• Notes Set 19: Baseband, AM, and FM analog receivers.

- Signal/noise ratio. Microphone preamp example. AM-(DSB-SC) receiver
- example. FM, nonlinear, and "twisted" modulation.
- FM radio sensitivity example. `

ECE ____ Notes Set 17

Analog Communications Systems

Here the analysis & concepts are fairly straightforward.

We could take the case of FM or AM Transmission; I will instead work 3 problem examples;

* DSB radio receiver sensitivity.
* S/N ratio of a microphone preamplifier.
* FM Broadcast radio.

Note that we could also work problems in AM & FM radio, but:

= AM radio is inefficient, plus the analysis is a trivial extension of the DSB case.

= FM radio with a large phase deviation is a nonlinear ("twisted") modulation method whose treatment must be lengthy.

First example - baseband S/N analog problem

Microphone preamplifier

typical moving-coil microphone

~1mV output "for some "standard"

acoustic input power level "0 dB_H or ..."

The generator impedance is ~200Ω resistive

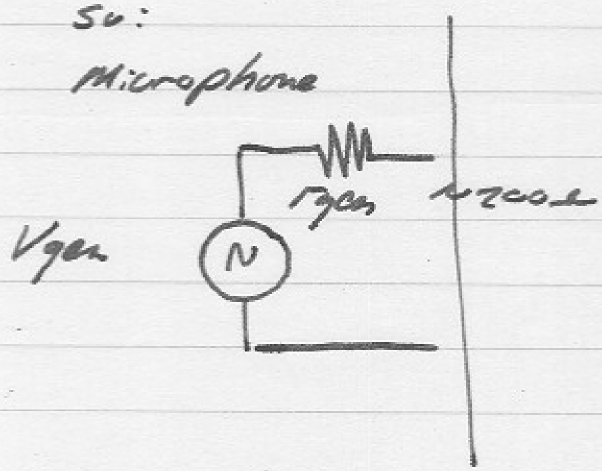
with ~~an~~ a wiring inductance L. We will

ignore the latter & refer the interested student

to a acoustic engineering handbook.

So:

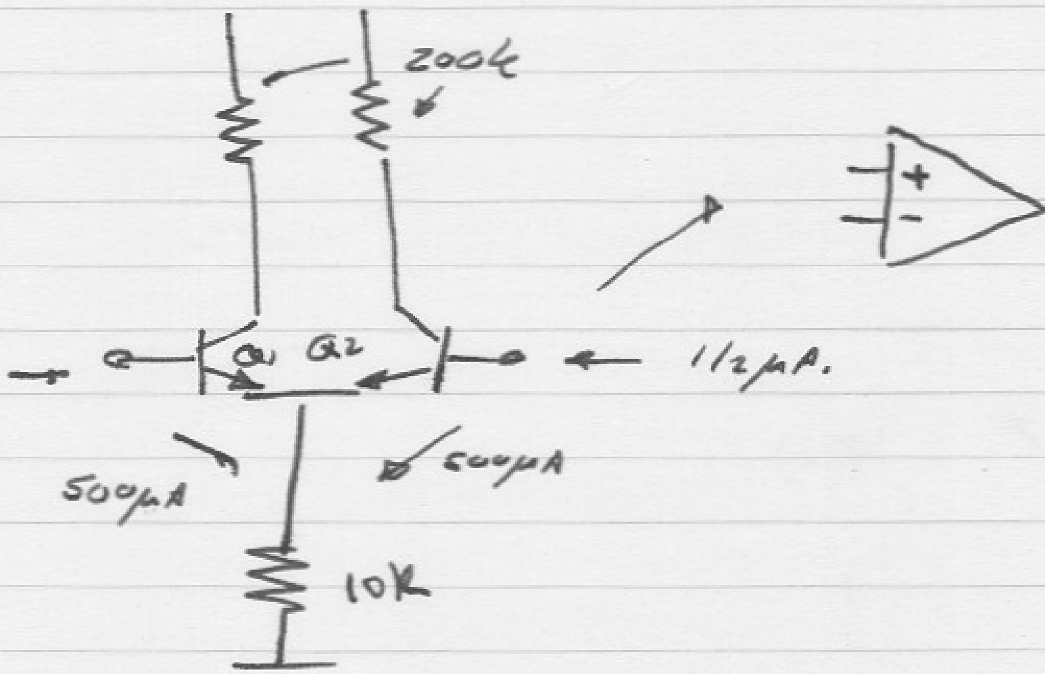
Microphone



$v_{gen} = 2\,mV$ @ $P_{acoustic}$ is $0\ dB_A$.

need to calculate the S/N ratio due to the preamplifier for a standard $0\ dB_A$ reference level...

