omega (2/3) \tau_{sc} \right]$$

to compute the short-circuit noise current we set $v=0$.

$$J_c(0) = \frac{-E_2 \cdot G}{\tau_{sc} (1 - j\omega (2/3) \tau_{sc})}$$

while $J_T = J_c(0) + j\omega \epsilon E_2$

$$- \underline{I_c}(0) \cdot \frac{\tau_{sc}}{\epsilon} \cdot (1 - j\omega(2/3)\tau_{sc}) = \underline{\epsilon}_2$$

So

$$- \underline{I_c}(0) \frac{\tau_{sc}}{\epsilon} \left[1 - j\omega(2/3)\tau_{sc} \right] = \underline{\epsilon}_1 + \frac{\tau_g}{\epsilon} \underline{I_c}(0)$$

hence

$$- \underline{\epsilon}_1 = \underline{I_c}(0) \left[\frac{\tau_{sc}}{\epsilon} (1 - j\omega \frac{2\tau_{sc}}{3}) + \frac{\tau_g}{\epsilon} \right]$$

but we also have:

$$\underline{I_c}(0) = \frac{G_1 \underline{\epsilon}_1}{1 + j\omega\tau_g} + \frac{\delta \underline{I_c}}{1 + j\omega\tau_g}$$

So

$$I_c(s) (1 + j\omega \tau_g) = G_1 E_1 + \delta I_t$$

So

$$G_1 E_1 = I_c(s) (1 + j\omega \tau_g) - \delta I_t$$

combine to get:

$$\delta I_t - I_c(s) (1 + j\omega \tau_g) = G_1 I_c(s)$$

$$\cdot \left[\frac{\tau_g}{\epsilon} + \frac{\tau_{sc}}{\epsilon} (1 - j\omega 2 \frac{\tau_{sc}}{3}) \right]$$

$$\left(\frac{-T_c(\omega)}{S-T_r} \right)^{-1} = 1 + j\omega T_g + \frac{G_1 T_g}{\epsilon} + \frac{G_1 T_{sc}}{\epsilon}$$

$$+ j\omega \left(\frac{2}{3} \right) \frac{G_1 T_{sc}}{\epsilon} \cdot T_{sc}$$

$$= 1 + (G_1 / \epsilon) (T_g + T_{sc})$$

$$+ j\omega T_g = j\omega \left(\frac{2 T_{sc}}{3} \right) \frac{G_1 T_{sc}}{\epsilon}$$

$$= \left[1 + (G_1 / \epsilon) (T_g + T_{sc}) \right]$$

$$\cdot \left[1 + j\omega \left(\frac{T_g + (2 T_{sc} / 3) G_1 T_{sc} / \epsilon}{1 + (G_1 / \epsilon) (T_g + T_{sc})} \right) \right]$$

Multiply by areas to get currents. Then
the short circuit noise current spectral density is:

$$\frac{d \langle I_n I_n^* \rangle}{df} = \frac{2q I_{dc}}{\left(1 + \frac{G_1}{\epsilon} (T_{sc} + T_g)\right)^2 \left(1 + \omega^2 \frac{T_g - (2T_{sc}/3) G_1 T_{sc}/\epsilon}{1 + (G_1/\epsilon)(T_g + T_{sc})}\right)^2}$$

note that in the negative differential
resistance region, G_1 is negative, and the
noise current is larger than shot noise.

... similar comments apply to $\frac{T_g - (2T_{sc}/3) G_1 T_{sc}/\epsilon}{1 + (G_1/\epsilon)(T_g + T_{sc})}$

I have now spent too much time on RTD
noise, and so will guess without proof
some relationships without carrying through the
math. Specifically, the time constant written
in terms of τ_e & τ_{sc} for the $y(\omega)$ characteristics
should be the same as the time constant
given in terms of τ_g & τ_{sc} for the noise
characteristics.

Specifically, if the well has a net conductance

$$G_1 = \frac{\partial I}{\partial E}$$

Then the RTD net conductance is

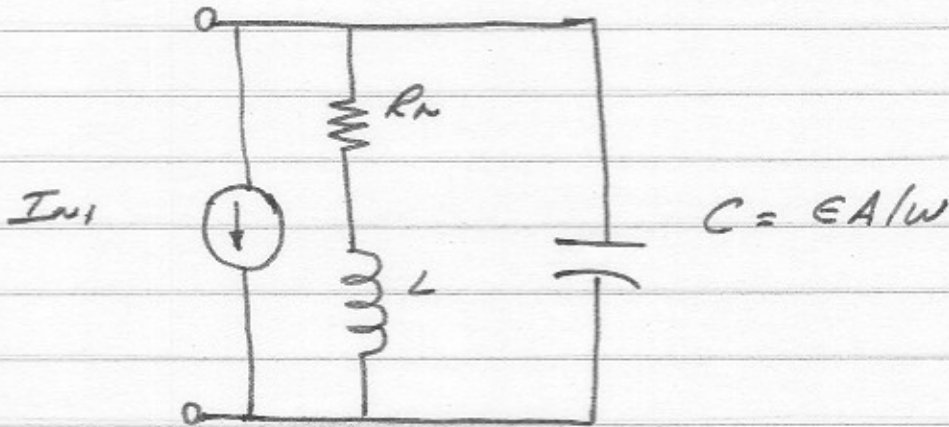
$$G_{net} = \frac{G_1}{1 + \frac{G_1}{E} (\tau_{sc} + \tau_g)} \cdot \frac{1}{W}$$

This is simply a voltage divider formula between the well conductance G_1 & the space-charge resistance,

$$\tau_{sc} \cdot W / E \text{ \& \ } \tau_g \cdot W / E.$$

Alternatively, compare the well conductance time constant $E / G_1 = \left[\frac{\partial I}{\partial E} \right]^{-1} E$ to the transit times...

