• Notes Set 13: Circuit noise analysis: overview and "benchmarks"

- Recap of notes set 6: 2-port noise models, etc.
- circuit noise analysis. Total input referred noise voltage, total input referred noise current.
- Short circuit input noise voltage. Open circuit input noise current
- Noise figure, minimum noise figure and optimum source impedance.
- signal-noise analysis by different communities: voltage method, current method, power method, temperature method, and equivalence of all of these.
- appropriateness of each method to various circuit/system problems

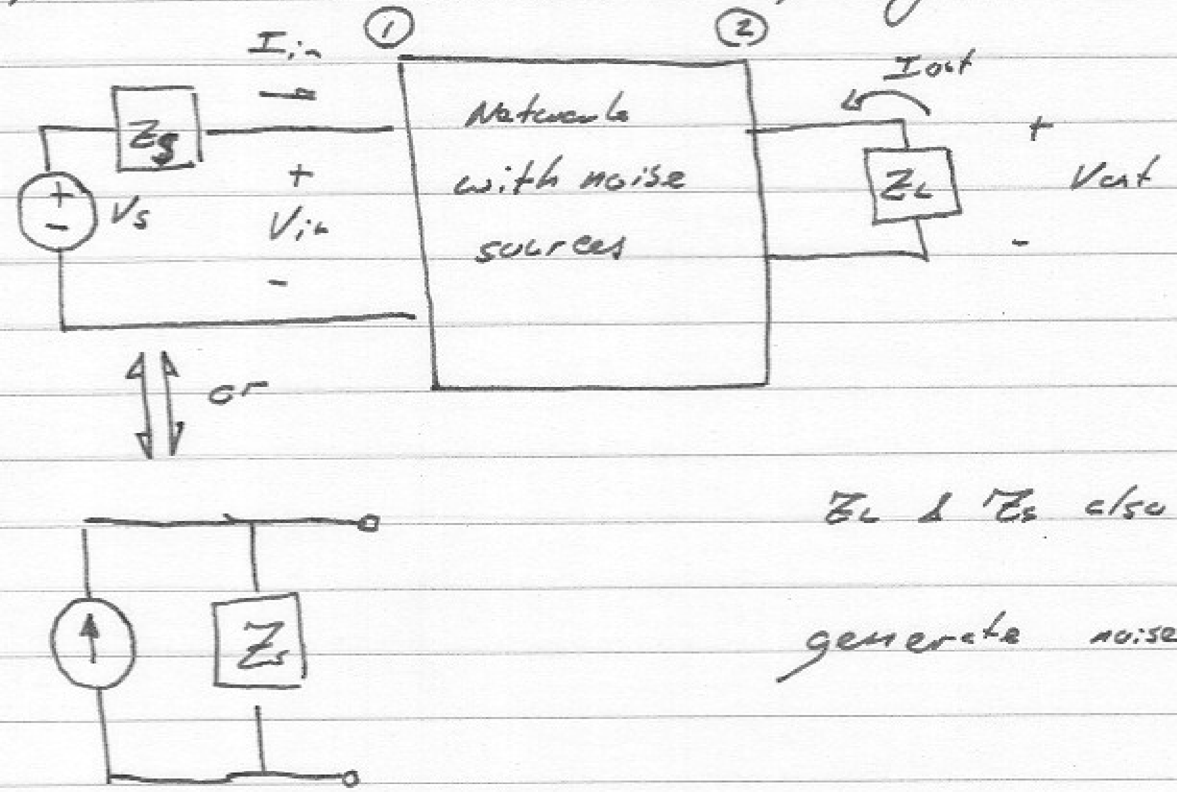
ECE ___ Notes set 13

Now we come to circuit noise analysis. Here the style of the material is quite different:

There are far fewer new ideas in this section. There is however a great deal to learn on how to work such problems effectively and efficiently. Basically, there are no new relationships to learn; we just need to know how to solve problems.

