Notes Set 19: Baseband, AM, and FM analog receivers.

- Signal/noise ratio. Microphone preamp example. AM-(DSB-SC) receiver example. FM, nonlinear, and “twisted” modulation.
- FM radio sensitivity example.
Analog Communications Systems

Here the analysis & concepts are fairly straightforward.

We could take the case of FM or AM Transmission; I will instead work 3 problems examples;

* DS13 radio receiver sensitivity.
* Sin rate of a microphone preamplifier.
* FM Broadcast radio.
Note that we could also work problems in AM & FM radio, etc.

- FM radio is inefficient; plus, the analysis is a trivial extension of the DB case.

- FM radio with a large phase deviation is a non-linear ("twisted") modulation method whose treatment must be lengthy.
First example - baseband S/W analog problem

Microphone preamplifier

Typical moving-coil microphone

~1mV output for some "standard"

Acoustic input power level "0 dB re..."

The generator impedance is nearly resistive

with an a curving inductance k. We will
ignore the letter k refer the interested student
to a acoustics engineering handbook.
$V_{ym} = 2 mV \quad @ \quad 20$ Pa

need to calculate the S/N ratio due to the preamplifier for a standard 0 dB reference level...
Perhaps our preamplifier has a bipolar differential input...

This is the input stage of an LM381 audio-op-amp, e.g. an op amp designed for low-noise audio applications. Note the resistive biasing.
Note specifically that this stage
- uses resistive biasing.
- uses $R = 1000$ ohms transistors.
- provides external adjustment of $I_C$ for noise minimization.

I will assume a 500 ohm base resistance.

Skipping a detailed noise treatment, our noise model looks like so:

```
\[ \text{Signal} \rightarrow 2R \rightarrow R_C \rightarrow \text{Input} \]
```

\[ 2R, \text{Input} \]

\[ + \]

\[ - \]

\[ 2R, \text{Input} \]
An op-amp is connected like so:

\[ \frac{R_{\text{gen}}}{R_2} \text{ to give gain } \frac{1 + R_2}{R_1} \]

The input noise voltage is:

\[ \frac{\sigma(V_{\text{in}})}{\sqrt{I}} = 4kT \left( \frac{2R_6 + R_2 + R_1R_2}{R_6 + R_{\text{gen}}} \right) + \frac{2gI_6}{B} \left( \frac{1}{R_6 + R_{\text{gen}}} + \frac{1}{R_6 + R_1R_2} \right) \]

\[ = 2 \times 10^{-17} V^2/Hz \quad R_{\text{gen}} = 500 \Omega \]

\[ 2.4 \times 10^{-17} \quad R_{\text{gen}} = 5000 \Omega \]

\[ 8.6 \times 10^{-17} \quad R_{\text{gen}} = 20,000 \Omega \]

\[ 4.27 \times 10^{-16} V^2/Hz \quad R_{\text{gen}} = 50,000 \Omega \]

\[ 3.6 \times 10^{-17} V^2/Hz \quad R_{\text{gen}} = 10,000 \Omega \]
There is an optimization, which I will not show with respect to the transformer ratio 5:1. We will use a 10X step-up transformer at the preamplifier.

\[ v \rightarrow 5v \rightarrow \text{to preamplifier.} \]

Arbitrarily choosing the preamplifier as the reference plane, 0 dBm now produces 70 mV and the generator impedance is \[ 200 \Omega (5)^2 = 5 \Omega. \]
At $V_{son} = 5kV$, $2(V_{n}V_{n}^{+}) = 2.9 \times 10^{-17} V^2/Hz$.  

$V_{signal} = 10mV$ at 0dBA

The remainder is language:

$$S = \frac{(10 \text{mV})^2}{N} = 124 \text{ dB (1kHz)} \text{ for 0dBA input.}$$

In a 20-20kHz bandwidth,

$$S = \frac{(10 \text{ mV})^2}{N} = 81 \text{ dB 20kHz bandwidth 0dBA input.}$$
We can also work with equivalent power:

Equivalent background acoustic signal level

= -124 dB A (14e)

This means that if we have a 14e wide filter, we will get unity sin ratio at -124 dB A.

With a 10 kHz wide filter, we will get unity sin ratio at -119 dB A, or that with a 1 kHz wide filter a 10 dB sin ratio requires -84 dB A acoustic power. So specifically, over the 20 kHz audio bandwidth,

Equivalent background acoustic noise level

= -81 dB A (20 kHz)
Of course, this was an easy example. Problems are complicated by

- frequency dependent noise due to source & amplifier reaction.

- frequency dependent sensitivity to the effects of noise.

For the letter, e.g. with sound, the noise is integrated over the effective (very-not-flat) frequency response of the human ear. This is called A-weighted. Signal/Noise.

Similar analysis for TV picture SNR, transfer SNR in instrumentation, etc.
Second Example.

Double-Sideband, Suppressed Carrier receiver system.