RTD Model:



$$G_N = \frac{1}{R_H} = \frac{A}{w} \cdot \frac{\partial I}{\partial \epsilon} \Big|_{\text{well}} \cdot \frac{1}{1 + \frac{1}{\epsilon} \frac{\partial I}{\partial \epsilon} (\tau_{sc} + \tau_g)}$$

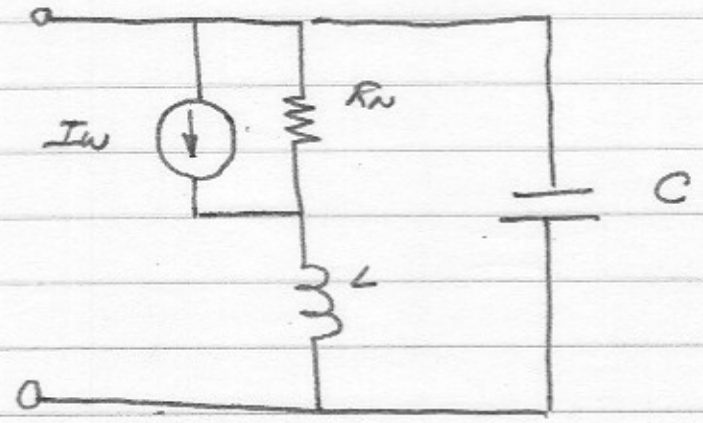
$$\frac{\partial I}{\partial \epsilon} = G_1$$

$$L = \tau / G = \frac{1}{G} \left[\frac{\tau_g - (2\tau_{sc}/3) G_1 \tau_{sc} / \epsilon}{1 + (G_1 / \epsilon) (\tau_{sc} + \tau_g)} \right]$$

spectral density of I_{ns} given above

both G_N & L are negative in the N. D. R. region.

We can model more cleanly like so:

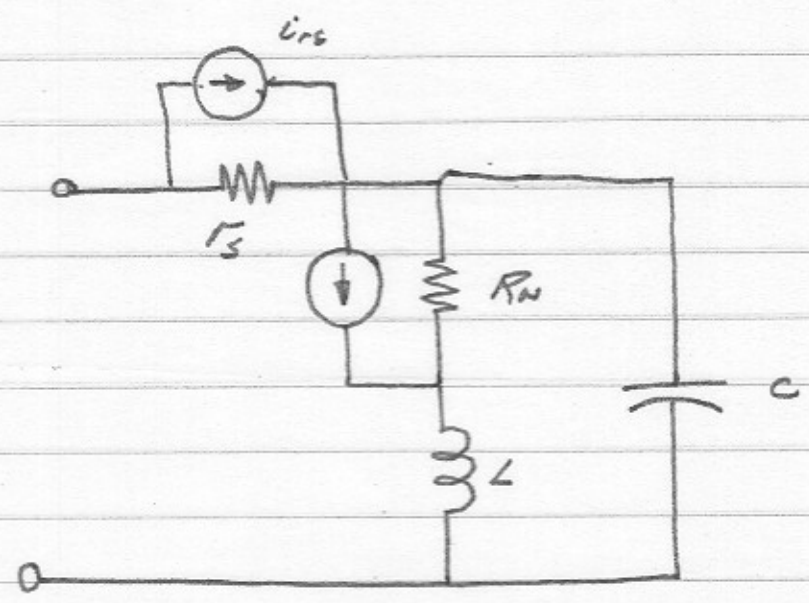


here, I_w has a spectral density:

$$\frac{2}{2f} \langle I_n I_n^* \rangle = 2g I_{dc} \left[\frac{1}{1 + \frac{G_1}{E} (\tau_{sc} + \tau_g)} \right]^2$$

$> 2g I_{dc}$ in the N.D.R. region.

We must also add a parasitic series resistance and its noise:



$$\frac{d \langle i_{rs} i_{rs}^* \rangle}{dt} = 4kT/r_s, \text{ other parameters given earlier.}$$

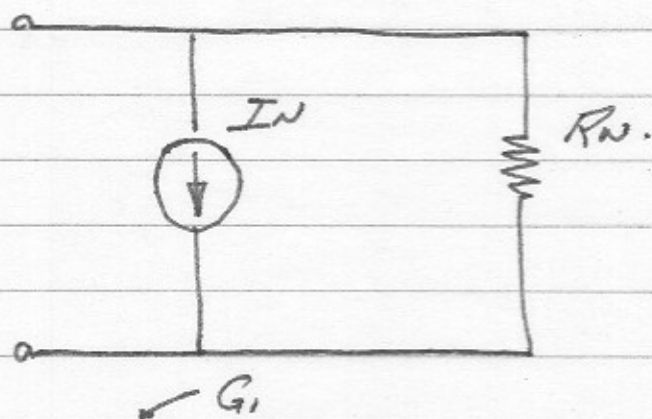
Note RTDs have a negative resistance cutoff frequency

f_{max} :

$$f_{max} \approx \begin{cases} \frac{1}{2\pi \sqrt{R_N R_S}} C & \text{if } \sqrt{R_N R_S} C \gg \tau_g + \tau_{sc} \\ \frac{1}{2\pi} \frac{1}{\sqrt{R_N C}} \frac{1}{\sqrt{R_S C}} \frac{1}{\sqrt{\tau_g}} & \text{if } \sqrt{R_N R_S} C \gg \tau_g + \tau_{sc} \text{ and } \tau_g \gg \tau_{sc}. \end{cases}$$

I will not attempt a frequency-dependent noise analysis here. Lets restrict ourselves to $f \ll f_{max}$ and $r_s \ll |R_n|$.

Then our device model is



$$(R_n)^{-1} = \frac{A}{W} \frac{\partial I}{\partial E_{well}} \frac{1}{1 + \frac{1}{E} \frac{\partial I}{\partial E_w} (T_{sc} + T_g)} = G_n$$

$$I_n: S_{I_n I_n}(f) = \frac{2q I_{dc}}{\left[1 + \frac{1}{E} \frac{\partial I}{\partial E_w} (T_{sc} + T_g) \right]^2}$$

We can write:

$$S_{I_N I_N}(f) = 4k T_{eq} |G_N|$$

From which:

$$T_{eq} = \frac{2q J_{dc} \cdot W}{\left[1 + \frac{1}{\epsilon} \frac{\partial J}{\partial \epsilon W} (\tau_{sc} + \tau_g) \right]} \frac{1}{\frac{\partial I}{\partial \epsilon}}$$

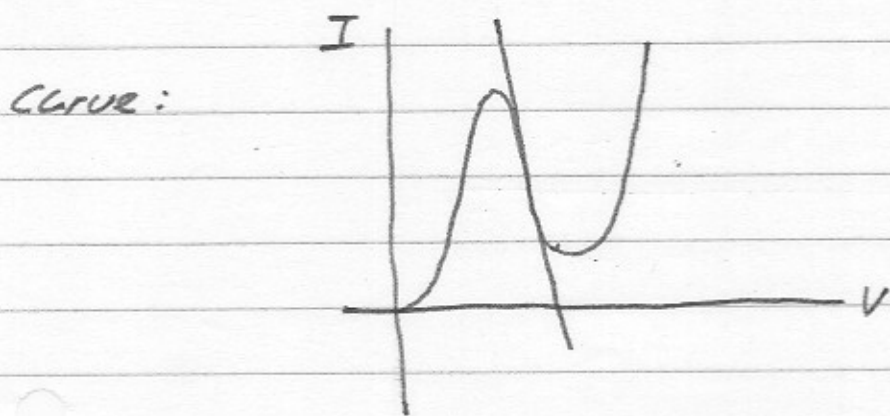
... we see immediately that the RTD space-charge layer thickness w should be as small as possible consistent with obtaining sufficient f_{max} .