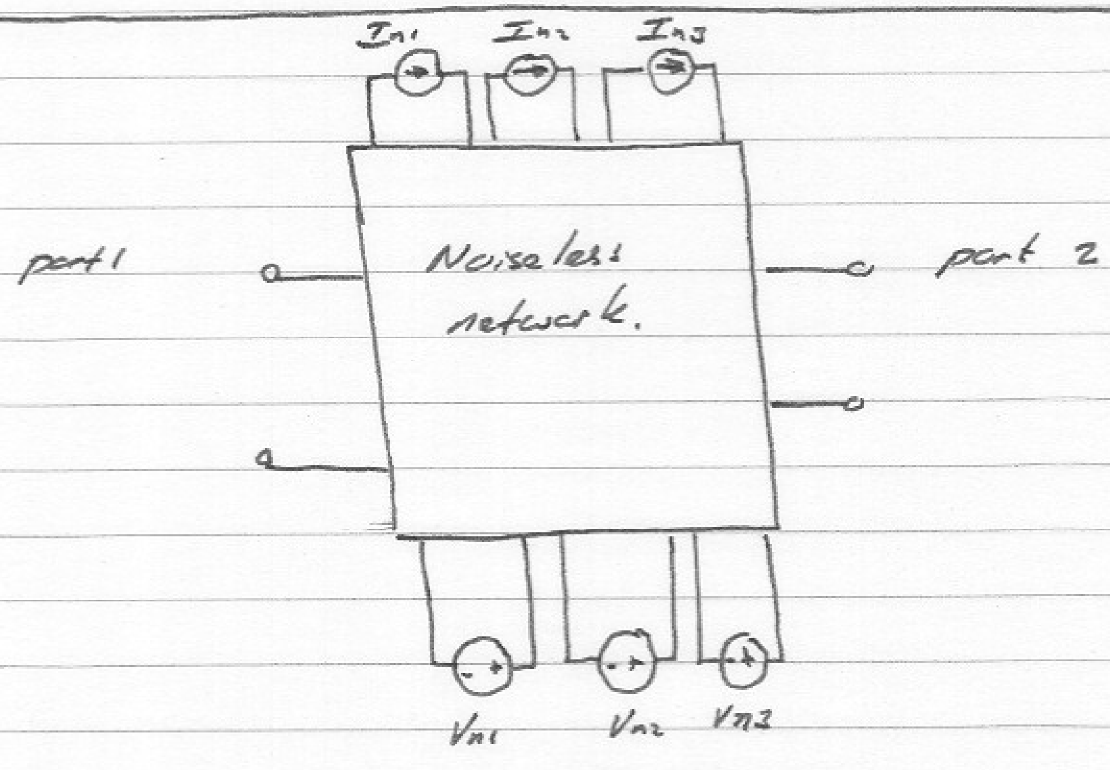
Now remember relationships given earlier:

(A)

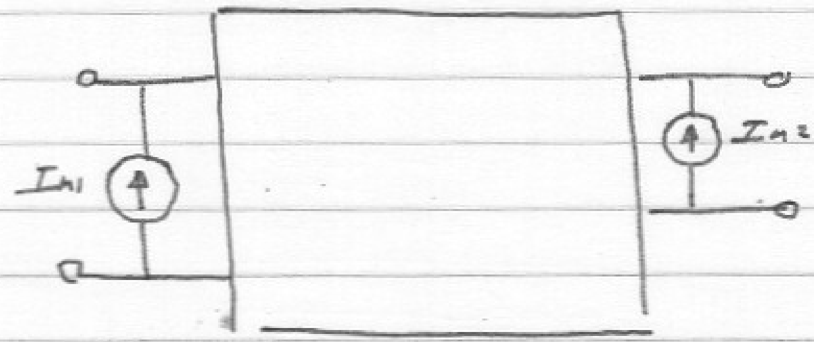


Z_L & Z_s also may generate noise.

(B)

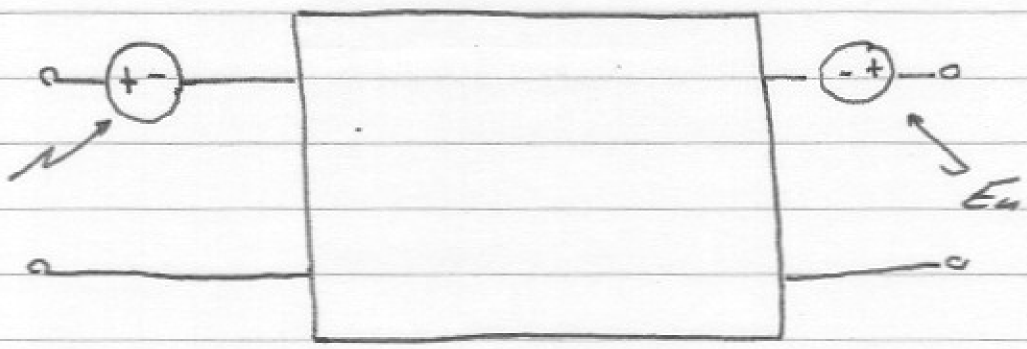


(4) In principle we then solve for the effect of the many noise generators to find



