Notes Set 15: FET minimum noise figure

- proof (and disproof...) of Fukui’s FET noise relationships
- impact of DC power constraints on low noise FET radio receivers
Consisting of:

1) Derivation of the Flick Minum
   noise figure of a FET, together with
   some corrections.

2) Comments regarding minimum noise figures
   obtainable for receivers operating with a
   DC power constraint.
How to build a CMOS LNA:

No on-chip inductors:
chip bonding.

-tuning network for noise match on hybrid-

what does the rest of the circuit look like:

antuned resistor-feedback stages. High circuit density.
what are the other issues?

Minimum noise figure with very low d.c. power and Zopt = \( \text{device peripher} \)

So optimum source impedance varies inversely with d.c. power consumption.

High ratios of impedance transformation are desirable... what do we do about this?

- Off-water impedance matching...
Some thoughts on impedance matching:

One way to transform impedances up, so that a low $Z_{opt}$ at the transistor terminals can be matched to a much higher antenna impedance:

\[ \text{in the limit } \omega C_2 >> \sqrt{L_2 R_{load}}, \text{ impedances are scaled by the square of the voltage division ratio.} \]

Unfortunately, we wish to scale a high $R_{opt}$ at the transistor to a much lower generator impedance...
we can, of course, do this transformation also:

\[ \text{Antenna} \quad 75 \Omega \quad C_1 \quad C_2 \quad \text{"Rpt"} \]

or the second implementation:

\[ \text{L}_1 \quad \text{L}_2 \quad \text{C} \quad \text{"Rpt"} \]

this looks attractive in the sense that only 1 connection need be made to the device. \( L_1 \) & \( L_2 \) become inductive microstrip elements on the hybrid, while \( C \) is an also protectably off-water.
Red questions would be:

- What values of L, L2 are required?
- How big are the resulting microstrip lines?
- What Q will the lines have, & how big
  an effect will this have on freq.
- How does the above limit the minimum
  feasible device size, hence power consumption?

Tasks:
- Develop estimated model of the low noise
  size mask.

- Do an elementary cad exercise!
Note that the radio receiver must have considerable pre-selection before mixing if it is to operate spurious free. It is particularly convenient for the input noise-matching network to perform this function...
Design approaches using on-water inductors:

Propose to look at this:

2nd stage progress...

and also propose to look at this:

Off water

On water

and comment that feasibility of approach depends on inductors. Will use on-water soon or input if train allows it. Not if otherwise.
\[ S_{out} = 4kT R_g \cdot \left[ \frac{1}{R_g + R_i + X_{gs} + X_g} + \frac{Q_i}{\omega C_{gs}} \right]^2 \cdot \frac{R_{ds}}{2} \]

\[ N_{out} = 4kT (R_g + R_i) \cdot \left[ \frac{1}{R_g + R_i + iX_{gs} + iX_g} + \frac{Q_i}{\omega C_{gs}} \right]^2 \cdot \frac{R_{ds}}{2} \]

\[ F = 1 + \frac{4kT Q_i (R_i/R_g)}{R_g} + \sum Q_m \left( R_i + R_g + iX_g + \frac{i}{\omega C_{gs}} \right) \cdot \frac{i}{\omega \eta^2} \]
exact expression for noise figure with

$$E_{en} = R_g + jX_g.$$  

$$F = 1 + \frac{R_i}{R_g} + \left(\frac{\omega}{\omega_p}\right)^2 \frac{E_{gm}}{R_g} \left| R_i + R_g + jX_g + \frac{1}{j\omega C_g}\right|^2$$

So what is the minimum noise figure?