This point is new. Esaki diodes have 10:1 to 100:1 thinner space-charge layers than do typical RTDs. RTDs are much noisier. Madhaker needs to clean up this analysis & publish it!

We note that the term

$$1 + \frac{1}{\epsilon} \frac{\partial I}{\partial E} / \frac{G_1}{W} (\tau_{sc} + \tau_g) \quad \text{is not observable.}$$

Since we can measure G_N from the DC I-V



... we want to write in terms of this parameter

$$G_N = \frac{G_1 / W}{1 + \frac{G_1}{\epsilon} (\tau_{sc} + \tau_g)}$$

From which:

$$G_1 = \frac{W G_N}{1 - \frac{W G_N}{C} (\tau_{sc} + \tau_g)}$$

note the difference in units:

$$G_N \triangleq \frac{\partial I}{\partial V} / \text{device} ; \quad G_1 \triangleq \frac{\partial I}{\partial \epsilon} / \text{well}$$

$$G_1 = \frac{W \cdot G_N}{1 - \frac{G_N}{C} (\tau_{sc} + \tau_g)}$$

here G_N is the externally
observable negative conductance &
 C the capacitance.

writing: $T_{rc} \triangleq \text{PRT RNC},$

negative in the N.D.R. region,

$$G_1 = \frac{W G_N}{1 - (\tau_{sc} + \tau_g) / T_{rc}}$$

$ G_1 $	}	$> G_N $	positive-resistance region
		$< G_N $	negative resistance region.

Now

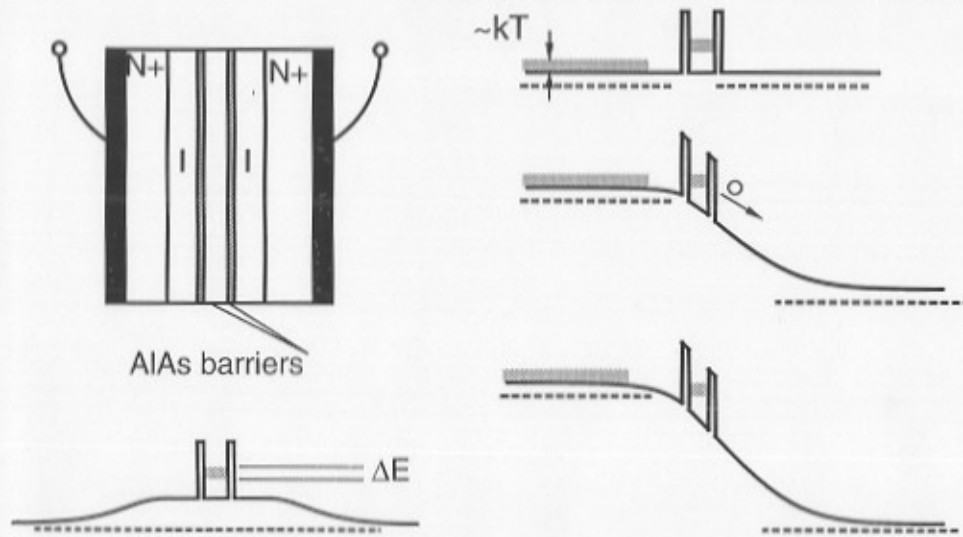
$$\left[\frac{1}{1 + \frac{G_1}{E} (\tau_{sc} + \tau_g)} \right] = 1 - (\tau_{sc} + \tau_g) / T_{rc}$$

and

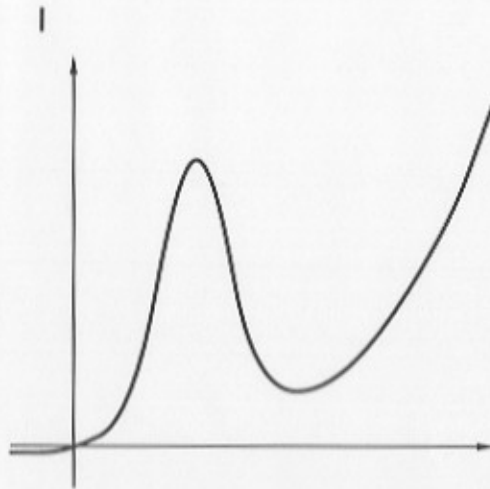
$$\frac{G_1}{1 + \frac{G_1}{E} (\tau_{sc} + \tau_g)} = W G_N !$$

Resonant Tunneling Diodes

(Sollner, MIT Lincoln Labs)
(Esaki)