or

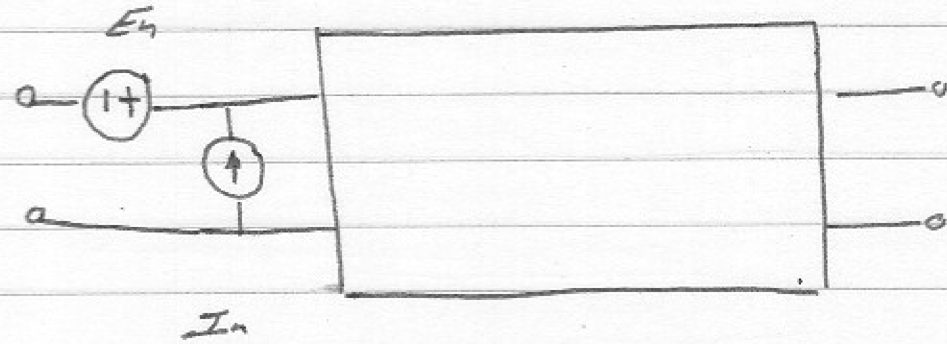
Note: These are measurable



where (E_{n1}, E_{n2}) / (I_{n1}, I_{n2}) are the open circuit/ short circuit noise voltages/currents at the 2 ports. They are correllated.

D one then transforms these to

the far more useful $E_n - I_n$ model



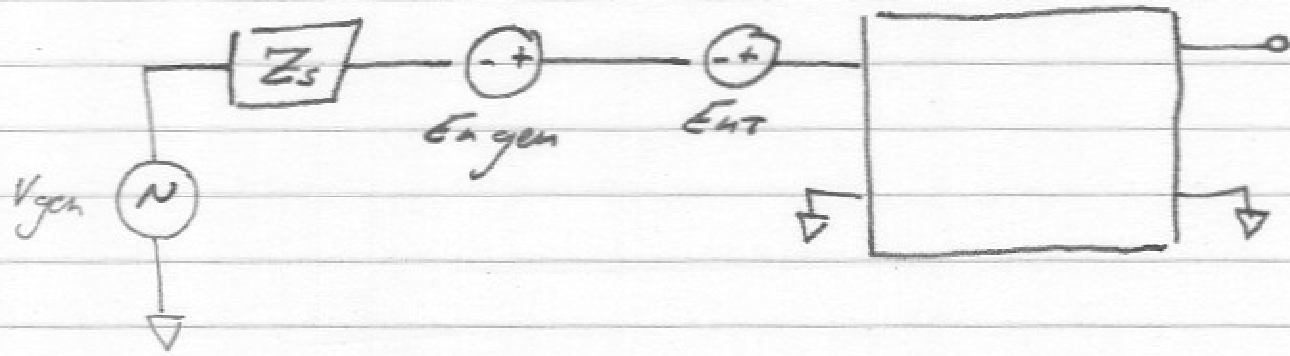
I_n and E_n are:

I_n : equivalent open-circuit input noise current

E_n: equivalent short-circuit input noise voltage.