Perhaps our preamplifier has a bipolar differential input...



This is the input stage of an LM381 Audio-op-amp, e.g. an op amp designed for low-noise audio applications. Note the resistive biasing.

Note specifically that this stage

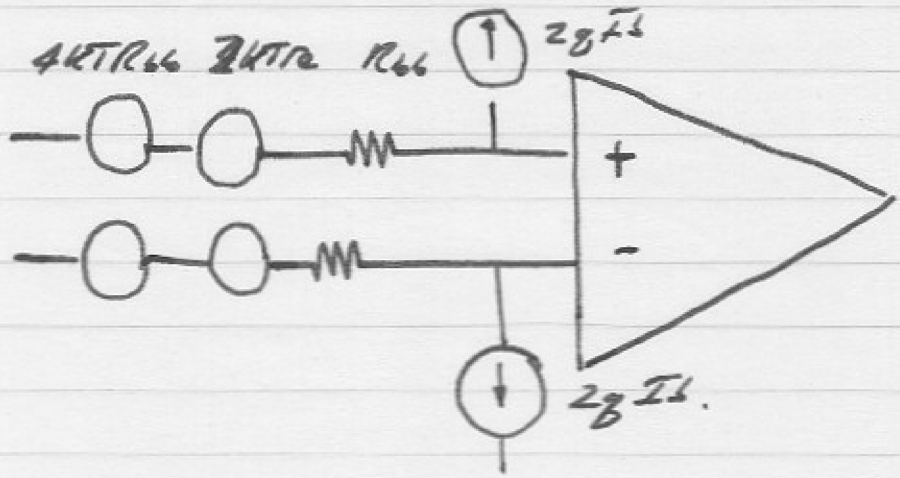
- uses resistive biasing.

- uses $\beta = 1000$ transistors.

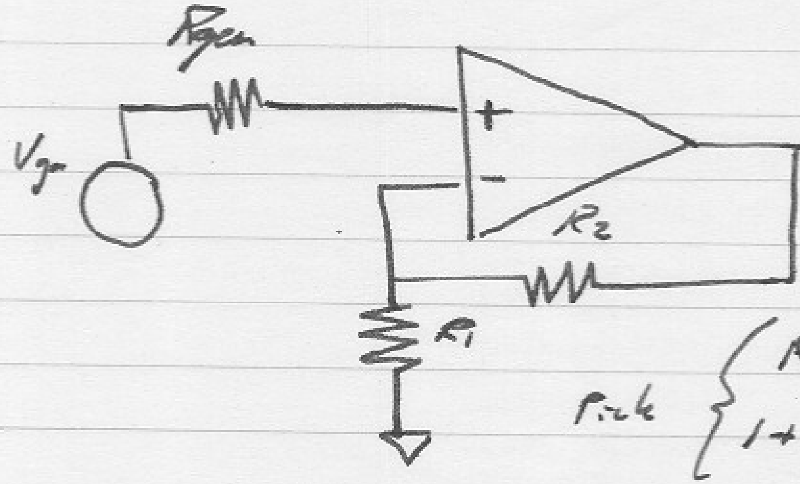
- provides external adjustment of I_c for noise minimization.

I will <u>assume</u> a $500\,\Omega$ base resistance.

Skipping a detailed noise treatment, our noise model looks like so:



An op-amp is connected like so:



to give gain $1 + \frac{R_2}{R_1}$

Pick $\left\{ \begin{array}{l} R_1 \| R_2 = 100\,\Omega \\ 1 + \frac{R_2}{R_1} = 100 \end{array} \right\}$

The input noise voltage is:

$$\frac{\partial \langle E_n E_n^* \rangle}{\partial f} = 4kT\left(2R_{bb} + r_e + r_1 \| r_2\right)$$

$$+ 2qI_b\left((R_{bb} + R_{gen})^2 + (R_{bb} + R_1 \| R_2)^2\right)$$

$= 2.(10^{-17})\ V^2/Hz$ $\quad R_{gen} = 500\,\Omega$.