Clearly, we pick $jX_g = -\frac{1}{j\omega C_g}$

Let's call this $F_x$. For $F$ with the right value of source reactance...
\[ F_x = 1 + \frac{R_i}{R_g} + \left( \frac{\omega}{\omega_r} \right)^2 \frac{\Pi \gamma_m}{R_g} \left( R_i + R_g \right)^2 \]

Now, what value of \( R_g \) minimizes \( F_x \)?

\[ F_x = 1 + \frac{R_i}{R_g} + \left( \frac{\omega}{\omega_r} \right)^2 \frac{\Pi \gamma_m R_i}{R_g} + \left( \frac{\omega}{\omega_r} \right)^2 \frac{\Pi \gamma_m R_g}{R_g} + \left( \frac{\omega}{\omega_r} \right)^2 \frac{\Pi \gamma_m (2R_i)}{R_g} \]

\[ F_x = \left[ 1 + \left( \frac{\omega}{\omega_r} \right)^2 \frac{\Pi \gamma_m 2R_i}{R_g} \right] + \left[ R_i + \left( \frac{\omega}{\omega_r} \right)^2 \frac{\Pi \gamma_m R_i}{R_g} \right] \frac{1}{R_g} \]

\[ + \left[ \left( \frac{\omega}{\omega_r} \right)^2 \frac{\Pi \gamma_m}{R_g} \right] R_g \]

Aha, we can do calculus by inspection!

\( F_x \) minimized when the last 2 terms are equal!
hence:

\[ \sqrt{\left[ R_i + \left( \frac{\omega}{w_f} \right)^2 \frac{\Gamma g_m}{\mathcal{E} g_m} R_i^2 \right]} \frac{1}{R_g} = \sqrt{\left\{ \left( \frac{\omega}{w_f} \right)^2 \mathcal{E} g_m \right\} R_g} \]

\[
R_g^2 = R_i + \left( \frac{\omega}{w_f} \right)^2 \frac{\Gamma g_m R_i^2}{(\omega/w_f)^2 \mathcal{E} g_m}
\]

\[
= \frac{R_i \left( \frac{w_f}{\omega} \right)^2}{\mathcal{E} g_m} + R_i^2
\]

\[
R_g = \sqrt{R_i^2 + \frac{R_i \left( \frac{w_f}{\omega} \right)^2}{\mathcal{E} g_m}}
\]
So here is the whole answer:

\[ F_{\text{min}} = 1 + \left( \frac{\omega}{\omega_t} \right)^2 \Gamma \eta m 2 \Gamma \]

\[ + 2 \Gamma \eta m \left( \frac{\omega}{\omega_t} \right)^2 \sqrt{R_i^2 + \frac{R_i}{\Gamma \eta m} \left( \frac{\omega_t}{\omega} \right)^2} \]

\[ iX_{\text{opt}} = \frac{1}{i\omega C_{\text{opt}}} \]

\[ R_{\text{opt}} = \sqrt{R_i^2 + \frac{R_i}{\Gamma \eta m} \left( \frac{\omega_t}{\omega} \right)^2} \]
Now let’s look at this:

\[ R_i \text{ is modelling gate resistance, source & channel resistance.} \]

\[ R_i \approx 1.9 \mathrm{m} \quad \theta^* = \begin{cases} 1.5 & \text{Capacitive gate, strong 20 section} \\ 0.75 & \text{Grd. Ch.} \end{cases} \]

Use \( \theta^* = 1 \)

\[
F_{\text{min}} = 1 + 2 \left( \frac{\omega}{\omega_t} \right)^2 + 2 \left( \frac{\omega}{\omega_t} \right)^2 \sqrt{1 + \left( \frac{\omega_t}{\omega} \right)^2} 
\]

\[
\frac{1}{s \sqrt{1 + \left( \frac{\omega_t}{\omega} \right)^2}} \quad R_{\text{cp}} \approx \frac{1}{s \omega \epsilon_{\text{gs}}} 
\]