I_n & E_n are generally correllated.

c) For a specific ~~Ngi~~ generator impedance:



$$\frac{d}{df}\langle E_{nT} E_{nT}^*\rangle = \frac{d}{df}\langle E_n E_n^*\rangle$$

$$+ \|Z_s\|^2 \cdot \frac{d}{df}\langle I_n I_n^*\rangle$$

$$+ 2\,\mathrm{Re}\left\{ Z_s^* \frac{d}{df}\langle E_n I_n^*\rangle \right\}$$

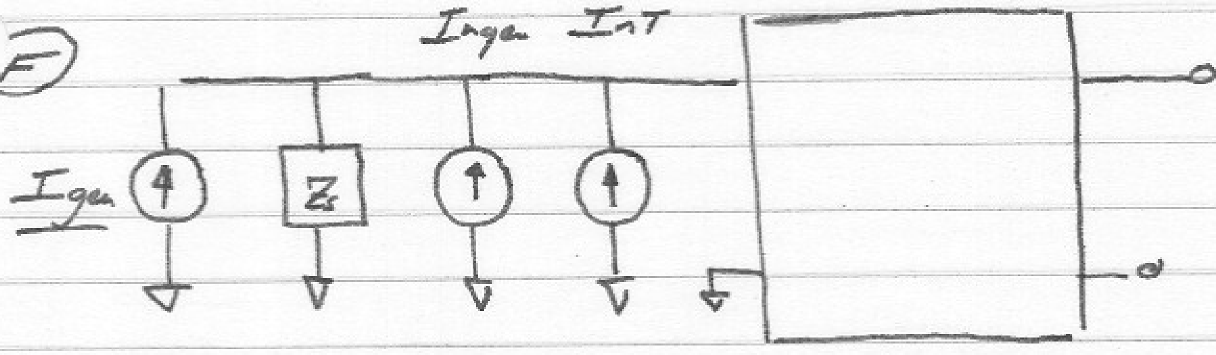
- derived from $E_{nT} = E_n + Z_s I_n$.

- $E_{n\,gen}$ is the generator noise;

$$\frac{d}{df}\langle E_{n\,gen} E_{n\,gen}^*\rangle \neq 4kT\,\mathrm{Re}\{Z_s\}$$!

<u>unless</u> the generator is a simple resistive network !!!

F



$$\frac{d}{df}\langle I_{nT} I_{nT}^*\rangle = \frac{d}{df}\langle I_n I_n^*\rangle + \left\|\frac{1}{Z_s}\right\|^2 \cdot \frac{d}{df}\langle E_n E_n^*\rangle + 2\,\mathrm{Re}\left[\frac{1}{Z_s^*}\,\frac{d}{df}\langle I_n E_n^*\rangle\right]$$

- derived from $I_{nT} = I_n + E_n / Z_s$

- Ingen is generator noise, again not necessarily with $P_{AV} = kT$.