Here the transmitted signal

\[ u_i(t) = U_m(t) \cdot \cos (\omega_c t) \cdot K_i \]

the received signal in the presence of noise

\[ u_r(t) = K_2 U_m(t) \cos (\omega_c t) \]

\[ K_2 \text{ very small} \]
The power in the received signal is random, with

\[
\langle P_r \rangle = \frac{k^2/2}{\text{Re}\{Z_{ant}\}} \langle |u_m(t)|^2 |u_a(t)|^2 \rangle
\]

... the denominator being the antenna resistance.
Let's take $\frac{\partial}{\partial t} \langle v_n v_{n+1}^* \rangle$ to be uniform over a

$\pm 20$ kHz bandwidth $= \pm \Delta f_m$

The receiver looks like so:
The Friis Noise formula gives

\[ \text{F_{receiver}} = \text{F_{NA}} + \frac{\text{F_{Mixer}} - 1}{\text{G_{NA}}} + \frac{\text{F_{IF}} - 1}{\text{G_{Mixer}}} + \ldots \]

Let's take \( \text{F_{NA}} = 2 \) dB, \( \text{G_{NA}} = 20 \) dB.

- \( \text{F_{Mixer}} = 3 \) dB, \( \text{G_{Mixer}} = -3 \) dB.
- \( \text{F_{IF}} = 6 \) dB.

Which gives \( \text{F_{receiver}} = 2.3 \) dB.

The receiver is 20 kHz bandwidth.
\[ S = \frac{2 \langle P \rangle}{\mathcal{N} \sigma^2 T \text{ receiver} \cdot \Delta f} \]

Since the signal power is uniform over a \( \Delta f = 20 \text{ kHz} \) bandwidth...

\[ S = \frac{\langle P \rangle}{\mathcal{N} \sigma^2 T \cdot \Delta f} \quad \text{for} \quad \Delta f = 20 \text{ kHz}. \]

Again, there are many ways to summarize this. If we have, e.g., some minimum \( \text{S/N ratio}, \) let's say \( \text{40 dB}, \) then the minimum receiver power is...
\[
\frac{<P_r>}{P_{r_{\text{min}}}} = kT F_r \cdot \Delta f + 40 \text{dB}
\]

Minimum

\[
= -173.8 \text{ dBm (1/Hz)} \quad kT
\]

\[
+ 2.3 \text{ dB} \quad F_r
\]

\[
+ 43 \text{ dB} \quad 10 \log \left( \frac{\Delta f}{10^6} \right)
\]

\[
+ 40 \text{ dB} \quad \text{s/n}
\]

\[
<\frac{P_r}{P_{r_{\text{min}}}} > = -88.53 \text{ dBm}^2 \quad @ \quad 40 \text{ dB s/n}
\]

\[
= 1.4 \text{ mW}
\]

FM radio requires a much smaller s/n ratio at the carrier for a given audio s/n ratio. Sensitivity then are

\[
\sim 10 \text{ dBf} = 10^{-14} \text{ W} = 10^{-11} \text{ mW} = -110 \text{ dBm}
\]
Let's do it more realistically:

- Mapping function is:

\[
\begin{align*}
U_2(t) &= k X_1(V_m) \cos \omega t + k X_2(V_m) \sin \omega t \\
X_1 &= \cos \left( \frac{V_m}{V_{max}} \Theta_{max} \right), \quad X_2 = \sin \left( \frac{V_m}{V_{max}} \Theta_{max} \right)
\end{align*}
\]

Once again, a picture is much more helpful...
In the plane ("vector space") of \( \mathbf{v} \) and \( \mathbf{v'} \) the message moves around a circle.

\[ \text{Note that if the maximum phase angle is } \pm \Theta_{\text{max}} \text{ radians, or } \pm (2\pi)^{-1} \Theta_{\text{max}} \text{ times round a circle, a message results in much more rapid variation of } \mathbf{v}, \mathbf{v'}. \]
Full-Amplitude message of highest allowed frequency:

\[ V_{\text{max}} \cos \omega t \]

Approximate representation of message

\[ V_{\text{max}} \]

\[ \text{triangle wave} \]

V变化至 amplitude

\[ \ldots \]

\[ \frac{1}{2f_m} \]

\[ = \frac{1}{2 \left( \frac{\omega_m}{2\pi} \right)} \]

\[ = \frac{\pi}{\omega_m} \]
Ah! peak modeled (transmitted) signal bandwidth is

\[ \Delta \omega = \pm \frac{2 \omega_{max}}{2\pi} \]

\[ = \frac{2 \omega}{\pi} \omega_{max} \text{ fm} \]

so the bandwidth is greatly increased.

Receiver bandwidth must be \[ \frac{2 \omega_{max}}{\pi} \text{ fm} \]

when people saw this, they said the FM system must be noisier...