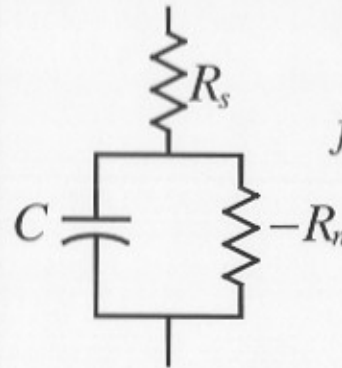
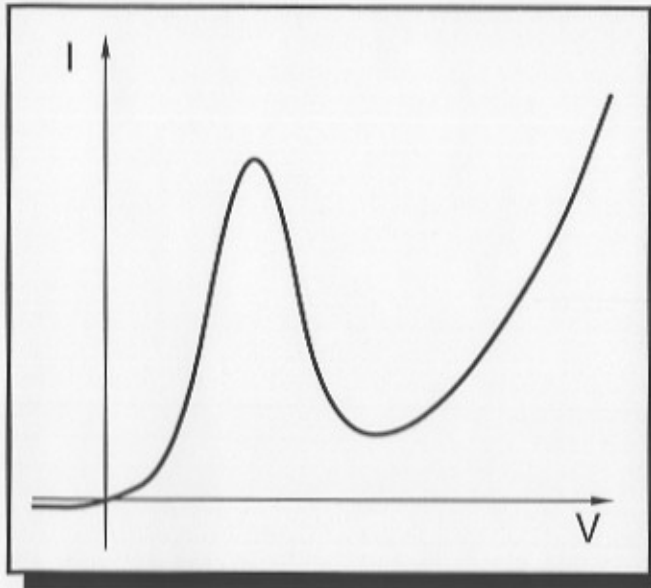
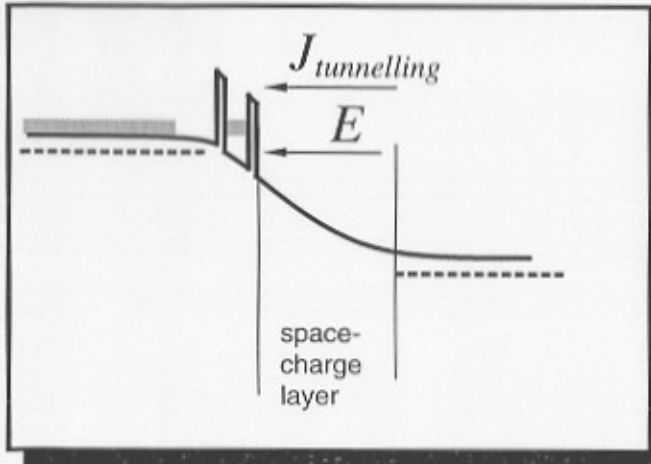


Strong current flow when electron reservoir and confined state are aligned in energy.



Narrower AlAs barriers decrease the electron trapping time, increasing ΔE , and increasing the peak current density. Increasing the emitter doping increases the electron supply and therefore also increases the current density.

Parameters Determining RTD f_{\max}



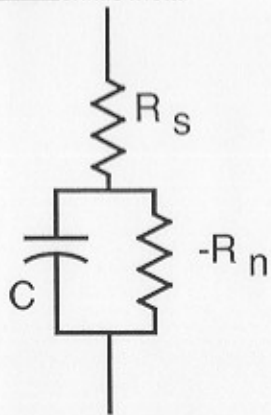
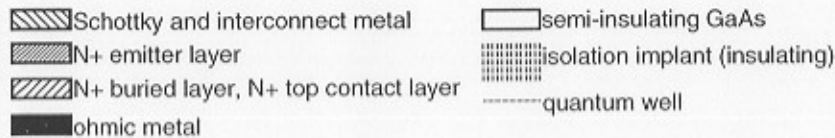
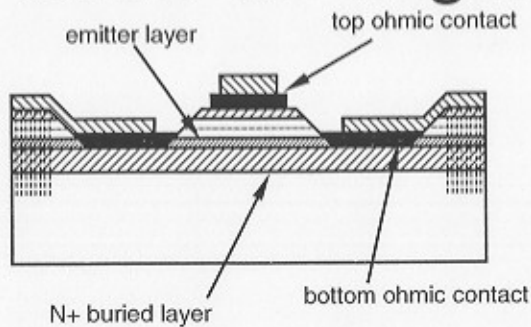
$$f_{\max} = \frac{1}{2\pi R_n C} \sqrt{\frac{R_n - R_s}{R_s}}$$

$$\frac{1}{R_n C} = \frac{1}{\epsilon} \frac{dJ}{dE} \quad \text{-Maximized by designing for high current density (reliability limits)}$$

- R_s then limits f_{\max} :

InAs/AISb RTD's have very low ($10^{-7} \Omega\text{-cm}^2$) contact resistance, & have attained $f_{\max}=1.3$ THz.

RTDs as negative-resistance oscillators



Coupled to the conjugate impedance $-jX(\omega)$, the RTD will oscillate if $R(\omega) < 0$

$$Z(\omega) = R_s + \frac{-R_n}{1 - j\omega R_n C}$$

$$= R(\omega) + jX(\omega)$$

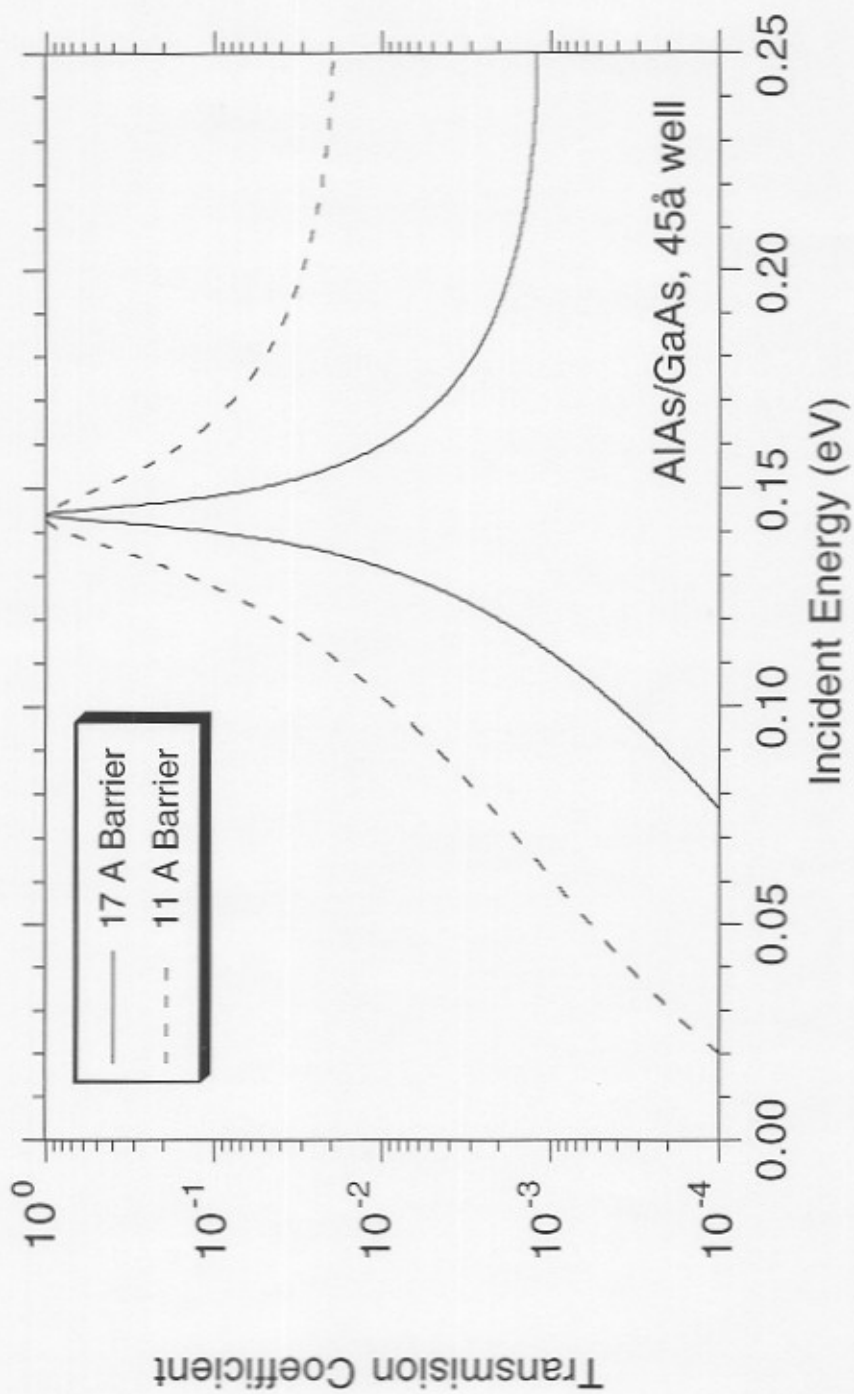
$$R(\omega) = R_s - \frac{R_n}{1 + \omega^2 R_n^2 C^2}$$