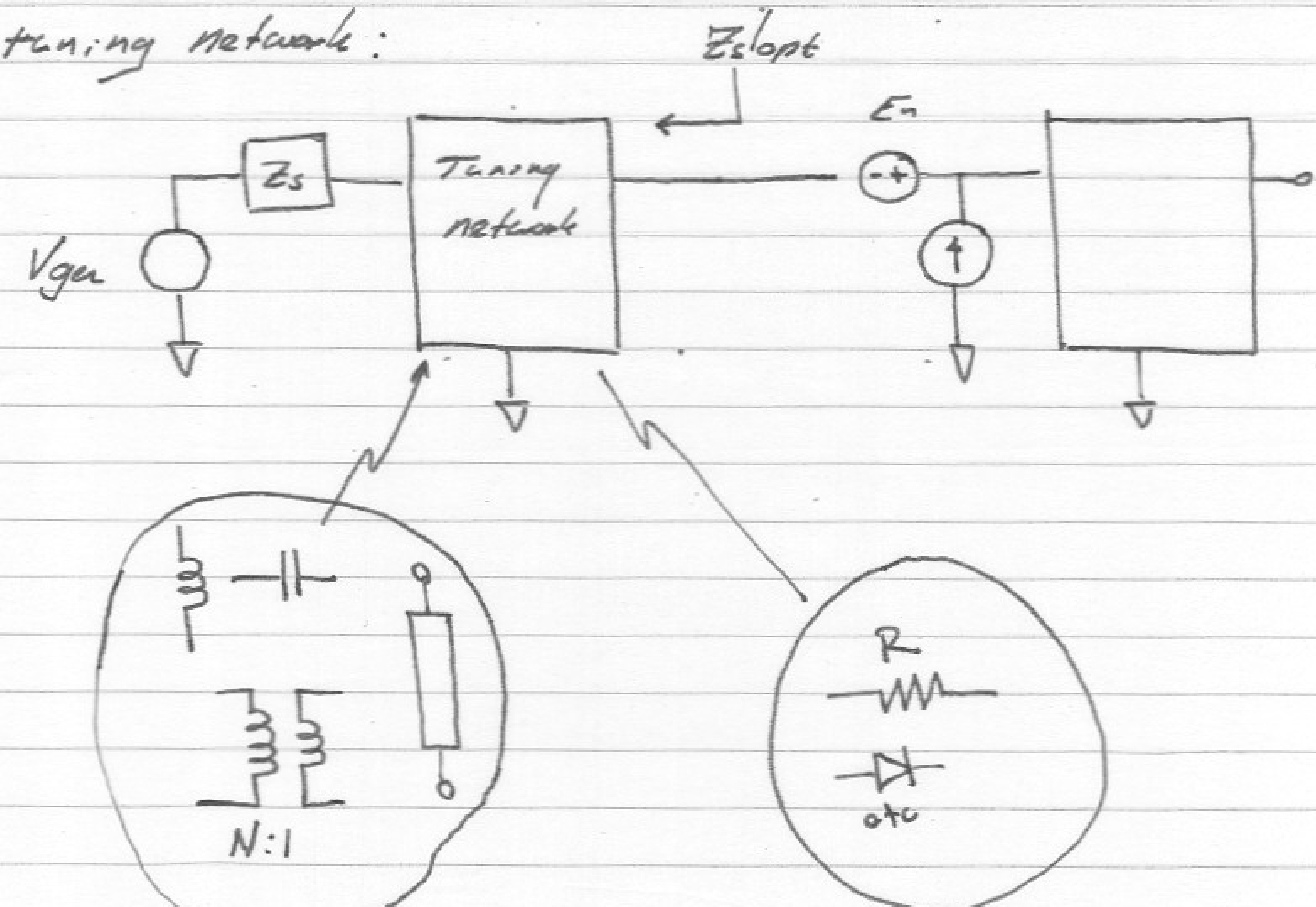
G From either E or F, if we want to calculate noise figures, we can

$$F = 1 + \frac{\frac{\partial}{\partial f}\langle V_n V_n^* \rangle + Z_g Z_g^* \frac{\partial}{\partial f}\langle I_n I_n^* \rangle + 2\,\mathrm{Re}\left[Z_g^* \frac{\partial}{\partial f}\langle E_n I_n^* \rangle\right]}{4kT\ \mathrm{Re}[Z_g]}$$

and noise temperature:

$$T_{eq} = \frac{\frac{\partial}{\partial f}\langle V_n V_n^* \rangle + Z_g Z_g^* \frac{\partial}{\partial f}\langle I_n I_n^* \rangle + 2\,\mathrm{Re}\left[Z_g^* \frac{\partial}{\partial f}\langle E_n I_n^* \rangle\right]}{4 \cdot k \cdot \mathrm{Re}[Z_g]}$$

(H) We were able to show that if we ~~match~~ tune the generator impedance using a Lossless (e.g. Noiseless) input tuning network:



ok

Not ok
- absorbs power
- generates noise.

...Then the Minimum Noise figure is:

$$F = 1 + \frac{1}{4kT}\left\{ 2\cdot\sqrt{\frac{\partial}{\partial f}\langle V_n V_n^*\rangle \cdot \frac{\partial}{\partial f}\langle I_n I_n^*\rangle - \left(Im\left(\frac{\partial}{\partial f}\langle V_n I_n^*\rangle\right)\right)^2} + 2\cdot Re\left[\frac{\partial}{\partial f}\langle V I^*\rangle\right]\right\}$$

where

$$Z_{opt} = R_{opt} + j\,X_{opt}$$

$$R_{opt}^2 = \frac{\frac{\partial}{\partial f}\langle V_n V_n^*\rangle}{\frac{\partial}{\partial f}\langle I_n I_n^*\rangle} - \left[\frac{Im\left(\frac{d}{df}\langle V_n I_n^*\rangle\right)}{\frac{\partial}{\partial f}\langle I_n I_n^*}\right]^2$$

and

$$X_{opt} = -\frac{Im\left[\frac{\partial}{\partial f}\langle V_n I_n^*\rangle\right]}{\frac{\partial}{\partial f}\langle I_n I_n^*\rangle}$$

Pause, and make 2 observations:

1) Whether we choose to work with $\langle E_n, I_n \rangle$, E_{nT}, I_{nT}, or F or T_{eq} is very much a matter of choice, based upon the relative convenience of each tool & upon the traditions of the field involved.