... Armstrong said no...
The received signal voltage (antenna impedance $Z_0$) is

$$V_r(t) = K_r \Re(V_m) \cos \omega t + K_r \Im(V_m) \sin \omega t$$

$$+ V_{\text{noise}}(t)$$

$$K_r^2 / 2Z_0 = P_{\text{rec}}.$$  

Noise: Variance $(Z_0) K TF B_{\text{rec}}$ in bandwidth $B_{\text{rec}}$.

$$E_{\text{noise}}^2 = \frac{4}{\pi} \Theta_{\text{max}} \sin \Theta_{\text{max}}$$

Angular deflection due to noise: $\Delta \Theta_{\text{noise}} = \frac{E_{\text{noise}}}{K_r}$

Noise voltage on recovered signal: $\Delta V_{\text{noise}} = V_{\text{max}} \cdot \Delta \Theta_{\text{noise}}$

...where the maximum signal is $V_{\text{max}}$...
\[ \frac{S}{N} = \frac{V_{\text{max}}^2}{(V_{\text{max}} \cdot \Delta \theta_{\text{noise}} / \Theta_{\text{max}})^2} = \frac{\Theta_{\text{max}}^2}{\Delta \Theta_{\text{noise}}} \]

Max Signal

\[ = \frac{\Theta_{\text{max}}^2}{V_n^2} \cdot K^2 \]

\[ = \frac{\Theta_{\text{max}}^2}{\frac{2}{\pi} \Theta_{\text{max}} \cdot f_m \cdot K T F \cdot B_0} \]

\[ = \Theta_{\text{max}} \left\{ \frac{1}{\frac{2}{\pi} f_m \cdot K T F} \right\} \]

\[ \frac{2}{\pi} \]

This is \( \Theta_{\text{max}} \) better than DSB?

E.g. direct linear analog modulation.
Threshold will occur when

\[ \Theta_{\text{max}} \cdot \frac{\sigma_{\text{in}}}{\sigma_{\text{out}}} = \text{Prec} \cdot 2 \Theta_{\text{max}} \cdot KTF \cdot \text{fn} \]

\[ \text{Prec} = \frac{\Theta_{\text{max}}}{KTF \cdot \text{fn}} \]

\[ \frac{1}{N} \Theta_{\text{max}} \frac{1}{\pi} \left( \frac{\text{Prec}}{KTF} \right) \]

for \( \text{Prec} \gg \text{Threshold} \)

where

\[ \text{Threshold} = \frac{2 \Theta_{\text{max}} \cdot KTF \cdot \text{fn}}{\pi} \]
Example:

\* FM broadcast radio - approximate values

\* $f_m < 20 \text{ kHz}$  (15 kHz actually)

\* 200 kHz channel spacing...

$$200 \text{ kHz} = \Delta f = \frac{2 \Theta_{\text{max}}}{\pi} . 20 \text{ kHz}$$

$$\Theta_{\text{max}} = 5 \text{ mrad}.$$ This means 5 full rotations of the circle at maximum modulation...
Threshold power:

\[ P_{th} = \frac{0}{\pi} \cdot KTF \cdot f_m \]

... lets assume 3dB noise figure...

\[ P_{th} = 5 \cdot z \cdot KT \cdot f_m \]

\[ = 10 \log_{10} (5 \times 2) + 10 \log_{10} \left( \frac{KT \cdot 1 \text{ Hz}}{1 \text{ mW}} \right) \]
\[ + 10 \log_{10} \left( \frac{2 \times 4 \text{ Hz}}{1 \text{ Hz}} \right) \]

\[ = 10 \text{ dB} - 173.83 \text{ dBm} + 43 \text{ dB} \]

\[ = -120.83 \text{ dBm} \]

\[ = -150.83 \text{ dBm} \]

\[ = -0.83 \text{ dBm} \leftrightarrow \text{ dBm}; \text{ dB relative to } 1 \text{ fW} = 10^{-15} \text{ W}. \]
So our radio receiver is at threshold at -0.8 dBf. What signal power does it take to get a 40 dB 5 Hz

\[
\frac{S}{N} = 40 \text{dB} = \frac{\Theta_{\text{max}}}{\pi} \cdot \frac{1}{f_m} \cdot \frac{\text{Pre}}{kT}\frac{1}{F}
\]

\[
\text{Pre} = 40 \text{dB} + 10 \log_{10} \left\{ \frac{\pi}{5} \cdot \frac{kT}{f_m} \cdot f_m \right\}
\]

\[
= 40 \text{dB} + 10 \log_{10} \left\{ \frac{2}{5} \right\} + 10 \log_{10} \left\{ \frac{kT \cdot 1 \text{Hz}}{1 \text{mW}} \right\} + 10 \log_{10} \left\{ \frac{2.44 \text{dB}}{1 \text{Hz}} \right\}
\]

\[
= 40 \text{dB} + (-3.97 \text{dB}) + (-173.83 \text{ dBm}) + 43 \text{dB}
\]

\[
= -94.8 \text{ dBm}
\]

\[
= 25.2 \text{ dBm} \quad (330 \text{ fW})
\]
So we conclude:

\[ \approx -0.8 \text{ dBm at threshold.} \]

\[ \approx 5 \text{ dBm (3 kW) intelligible signal.} \]

\[ \approx 25 \text{ dBm (300 kW) Hi-Fi quality (40 dB) SNR) } \]

... Isn't this great!

Note that a picowatt is a lot of power for a radio receiver...

why? - well think of what we have learned... the universe ain't that hot anymore, so signals don't have to be very big.