Now let’s look at limiting behaviour:
\[
\omega \ll \omega_p : \quad \sqrt{1 + \left(\frac{\omega_p}{\omega}\right)^2} = \frac{\omega_p}{\omega}
\]

\[
F_{\text{min}} \sim 1 + 2 \left(\frac{\omega}{\omega_p}\right) + 2 \left(\frac{\omega}{\omega_p}\right)^2
\]

\[
\begin{align*}
\text{i} X_{\text{opt}} & \sim \frac{1}{\omega \text{gs}} \\
R_{\text{opt}} & \sim \frac{1}{9m} \frac{\omega_p}{\omega}
\end{align*}
\]

\[
\omega \gg \omega_p : \quad F_{\text{min}} \sim 1 + 4 \left(\frac{\omega}{\omega_p}\right)^2
\]

\[
\begin{align*}
\text{i} X_{\text{opt}} & \sim \frac{1}{\omega \text{gs}} \\
R_{\text{opt}} & \sim \frac{1}{9m}
\end{align*}
\]

in reality, we are probably not interested in \(\omega \gg \omega_p\), so let's focus on the first approximation.
\[ F_{\text{min}} \sim 1 + 2 \left( \frac{\omega}{w_p} \right) + 2 \left( \frac{\omega}{w_p} \right)^2 \]

\[ \omega_{\text{opt}} \sim 1/j\omega C_g \]

\[ R_{\text{opt}} \sim \left( 1/g_m \right) \left( w_p / \omega \right) \]

\( R_i = 1/g_m, \ D = 1 \)

\[ F_{\text{min}} \sim 1 + 2 \sqrt{D R_i g_m} \left( \frac{\omega}{\omega_p} \right) + 2 \left( D R_i g_m \right) \left( \frac{\omega}{\omega_p} \right)^2 \]

\[ \omega_{\text{opt}} \sim 1/j\omega C_{g_s} \]

\[ R_{\text{opt}} \sim \sqrt{\frac{R_i}{g_m}} \left( \frac{w_p}{\omega} \right) \]
Let's do a sanity check:

In GaAs, hertz with $f_r \sim 160$ kHz.

<table>
<thead>
<tr>
<th>$f_r$</th>
<th>$F$</th>
<th>$F_{dB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.133</td>
<td>0.5 dB</td>
</tr>
<tr>
<td>60</td>
<td>2.03</td>
<td>3.1 dB</td>
</tr>
</tbody>
</table>

10 GHz noise figure, SiGe:

<table>
<thead>
<tr>
<th>$f_r$</th>
<th>$F_{min}$</th>
<th>$X_{ref}$</th>
<th>$R_{pos}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 GHz</td>
<td>2.2 dB</td>
<td>(1/9m)</td>
<td>(1/9m)</td>
</tr>
<tr>
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<td>1.5 dB</td>
<td>(5/9m)</td>
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</tr>
</tbody>
</table>
Flkue gives

\[ F_{min} = 1 + K_i \frac{C_i s}{g_m} \sqrt{\frac{R_g + R_s}{g_m}} \]

\[ = 1 + \frac{K_i}{2\pi} \frac{C_i s}{w_i} \sqrt{\frac{g_m}{(R_g + R_s) g_m}} \]

\[ = 1 + \left( \frac{K_i}{2\pi} \right) \frac{C_i s}{w_i} \sqrt{\frac{g_m}{(R_g + R_s) g_m}} \]

where \( K_i \approx 0.16 \)
\( K_i h_{vp} = 0.03 \)

My expression is

\[ F_{min} = 1 + 2\sqrt{I\tau} \sqrt{(R_i + R_g + R_s) g_m} \left( \frac{w_i}{w_i} \right) \]

which correlates only if \( 2\sqrt{I\tau} < 0.03 \).

This is confusing! - check units!
IEEE TED July 1974

$f(k_f) : \quad F_0 = 1 + 2\pi K_f + C_{gs} \frac{\sqrt{R_y + K_s}}{g_m} \cdot 10^{-3}

K_f \approx 2.5
\theta \text{ in G/H, } g_m \text{ in } 1/\text{m}
C_{gs} \text{ in } \Omega.