Ultimately our goal is to find a $\boxed{\text{Signal to Noise Ratio}}$ and we can work towards this using E_{nT}, I_{nT}, F, T_{eq}, etc at our choice.

2) I have presented the development of ...

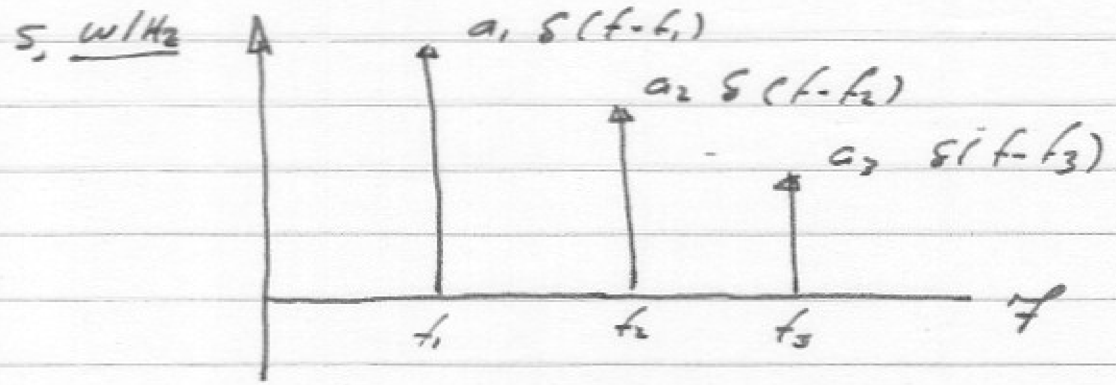
$$\left.\begin{array}{l}(E_{n1}, E_{n2}) \rightarrow (E_n, I_n) \rightarrow \left\{\begin{array}{l} E_{nT} \\ I_{nT} \\ F \rightarrow F_{min} \\ T_{eq} \rightarrow T_{eq\,min} \end{array}\right.\end{array}\right\} \rightarrow \frac{S}{N}$$

... as a sequence of steps. This is how a <u>computer</u> would do it. If you are writing <u>computer</u> <u>code</u>, this is how you should do it, too.

If you are doing <u>hand</u> <u>analysis</u>, you can usually work in <u>one step</u> to the particular answer you require.

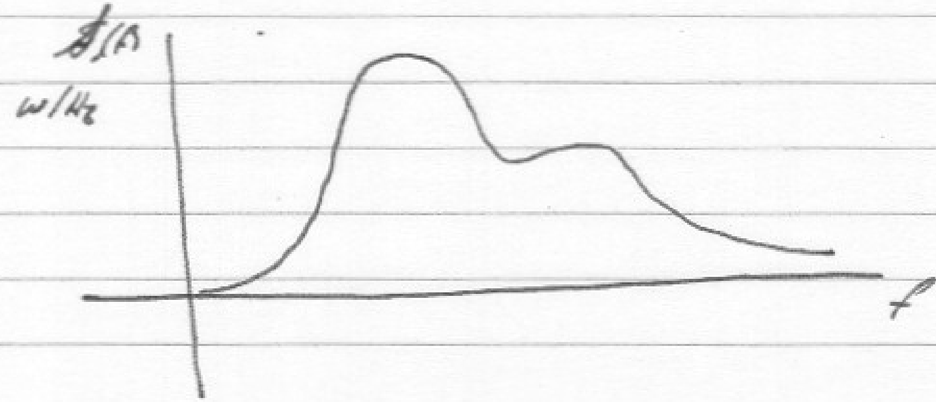
As I just mentioned, ultimately we are interested in a signal / noise ratio