$2.4(10^{-17})$ — $\quad R_{gen} = 5000\,\Omega$

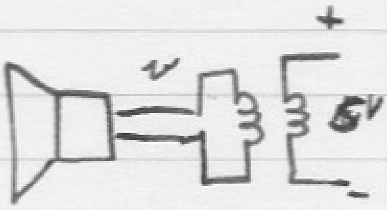
$8.6(10^{-17})$ $\quad R_{gen} = 20{,}000\,\Omega$

$4.27(10^{-16})\ V^2/Hz$ $\quad R_{gen} = 50{,}000\,\Omega$

$3.6(10^{-17})\ V^2/Hz.$ $\quad R_{gen} = 10{,}000\,\Omega$

There is an optimization, which I will not show, with respect to the transformer ratio.

We will use a ~~10:1~~ 5:1 step-up transformer at the preamplifier.



→ to preamplifier.

arbitrarily choosing the preamplifier as the reference plane, $0\,dB_0$ now produces <u>10 mV</u>

and the generator impedance is $200\Omega(5)^2 = 5k\Omega$.

At $R_{gen} = 5\,k\Omega$, $\frac{\partial \langle E_n E_n^* \rangle}{\partial f} = 2.4(10^{-17})\ V^2/Hz.$

$V_{gen}\big|_{Signal} = 10mV$ at 0dBa

The remainder is language:

$$\frac{S}{N} = \frac{(10\ mV)^2}{2.4(10^{-17})\ V^2/Hz} = 124\ dB\ (1Hz)$$ for 0dBA input.

In a 20-20kHz bandwidth.

$$\frac{S}{N} = \frac{(10\ mV)^2}{2.4(10^{-17})\ V^2/Hz \cdot 20kHz} = 81\ dB$$ 20kHz bandwidth 0dBA input.

We can also work with equivalent powers:

$$\boxed{\begin{array}{l}\text{Equivalent background acoustic signal } \underline{\text{level}} \\ \quad = -124\ dB_A\ (1\ Hz)\end{array}}$$

This means that if we have a 1 Hz-wide filter, we will get unity S/N ratio at $-124\ dB_A$, with a 10-Hz-wide-filter we will get unity S/N ratio at $-114\ dB_A$, or that with a 1 kHz wide filter a 10 dB S/N ratio requires $-84\ dB_A$ acoustic power. So specifically, over the 20 kHz audio bandwidth,

$$\boxed{\begin{array}{l}\text{Equivalent background acoustic noise level} \\ \quad = -81\ dB_A \quad (20\ kHz)\end{array}}$$

Of course, this was an easy example.

Problems are complicated by

- frequency dependent noise due to source & amplifier reactances.

- frequency-dependent sensitivity to the effects of noise.

For the latter, e.g. with sound, the noise is integrated over the effective (very-not-flat) frequency response of the human ear. This is called A-weighted. Signal / Noise.

Similar analysis for TV picture S/N, transducer S/N in instrumentation, etc.

Second Example.

Double-Sideband, Suppressed Carrier

receive system.

Were the transmitted signal

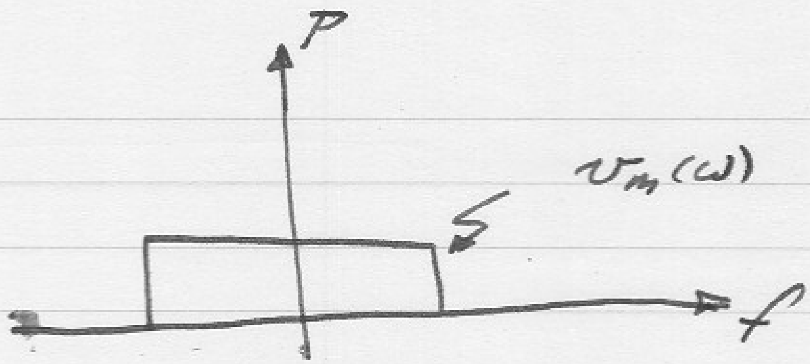
$$v_t(t) = v_m(t) \cdot \cos(\omega_c t) \cdot K_1$$