So going to MKS units:

\[
F_0 = 1 + 2\pi K_f \frac{\theta}{10^9} \cdot C_{gs} \frac{\sqrt{R_y + K_s}}{g_m} \cdot 10^{-3}
\]

\[
= 1 + 2\pi K_f \theta C_{gs} \sqrt{(R_y + K_s)} / g_m
\]

\[
F_0 = 1 + K_f \left( \frac{\omega}{\omega_T} \right) \sqrt{g_m (R_y + K_s)} / g_m
\]

\[
K_f \approx 2.5
\]

My derivation is:

\[
F_0 = 1 + 2\sqrt{\pi} \sqrt{g_m (R_y + K_s)} g_m \left( \frac{\omega}{\omega_T} \right)
\]
Note first that \( \sqrt{(R_i + R_j + R_k)^9} \)\
\[= \sqrt{((R_i)^9 + (R_j)^9 + (R_k)^9)} = \sqrt{1 + (R_j + R_k)^9} \]

so there is a slight inconsistency in the results in the 2 equations.

Beyond this small correction

Fukai: \( K_t = 2.5 \)

Rodwell \( K_t = 2 - \frac{17}{17} \)

\[\frac{17}{17} \approx 0.75 - 1.5 \]

= 1.7 to 2.4

Given the slightly bigger radical in my expression, the correlation is excellent!

I will get good fitted if I use \( \Pi = 1.5 \).
Falki: says:

\[ R_{opt} \sim K_3 \left[ \frac{1}{2g} + R_9 + R_9 \right] \]

\[ X_{opt} \sim K_9 \left[ \frac{1}{g_e g_s} = 0.06 \right] \sim K_9 = 1 \text{, for } g_s \]

Note that his expression for \( X_{opt} \) is exactly the same as mine, but that the \( R_{opt} \) doesn't have the same frequency dependence at all.

But note that Falki's expression is empirical.

Brian Hughes' expressions are given in:

IEE Trans MTI 93, Feb '93, pg 190

This agrees with mine.
<table>
<thead>
<tr>
<th>Transistor</th>
<th>gm/I ratio</th>
<th>equals $1/2(V_{gs}-V_{t})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FET gm</td>
<td>2.00E+00</td>
<td></td>
</tr>
<tr>
<td>FET ft</td>
<td>5.00E+10</td>
<td></td>
</tr>
<tr>
<td>Gamma</td>
<td>1.50E+00</td>
<td></td>
</tr>
<tr>
<td>Fet Ri</td>
<td>1.50E+02</td>
<td></td>
</tr>
<tr>
<td>Fet Cgs</td>
<td>2.12E-14</td>
<td></td>
</tr>
<tr>
<td>Signal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency</td>
<td>1.00E+10</td>
<td></td>
</tr>
<tr>
<td>Generator</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rg, optimum</td>
<td>7.50E+02</td>
<td></td>
</tr>
<tr>
<td>Xg, optimum</td>
<td>7.50E+02</td>
<td></td>
</tr>
<tr>
<td>Rg</td>
<td>2.00E+02</td>
<td></td>
</tr>
<tr>
<td>Xg</td>
<td>2.00E+02</td>
<td></td>
</tr>
<tr>
<td>Noise Performance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fmin</td>
<td>1.49E+00</td>
<td>Linear</td>
</tr>
<tr>
<td>Fmin</td>
<td>1.73E+00</td>
<td>Very simplified Expression</td>
</tr>
<tr>
<td>Fmin</td>
<td>1.61E+00</td>
<td>Slightly simplified Expression</td>
</tr>
<tr>
<td>Fmin</td>
<td>2.07E+00</td>
<td>Slightly simplified Expression</td>
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<tr>
<td>F</td>
<td>2.60E+00</td>
<td>Linear</td>
</tr>
<tr>
<td>F</td>
<td>4.15E+00</td>
<td>Exact expression</td>
</tr>
<tr>
<td>DC Power Consumption</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vcc</td>
<td>1.50E+00</td>
<td>Volts</td>
</tr>
<tr>
<td>Id</td>
<td>3.33E-03</td>
<td>amps</td>
</tr>
<tr>
<td>Pdc</td>
<td>5.00E-03</td>
<td>Watts</td>
</tr>
</tbody>
</table>