Signals may be discrete (deterministic), with an impulsive power spectrum



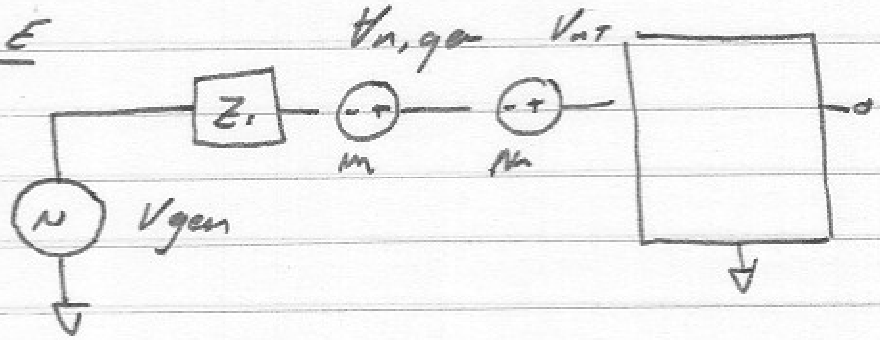
so each line has units, not of watts / Hz, but of watts.

often we are dealing with signals carrying information or modulation, which are therefore themselves random processes, and the signal has a power spectrum in W/Hz

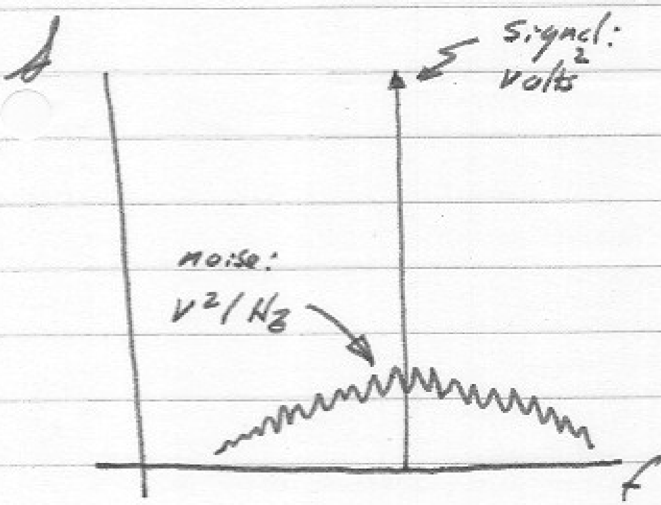


Total voltage method

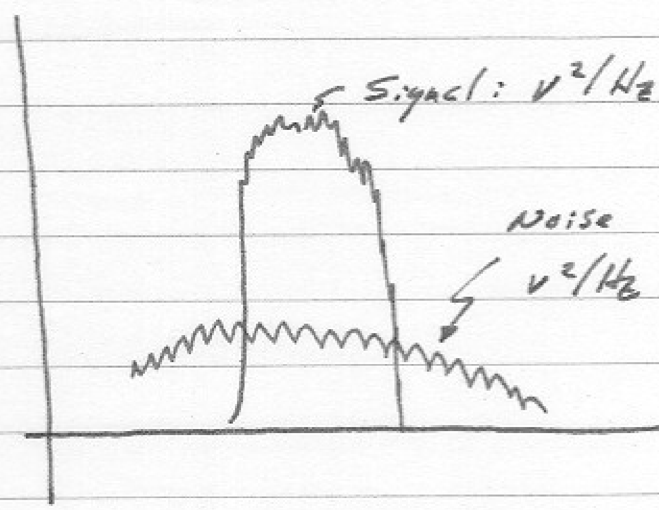
Case E



$$\frac{S}{N} = \frac{\frac{\partial}{\partial f}\langle V_{gen} V_{gen}^{*}\rangle}{\frac{\partial}{\partial f}\langle V_{ngen} V_{ngen}^{*}\rangle + \frac{\partial}{\partial f}\langle V_{nT} V_{nT}^{*}\rangle}$$



Discrete Signal



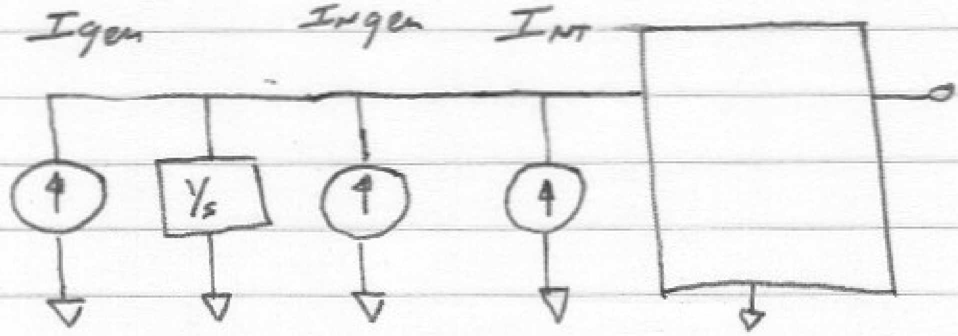
Random Signal.