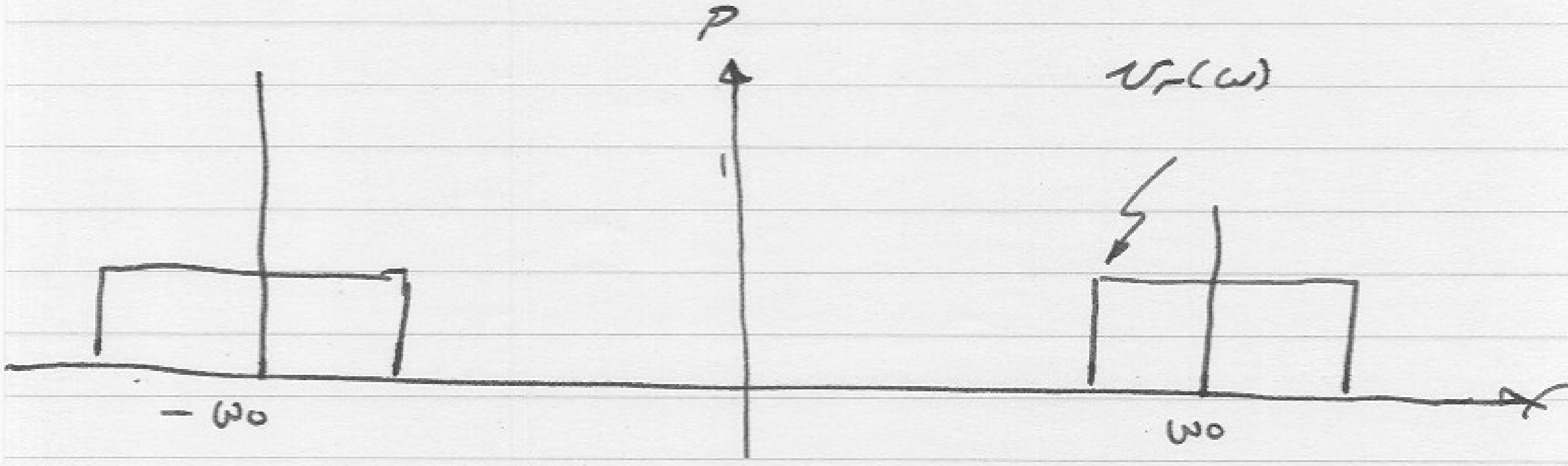
the received signal in the presence of noise

is

$$v_r(t) = K_2 \, v_m(t) \cos(\omega_c t)$$

K_2 very small





The power in the received signal is random, with

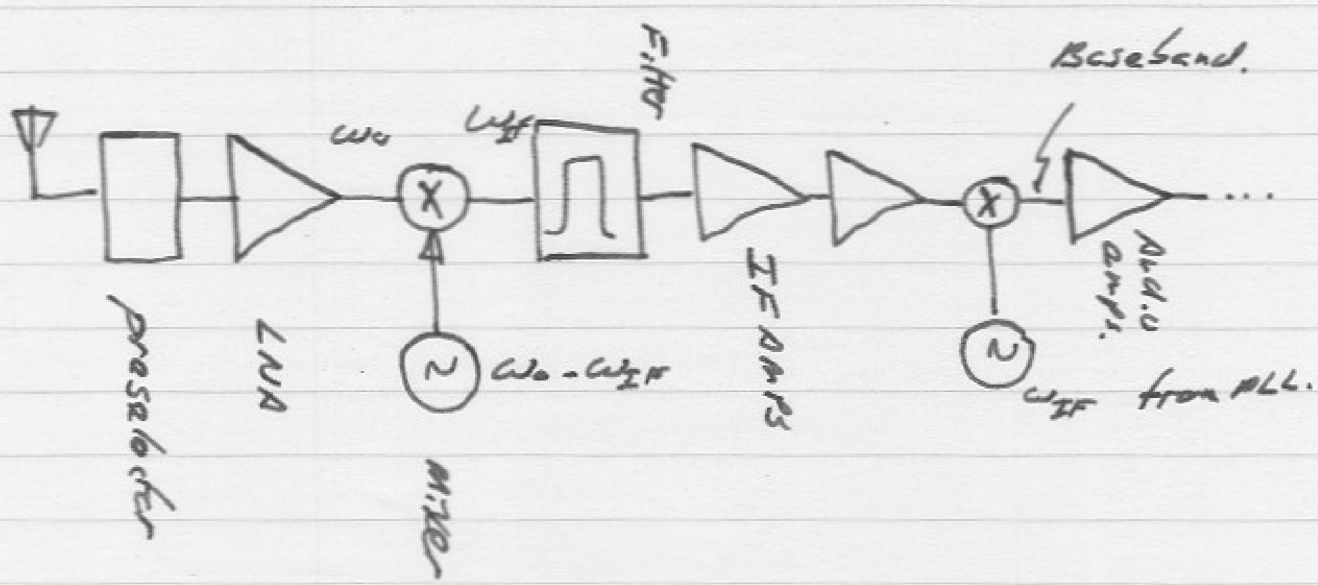
$$\langle P_r \rangle = \frac{K_2^2/2}{Re\{Z_{ant}\}} \langle v_m(t)\, v_m^*(t) \rangle$$

... the denominator being the antenna radiation resistance.