$$R(\omega_{\max}) = 0$$

$$\omega_{\max} = \frac{1}{R_n C} \sqrt{\frac{R_n - R_s}{R_s}}$$

RTD oscillators have been demonstrated to 712 GHz

Well Transmission Probability vs Energy



Quantum Well Inductance

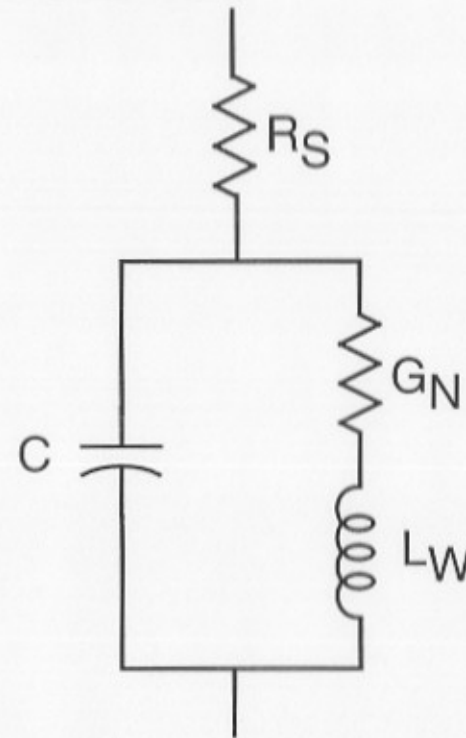
Transit time through well:

$$\tau = \frac{2\hbar}{\Delta E}$$

ΔE is the F.W.H.M. of the transmission probability

Equivalent inductance:

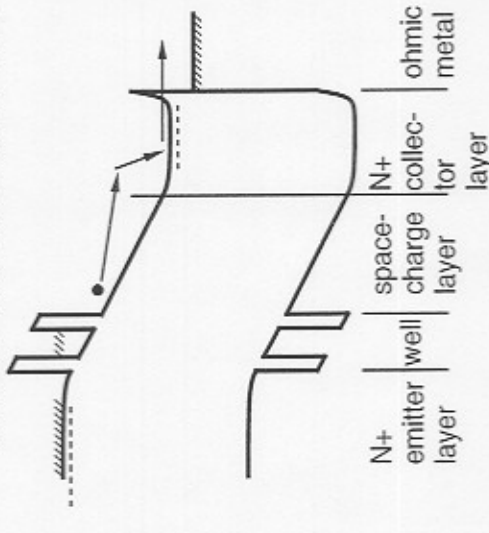
$$L_W = \frac{\tau}{G_N}$$



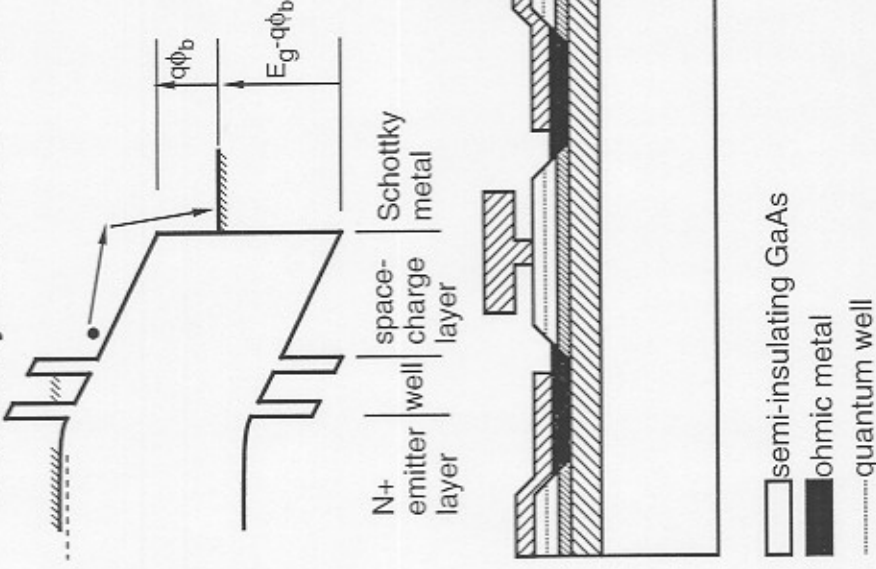
- In the negative resistance region G_N and L_W are negative.
- f_{\max} is decreased
- Space charge transit time $\tau_t = d_{\text{space-charge}} / v_{\text{electron}}$ also effects f_{\max} , but with $d_{\text{space-charge}} \cong 500 \text{ \AA}$, $v_{\text{electron}} \gg 10^7 \text{ cm/sec}$ and $\tau_t \approx 100 \text{ fs}$