S/N: dB (1Hz)

S/N dB.

Total Current Method

Case F

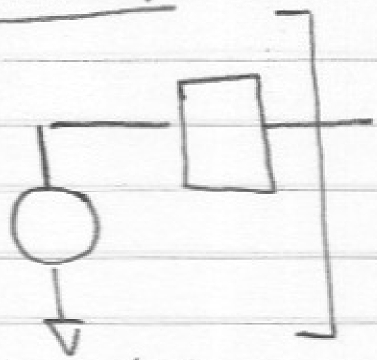


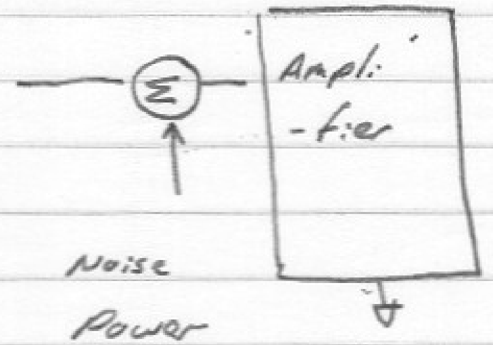
$$\frac{S}{N} = \frac{\frac{2}{\Delta f}\langle I_{gen} I_{gen}^{*}\rangle}{\frac{2}{\Delta f}\langle I_{NT} I_{NT}^{*}\rangle + \frac{2}{\Delta f}\langle I_{ngen} I_{ngen}^{*}\rangle}$$

(same pictures)

~~To~~tal Power Method

Case G





Noise
Power
adds power available

generator:

$\frac{\partial}{\partial f}\langle P_{av},\ \text{signal}\rangle$

$\frac{\partial}{\partial f}\langle P_{av\ \text{noise gen}}\rangle$

$kT\,(F-1)$

$$\frac{S}{N} = \frac{\frac{\partial}{\partial f}\langle P_{av}\ \text{signal}\rangle}{\frac{\partial}{\partial f}\langle P_{av\ \text{noise gen}}\rangle + kT\,(F-1)}$$

(same pictures)

Temperature Method

$\boxed{\text{Case } G(b)}$

$$\frac{S}{N} = \frac{\frac{\partial}{\partial f}\langle P_{av,\,signal}\rangle}{K T_{\substack{generator\\noise}} + K T_{eq,\,amplifier}}$$

... or ...

$$\frac{S}{N} = \frac{\overbrace{\frac{1}{K}\frac{\partial}{\partial f}\langle P_{av,\,signal}\rangle}^{\text{"signal temperature"}}}{T_{\substack{generator\\noise}} + T_{eq,\,amplifier}}$$

Simply: There are many roads to Rome.

We can work with noise voltages, currents, powers, or equivalent temperatures, but our goal is finally to <u>find a Signal / Noise ratio</u>.

Preferred method a matter of <u>convenience</u> & <u>tradition</u>

The traditions are as follows:

Radio receivers / Radar / etc, primarily terrestrial

Impedance matching easy. Powers ∴ relevant quantities. Antenna temperatures near 300°K.

→ Power Method / Noise figure.

Fiber optic receivers – Electronic kind

Here very broadband, so matching impossible over bandwidth.

Available ~~Noise~~ signal power, etc, ∴ not very useful idea.

Photodiode approximately an infinite impedance, so signal power not defined.

→ input referred noise current.

Similar: Geiger Counters, etc, for Nuclear instrumentation.

Low-Frequency Instrumentation

usually broadband → no matching

→ signal power concepts generally unhelpful.

Traditional method of looking at signal voltages using an oscilloscope → noise voltage method.

Radio astronomy & Satellite radio systems

See radio comments above. But antenna noise temperature between ~3 – 200 K, not 300 K. Equivalent Temperature method used.