Let's take $\frac{\partial}{\partial f}\langle v_n v_n^* \rangle$ to be uniform over a

$\pm$ 20 kHz bandwidth $= \pm\Delta f_m$

The receiver looks like so:



The FRIIS Noise formula gives

$$F_{receiver} = F_{LNA} + \frac{F_{mixer} - 1}{G_{AV\,LNA}} + \frac{F_{IF} - 1}{G_{AV\,LNA}\,G_{AV\,mixer}} + \cdots$$

lets take $F_{LNA} = 2\,dB$, $G_{A\,LNA} = 20\,dB$

$F_{mixer} = 3\,dB$ $G_{mixer} = -3\,dB$

$F_{IL} = 6\,dB$.

which gives $\underline{F_{receiver} = 2.3\,dB}$

The receiver is 20 kHz bandwidth.

$$\frac{S}{W} = \frac{\frac{\partial \langle P_r \rangle}{\partial f} \cdot \Delta f}{KT\, F_{receiver} \cdot \Delta f}$$

Since the signal power is uniform over a Δf $= 20$ kHz bandwidth, ...

$$\frac{S}{N} = \frac{\langle P_r \rangle}{KT\, F_r \cdot \Delta f} \qquad \text{for} \quad \Delta f = 20 \text{ kHz}.$$

again, there are many ways to summarize this. If we have, e.g., some minimum S/N ratio, lets say 40dB, then the minimum receiver power is.

$$\langle P_r \rangle \Big/_{\text{Minimum}} = kT F_r \cdot \Delta f \quad + \quad 40\,dB$$

$$= -173.8 \text{ dBm (1 Hz)} \qquad kT$$

$$+ 2.3 \text{ dB} \qquad F_r$$

$$+ 43 \text{ dB} \qquad 10\log_{10}\left(\frac{\Delta f}{1\,Hz}\right)$$

$$+ 40 \text{ dB} \qquad S/N.$$

$$\left.\begin{aligned} \langle P_r \rangle \Big/_{min} &= -88.53 \text{ dBm} \\ &= 1.4 \text{ nW} \end{aligned}\right\} @ \ 40 \text{ dB } S/N$$

FM radio requires a much smaller S/N ratio at the carrier for a given audio S/N ratio. Sensitivities there are ~ 10 dBf $= 10^{-14}$ W $= 10^{-11}$ mW $= -110$ dBm

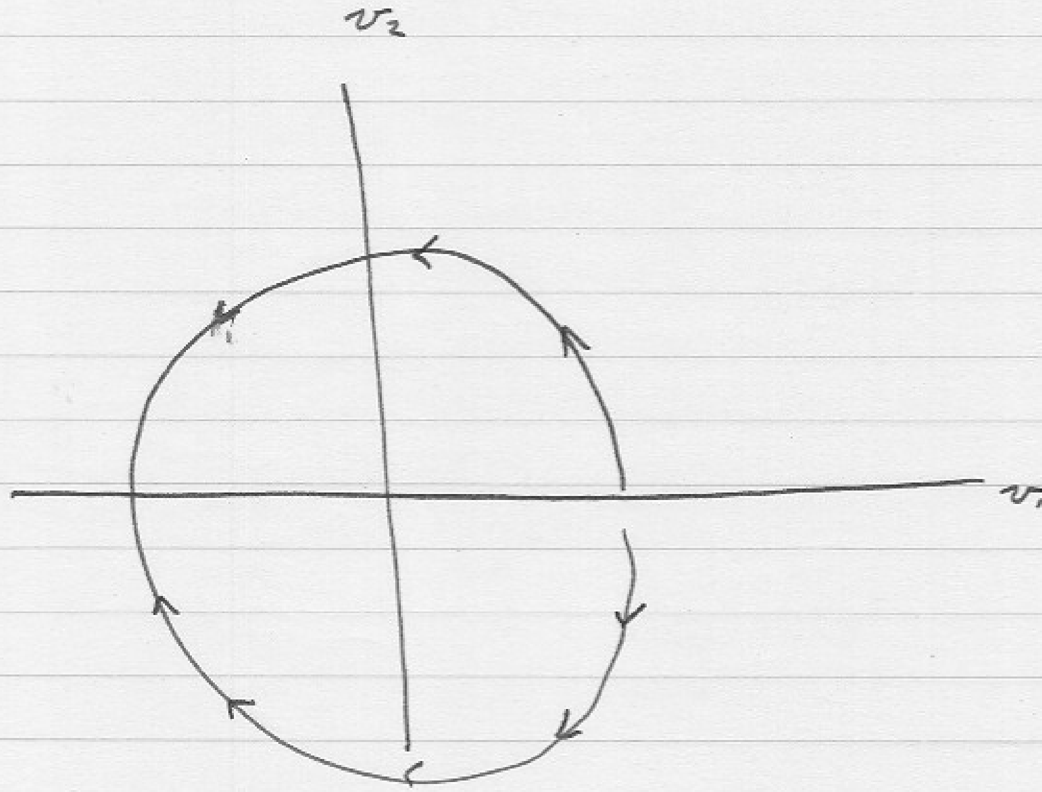
Lets do FM More realistically:

- Mapping function is:

$$v_t(t) = k\overbrace{x_1(v_m)}^{v_1}\cos\omega_0 t + \overbrace{k x_2(v_m)}^{v_2}\sin\omega_0 t$$

$$x_1 = \cos\left(\frac{v_m}{v_{max}}\Theta_{max}\right), \quad x_2 = \sin\left(\frac{v_m}{v_{max}}\Theta_{max}\right)$$

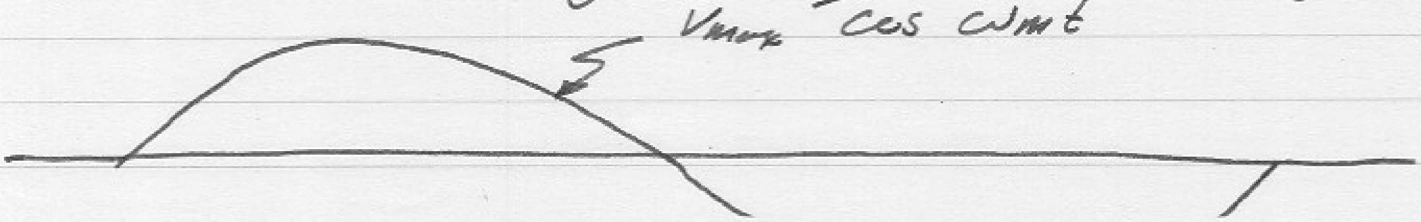
once again, a picture is much more helpful...



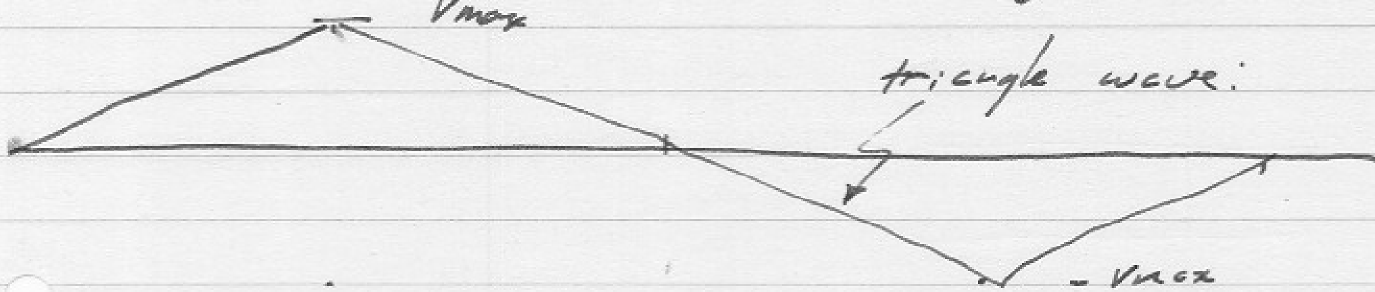
In the plane ("vector space") of v_1 & v_2 the message moves around & around a circle.