Schottky-Collector Resonant Tunnel-Diodes

ohmic-contacted RTD



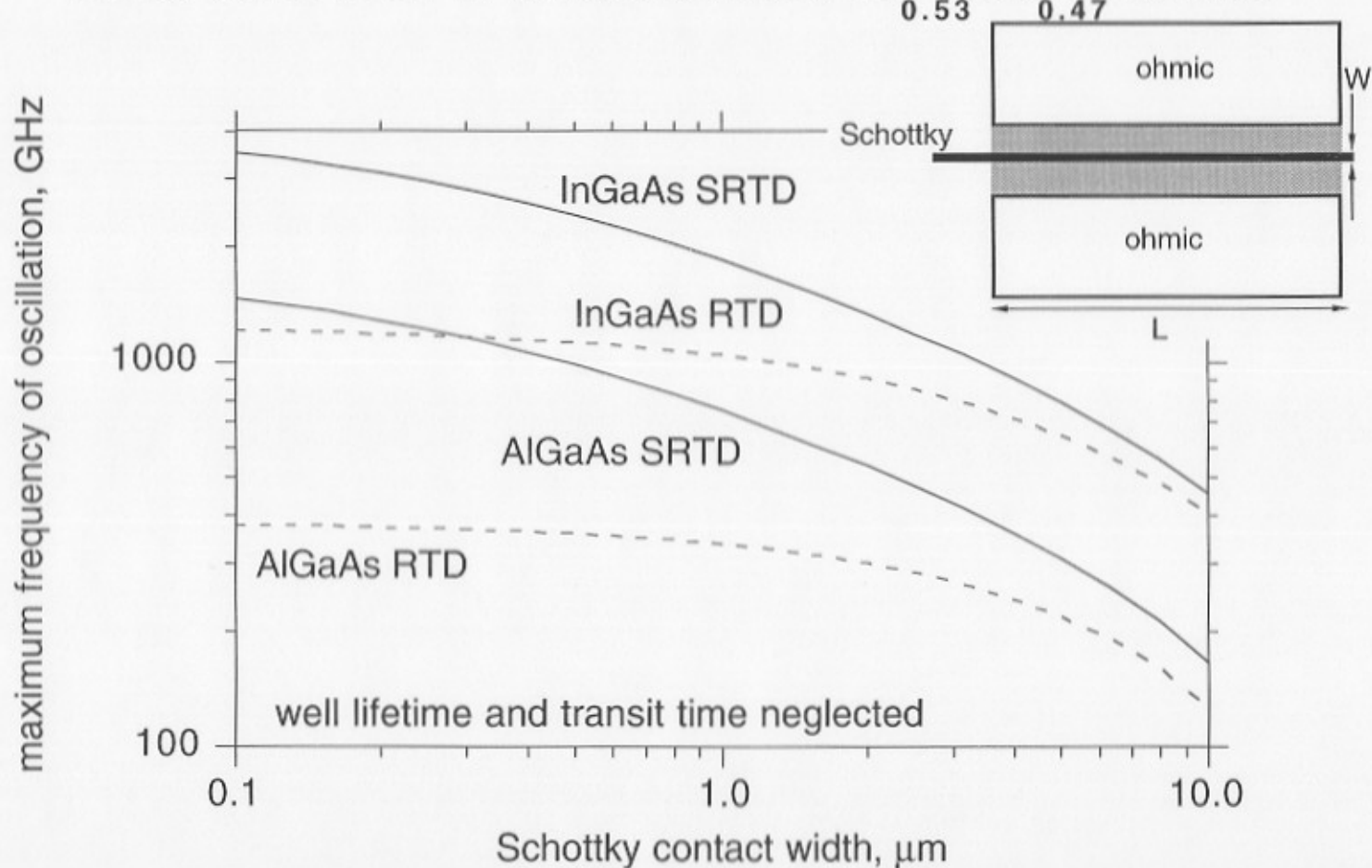
Schottky-collector RTD



Top ohmic contact is eliminated. Decreased R_s , increased f_{max}

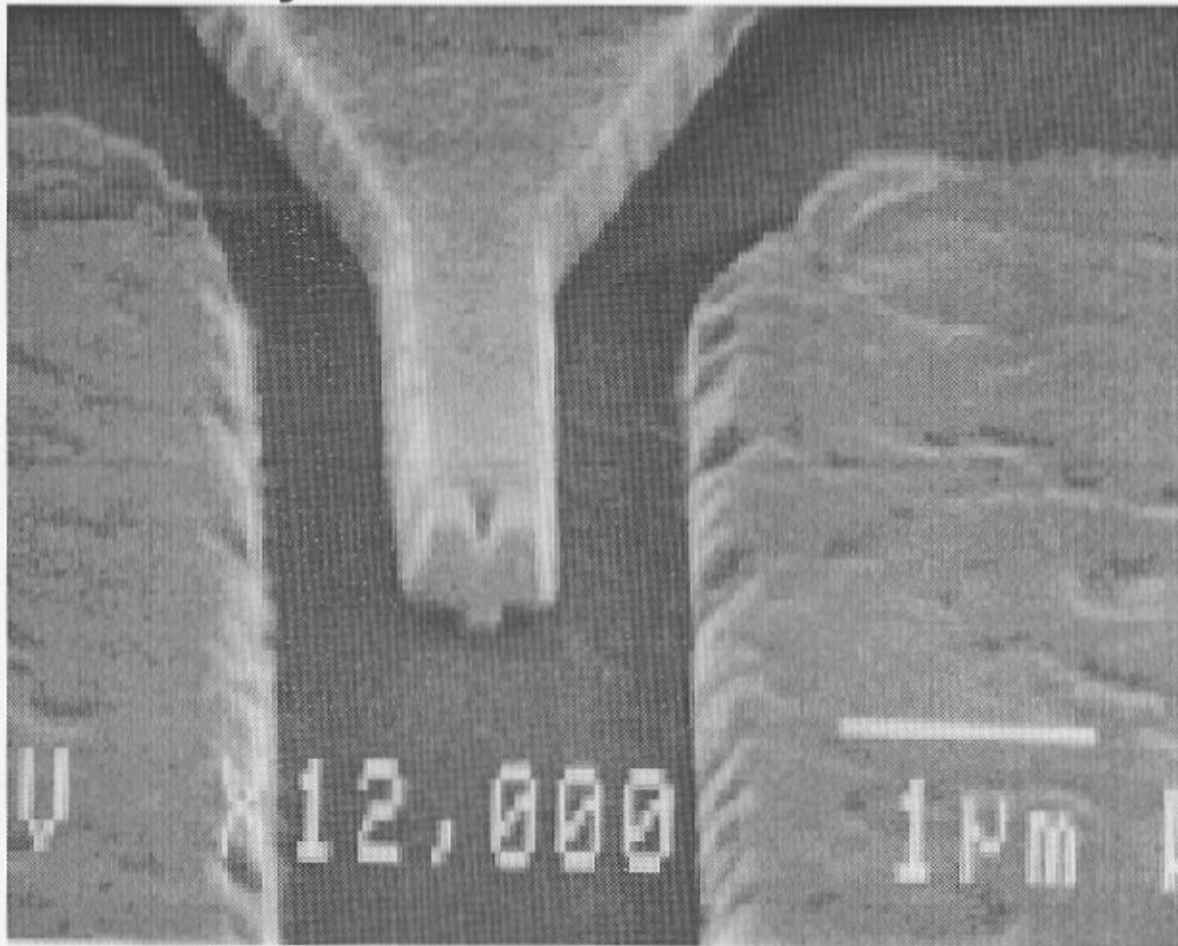
Bandwidth of Submicron Schottky-Collector RTDs