- **Frequency**: 10 GHz
- **FET ft**: 50 GHz
- **Rg**: 200 Ω
- **Xg**: 200 Ω
- **DC Power**: 5.0 mW
- **Fmin**: 2.1 dB
- **F**: 4.1 dB

2.068
4.150
To repeat

\[ F_{\text{min}} \approx 1 + 2 \sqrt{\frac{\pi}{2}} \sqrt{\frac{g_m (R_i + R_k + R_g)}{\omega r}} \]

\[ j X_{\text{opt}} = \frac{1}{-j \omega C_g s} \]

\[ R_{\text{opt}} = \sqrt{\frac{R_i}{\pi g_m}} \left( \frac{\omega r}{\omega} \right) \]

Approximate

\[ = \sqrt{\frac{R_i g_m}{\pi}} \frac{1}{\omega C_g s} \]

Less approximate

\[ F_{\text{min}} = 1 + \left( \frac{\omega}{\omega r} \right)^2 2 \pi g_m \sqrt{\frac{R_i}{\pi g_m}} \left( \frac{\omega r}{\omega} \right)^2 + R_i \]

\[ + \left( \frac{\omega}{\omega r} \right)^2 \pi g_m 2 R_i \]

\[ j X_{\text{opt}} = \frac{1}{-j \omega C_g s} \]

\[ R_{\text{opt}} = \sqrt{\frac{R_i + R_i}{\pi g_m}} \left( \frac{\omega r}{\omega} \right)^2 \]
Note that:

- no gate leakage shot noise has been modelled.

- Quadratic term in \((\omega l/\nu t)\) is real & observed.

- The optimum generator impedance, for \((\omega << \nu t)\),
  has equal resistive & inductive components,
  both of which have the same magnitude as the
  capacitive reactance of \(C_g\).

- This means that for \(\omega >> \nu t\), \(\omega << \nu t\)
  the generator impedance becomes very big.
The full expression for noise figure is:

\[ F = 1 + \frac{R_i + R_s + R_g}{R_{gen}} \]

\[ + \left( \frac{w}{w_T} \right)^2 \frac{I_{gm}}{R_{gen}} \left[ \left( R_i + R_s + R_g + R_{gen} \right)^2 \right. \]
\[ + \left. \left( \frac{R_{gen} - 1}{\omega C_{gs}} \right)^2 \right] \]

...This will vary as \( \sim \frac{1}{R_{gen}} \) for \( R_{gen} \ll R_{opt} \).
Let's consider a specific example:

Deep submicron MOSFET as \( \leq 1-10 \text{ GHz} \)

A low-power radio receiver. What noise figure can we obtain?

- Block diagrams of the receiver & preamplifier are shown on the next pages.

The problem:

- Difficult to transform to high impedances at microwave frequencies; \( \|Z_{out}\| < 300 \Omega \).

- With a fixed device \( g_m \) to \( I_d \) ratio, a dc power constraint becomes a constraint on \( g_m \).

- Antenna & device are then no longer noise-matched.
Emerging IC topology for monolithic mobile radio communications receivers (Berkeley, UCLA)
Assumptions:
\[
g_m = \frac{1}{I} \quad \frac{1}{2(V_{gs} - V_t)}
\]

\[g_m, (R_i + R_y + R_s) = 1\]

Noise Figure vs. DC Power Consumption

\[f = 50 \text{ GHz at } (V_{gs} - V_t) = 0.25 \text{ V}\]

\[|Z_{in}| \leq 300 \text{ } \Omega\]

\[V_{cc} = 1.5 \text{ V}\]

\[
\begin{align*}
\text{Noise Figure, dB} & \\
\text{Front-End DC Power Consumption, mW} & \\
& \text{5 GHz} \\
& \text{10 GHz} \\
& \text{2 GHz} \\
& \text{5 GHz} \\
& \text{10 GHz} \\
\end{align*}
\]