$\equiv$ Note that if the maximum phase angle is $\pm\theta_{max}$ radians, or $\pm(2\pi)^{-1}\theta_{max}$ times round a circle, a message $\boxed{V_{max}\cos\omega_m t}$ results in much more rapid variation of v_1 & v_2.

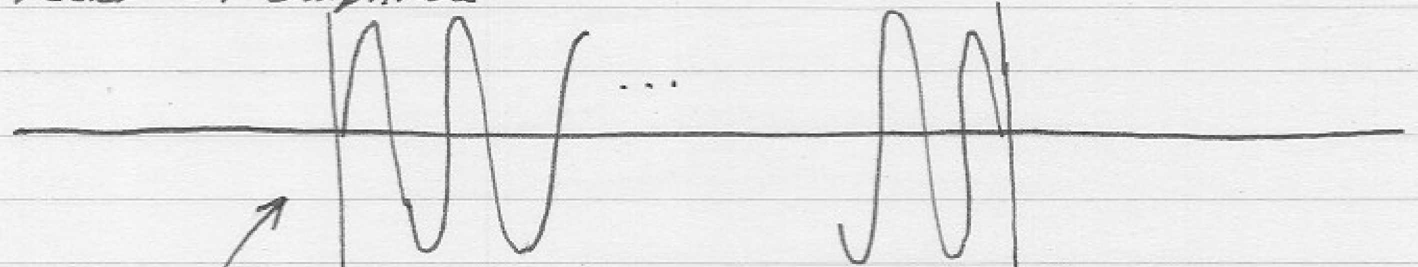
Full-Amplitude message of highest allowed frequency:



Approximate representation of message



Vector v_1 amplitude



goes through $\frac{2\Theta_{max}}{2\pi}$ cycles in time

$$\frac{1}{2f_m} = \frac{1}{2\left(\frac{\omega_m}{2\pi}\right)} = \frac{\pi}{\omega_m}$$

Aha!: peak modulated (transmitted) signal bandwidth is

$$\Delta f = \pm \frac{2\Theta_{max}}{2\pi} \cdot 2 f_m$$

$$= \frac{\cancel{2}}{\pi} \Theta_{max} f_m$$

so the bandwidth is greatly increased.

- receiver bandwidth must be $\frac{2}{\pi}\Theta_{max} f_m$. Hz -

when people saw this, they said the FM system must be noisier...

... Armstrong said no...

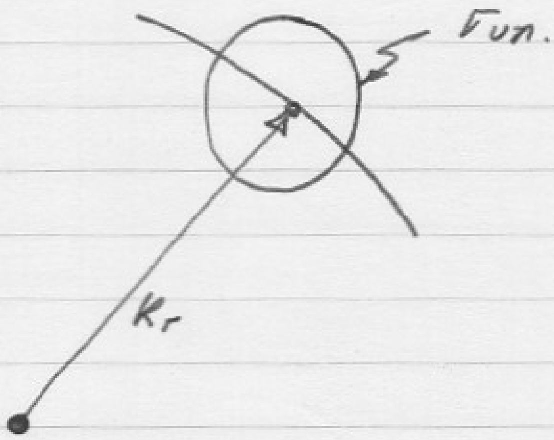
The received signal voltage (antenna impedance Z_0) is

$$v_r(t) = K_r\, \psi_1(V_m) \cos \omega_0 t + K_r\, \psi_2(V_m) \sin \omega_0 t$$
$$+\ v_{noise}(t)$$

$K_r^2 / 2Z_0 = P_{rec}.$

v_{noise}: variance $(Z_0) KTF\, B_{rec}$ in bandwidth B_{rec}.

$$\sigma_{v_n}^2 = \frac{4}{\pi}\, \Theta_{max}\, f_m \cdot KTF\, Z_0$$



angular deviation due to noise: $\Delta\Theta_{noise} = \dfrac{\sigma_{v_n}}{K_r}$

noise voltage on recovered signal: $\Delta v_{noise} = \dfrac{V_{max}}{\Theta_{max}} \cdot \Delta\Theta_{noise}$.