RTDs vs. SRTDs in AlAs/GaAs and In_{0.53}Ga_{0.47}As/AlAs



- Scaling to submicron dimensions increases RTD periphery/area ratio
- Periphery-dependent parasitic resistance terms driven towards zero (bottom ohmic contact resistance, buried N+ layer)

Schottky-Collector RTDs for 0.3-3 THz Oscillators



0.12- μm
AlAs/InGaAs/InP
device

- Schottky (metal) electron collector, 0.1 μm geometry.

- Greatly reduced series resistance, large increase in f_{max}

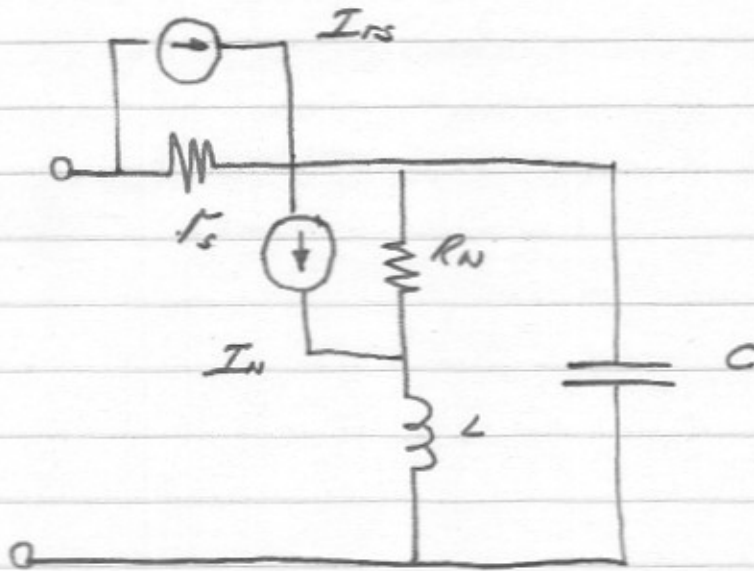
AlAs/GaAs SRTDs: 900 GHz f_{max} estimated

InGaAs/AlAs SRTDs: 2.2 THz f_{max} estimated

Application: 0.3-1.5 THz quasi-optical power oscillator arrays

So, in terms of externally observable parameters,

our RTD Model is:



L, C, r_s as before,

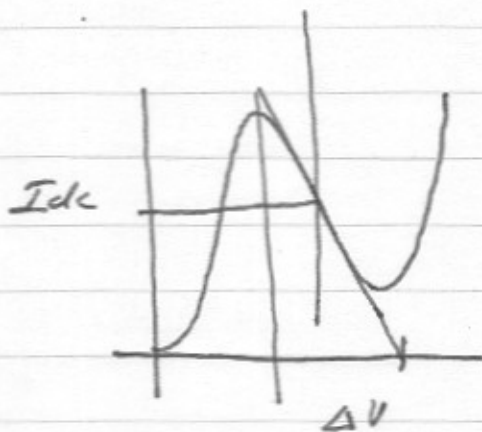
$$S_{I_{IN}I_{IN}}(f) = 2g I_{dc} \left[1 - (\tau_{sc} + \tau_g) / \tau_{rc} \right]^2$$

τ_{rc} negative in NDR!

so the "noise temperature" of R_N becomes:

$$T_{eq} = \frac{2q I_{dc}}{kT} R_N \left[1 - (\tau_{sc} + \tau_g) / \tau_{rc} \right]^2$$

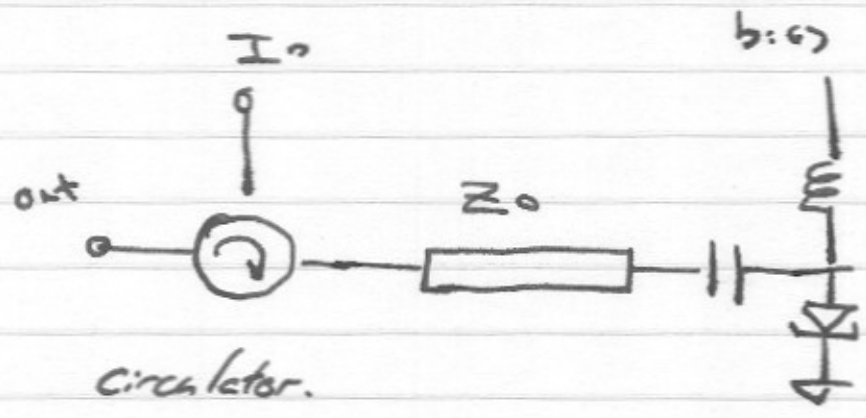
note that $R_N \propto W$!



low noise requires small W and C

high peak/valley current ratio...

Tunnel diodes are used as amplifiers thus:



Amplification is obtained from the

reflection gain, eg the reflection

magnitude is > 1 because the RTD

terminal impedance is negative.