... where the maximum signal is V_{max}...

so,

$$\frac{S}{N}\Big|_{\text{max signal}} = \frac{V_{max}^2}{(V_{max} \cdot \Delta\Theta_{noise} / \Theta_{max})^2} = \frac{\Theta_{max}^2}{\Delta\Theta_{noise}^2}$$

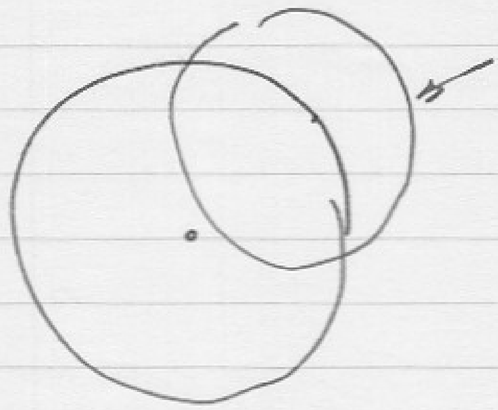
$$= \frac{\Theta_{max}^2}{\sigma_{v_n}^2} \cdot k_r^2 = \frac{\Theta_{max}^2}{\frac{2}{\pi}\Theta_{max} f_m \cdot kTF\, Z_0} \cdot 2 Z_0 P_{rec}$$

$$= \frac{\Theta_{max}}{\pi} \left\{ \frac{1}{f_m} \frac{P_{rec}}{kTF} \right\}$$

this is $\sim \frac{\Theta_{max}}{\pi}$ better than DSB!

e.g. direct linear analog modulation.

Threshold will occur when



e.g., ... $\sigma_{vn}^2 = kr^2$

$$\frac{2}{\pi}\,\Theta_{max}\cdot f_m\cdot kTF\,Z_0 = P_{rec}\cdot 2Z_0$$

$$P_{rec} = \frac{\Theta_{max}}{\pi}\,kTF\,f_m.$$

Summarize:

$$\frac{S}{N} \cong \frac{\Theta_{max}}{\pi}\,\frac{1}{f_m}\left(\frac{P_{rec}}{kTF}\right)$$

for $P_{rec} >> P_{threshold}$.

where

$$P_{threshold} \cong \frac{\Theta_{max}}{\pi}\cdot kTF\cdot f_m$$

Example:

~ FM broadcast radio - approximate values

* $f_m < 20\ kHz$ (15 kHz actually)

* 200 kHz channel spacing...

$$200\ kHz = \Delta f = \frac{2\ \theta_{max}}{\pi} \cdot 20\ kHz$$

$\rightarrow$ $\theta_{max} = 5\pi$ radians.

this means 5 full rotations of the circle at maximum modulation...

Threshold power:

$$P_{th} = \frac{\Theta_{max}}{\pi} \cdot kTF \cdot f_m.$$

... lets assume 3 dB noise figure ...

$$P_{th} = 5 \cdot 2 \cdot kT \; f_m$$

$$= 10\log_{10}(5 \times 2) + 10\log_{10}\left(\frac{kT \cdot 1\,Hz}{1\,mW}\right)$$

$$+ 10\log_{10}\left(\frac{20\,kHz}{1\,Hz}\right)$$

$$= 10\,dB - 173.83\,dBm + 43\,dB$$

$$= -120.83\,dBm$$

$$= -150.83\,dBW$$

$= -0.83\ dB_f \longleftrightarrow dB_f$; dB relative to $1\,fW = 10^{-15}\,W$.

So our radio receiver is at threshold at -0.8 dBf ! What signal power does it take to get a 40 dB S/N ?

$$\frac{S}{N} = 40\,dB = \frac{\Theta_{max}}{\pi} \cdot \frac{1}{f_m} \frac{P_{rec}}{KTF}$$

$$P_{rec} = 40\,dB + 10\log_{10}\left\{\frac{\pi}{5\pi}\,KTF^{2} \cdot f_m\right\}$$

$$= 40\,dB + 10\log_{10}\left\{\frac{2}{5}\right\} + 10\log_{10}\left\{\frac{kT \cdot 1Hz}{1mW}\right\} + 10\log_{10}\left\{\frac{20kHz}{1Hz}\right\}$$

$$= 40\,dB + (-3.97\,dB) + (-173.83\,dBm) + 43\,dB$$

$$= -94.8\,dBm$$

$$= 25.2\,dBf \quad (330\,fW)$$

so we conclude:

~ −0.8 dBf at threshold.

~ 5 dBf (3 fW) intelligible signal.
20 dB S/N

~ 25 dBf (300 fW) Hi-Fi quality (40 dB S/N)

... Isn't this great!

Note that a picowatt is a lot of power for a radio receiver...

why? - well think of what we have learned... the universe ain't that hot anymore, so signals don't have to be very big.