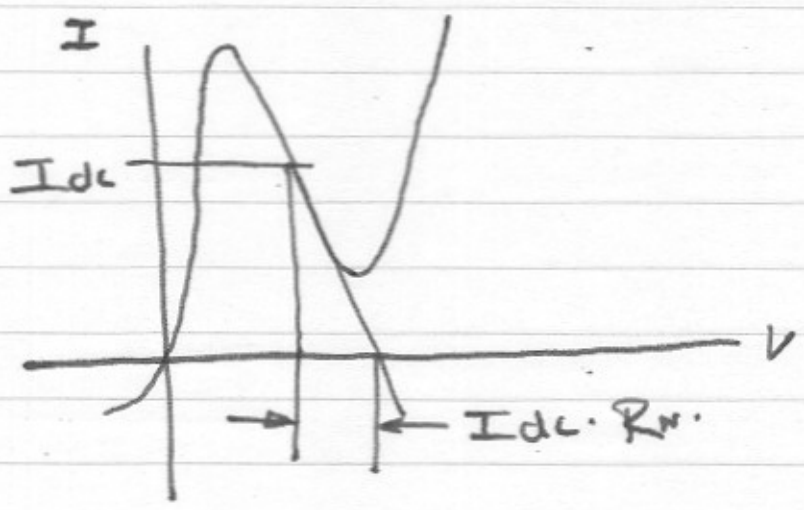
$$\Gamma = \frac{1 - G Z_0}{1 + G Z_0} ; |\Gamma| \rightarrow \infty \text{ as } G \rightarrow -1/Z_0$$

Noise Measure of the reflection amplifier

15

$$M = \frac{g I_{dc} R_n}{2kT_0} \left[1 - \frac{T_{sc} + T_g}{T_{rc}} \right]^2$$

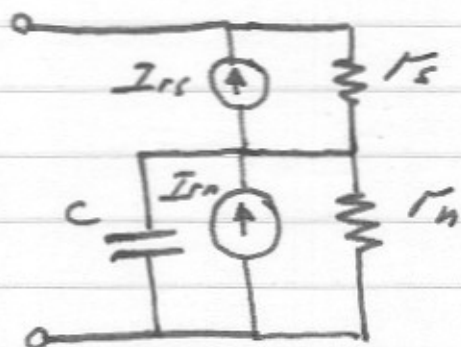
$$= \frac{I_{dc} R_n}{(2kT_0/g)} \left[1 - \frac{T_{sc} + T_g}{T_{rc}} \right]^2$$



Key parameters for low noise are high P_{out} and thin InAs space-charge region.

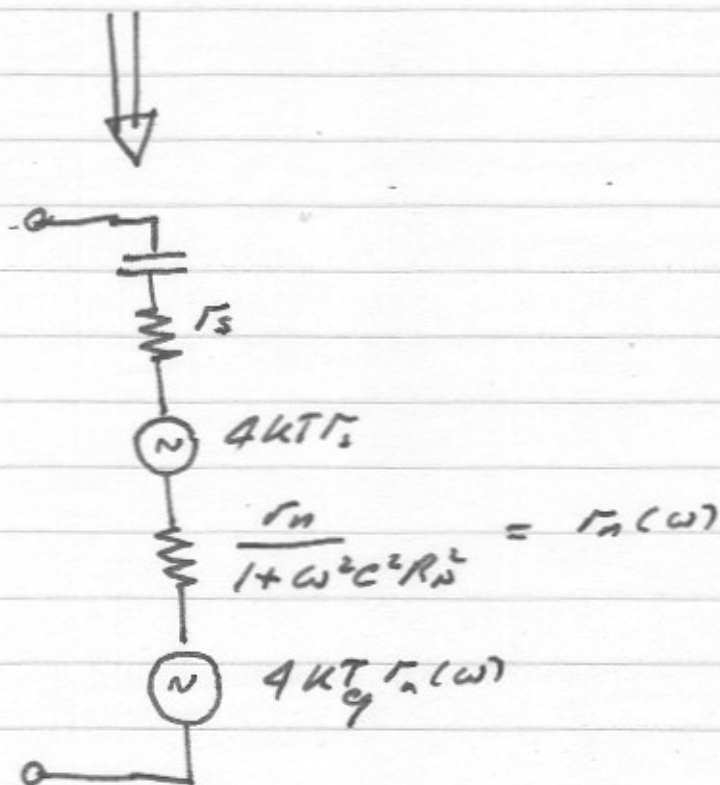
Let's find the frequency dependent noise

assuming $T_{gw} + T_{sc} \ll \sqrt{R_n R_s} C$.



$I_{rs}: 4kT/R_s$

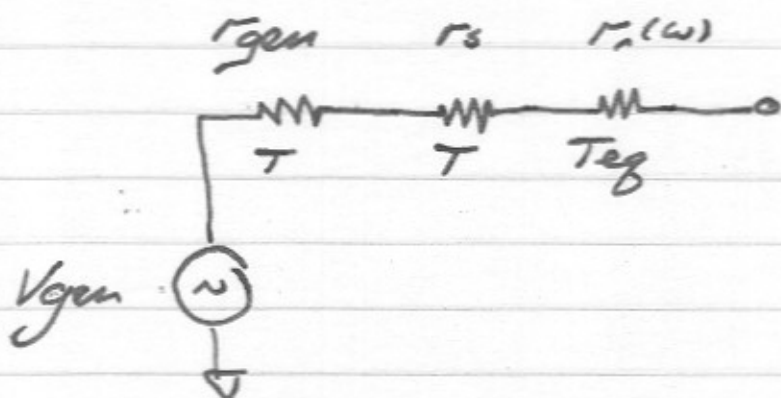
$I_{rn}: 4kT_{eq}/R_n$



$$M = \frac{|\Gamma_n(\omega) - \Gamma_s| + |\Gamma_n(\omega)| T_{eg} / T}{|\Gamma_n(\omega) - \Gamma_s|}$$

$$\approx \frac{|\Gamma_n| - \Gamma_s \omega^2 c^2 |\Gamma_n| + |\Gamma_n| T_{eg} / T}{|\Gamma_n| - \Gamma_s \omega^2 c^2 |\Gamma_n|}$$

Hook this in series with a (noisy) signal generator



$$F = \frac{4kT r_s + 4kT_{eg} r_n(\omega) + 4kT r_{gen}}{4kT r_s}$$

$$= 1 + \frac{r_{gen} + r_n(\omega) \cdot T_{eg}/T}{r_s}$$

gain is infinite for $r_{gen} + r_s + r_n(\omega) = 0$
 $r_{gen} = -r_n(\omega) - r_s$

$$F_{\infty} = 1 + \frac{|r_n(\omega) - r_s| + r_n(\omega) T_{eg}/T}{|r_n(\omega) - r_s|}$$