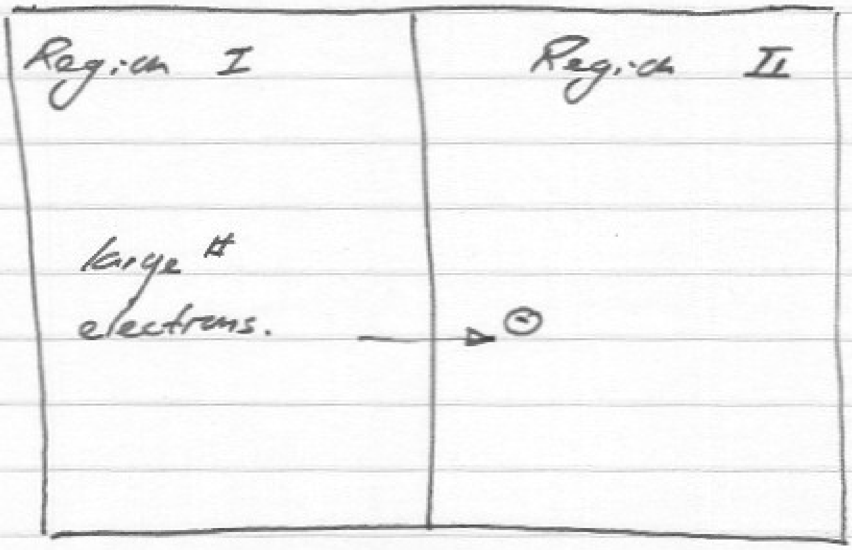
# • Notes Set 8: Shot Noise and Diffusion Noise

- independent events and shot noise derivation. Spectral densities
- The ultraviolet crisis revisited: indication of thermal correlation times ≈ h/kT
- Counting arguments
- Schottky diode: thermionic emission, shot noise vs bias, 2kTR relationship
- Diffusion noise: variations in fluxes
- diffusive transport and diffusive noise in PN junctions, physics, circuit models
- shot noise of generation-recombination, transit times, diffusion capacitance

ECE ______ Notes set 8

**Shot Noise:** Diodes & transistors.

Model:




suppose that we have, at time intervals $\tau_1$, some probability of emission $p_1$ from region I → II.

average electron flux $= p_1 \tau_1^{-1} = r$

average d.c. current $= q_1 (p_1 \tau_1^{-1})$

Lets break time into small time steps $\Delta t$.

1) then # of electrons in each $\Delta t$ is $\perp$ of # of electrons in other $\Delta t$'s.

2) $P_{\Delta t}$ { $n$ electrons) =

$$p_{\Delta t}(n) = p(n) = \binom{\Delta t/\tau}{n} p_1^{\,n} q_1^{(\Delta t/\tau)-1}$$

because there are $(\Delta t/\tau)$ trials each of probability $p_1$.

For convenience, lets make $(p\,\Delta t/\tau)\cdot q >> 1$, then we can apply the limiting theorem for a Gaussian to obtain:

$p_{\Delta t}(n)$ = gaussian with mean $p\Delta t/\tau$ & variance $\frac{\Delta t}{\tau}\cdot p\cdot q$.

If we have set up the problem so that $gp << 1$

then $g \sim 1$. and.

$p_\Delta(\tilde{n}) =$ Gaussian with mean $p\,\Delta t/\tau$

& variance $p.\,\Delta t/\tau$.

~~Eq. / th~~

Lets look at the mean and the average

values separately:

$\Delta \tilde{n} = \tilde{n} - \bar{\tilde{n}}$ = fluctuation.

$\bar{\tilde{n}}$ = mean electron flux in $\Delta T$.

$$\boxed{\text{Then } \Delta\tilde{n} \text{ is gaussian with variance } = \bar{\tilde{n}}}$$

How do we compute the power spectrum:

well $\overline{\Delta n_{\Delta t_1} \cdot \Delta n_{\Delta t_2}} = 0$

counts are uncorrelated between time intervals.

so the autocorrellation function is

$$R_{nn}(\Delta t_i, \Delta t_j) = (\bar{n}) \qquad \delta_{ij}$$

$$= \frac{\rho_1}{\tau_1} \cdot \Delta t\,(\delta_{ij}) = r \cdot \Delta t \cdot (\delta_{ij})$$

Take the Fourier Transform to calculate the power spectrum:

Limiting arguments at this point look like they will take more time than I can devote. We must take the limit $\Delta t \to 0$ while computing the power spectrum:

This will be:

$$S(\omega) = r$$

e.g. if the electron flux is gen. has an average rate $r$, then the ~~d~~ fluctuations in the electron flux will have power spectral density:

$$S(\omega) = r$$

and autocorrelation function

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S(\omega)\, e^{-j\omega\tau}\, d\omega = r \cdot \delta(\tau)$$

Note we are using a single-sided power spectral density here. If we use power spectral densities defined thus:

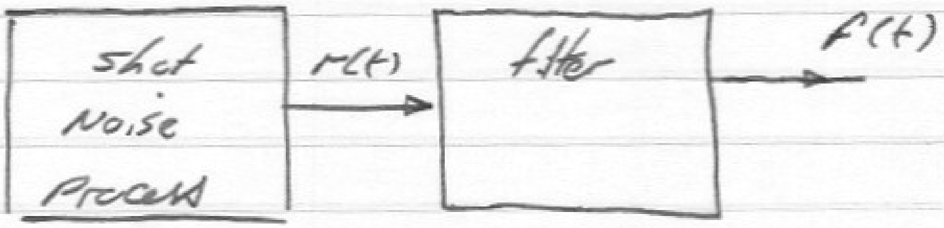
$$R(\tau) = \int_0^{+\infty} \frac{\partial}{\partial f}\langle \Delta\Gamma \Delta\Gamma^* \rangle \, e^{-j2\pi f \tau} \, df$$

then

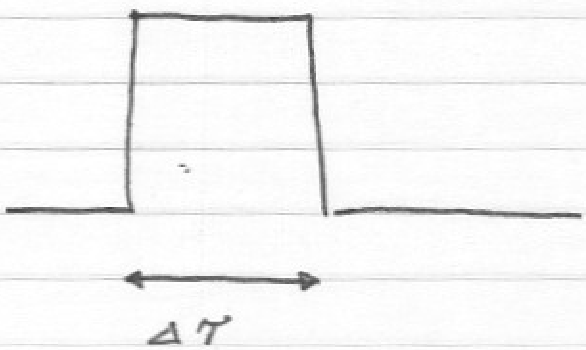
$$\boxed{\frac{\partial}{\partial f}\langle \Delta\Gamma \Delta\Gamma^* \rangle = 2 \cdot \Gamma}$$

The single-sided power spectral density is twice the flux.

There is another way of doing this:




the filters' impulse response will be as so:




again, the process is at rate $r$

counted over period $\Delta T$.

If we deal with the fluctuations $\Delta f$ & $\Delta r$,

then

$\Delta f =$ gaussian with variance $(r \cdot \Delta T)$

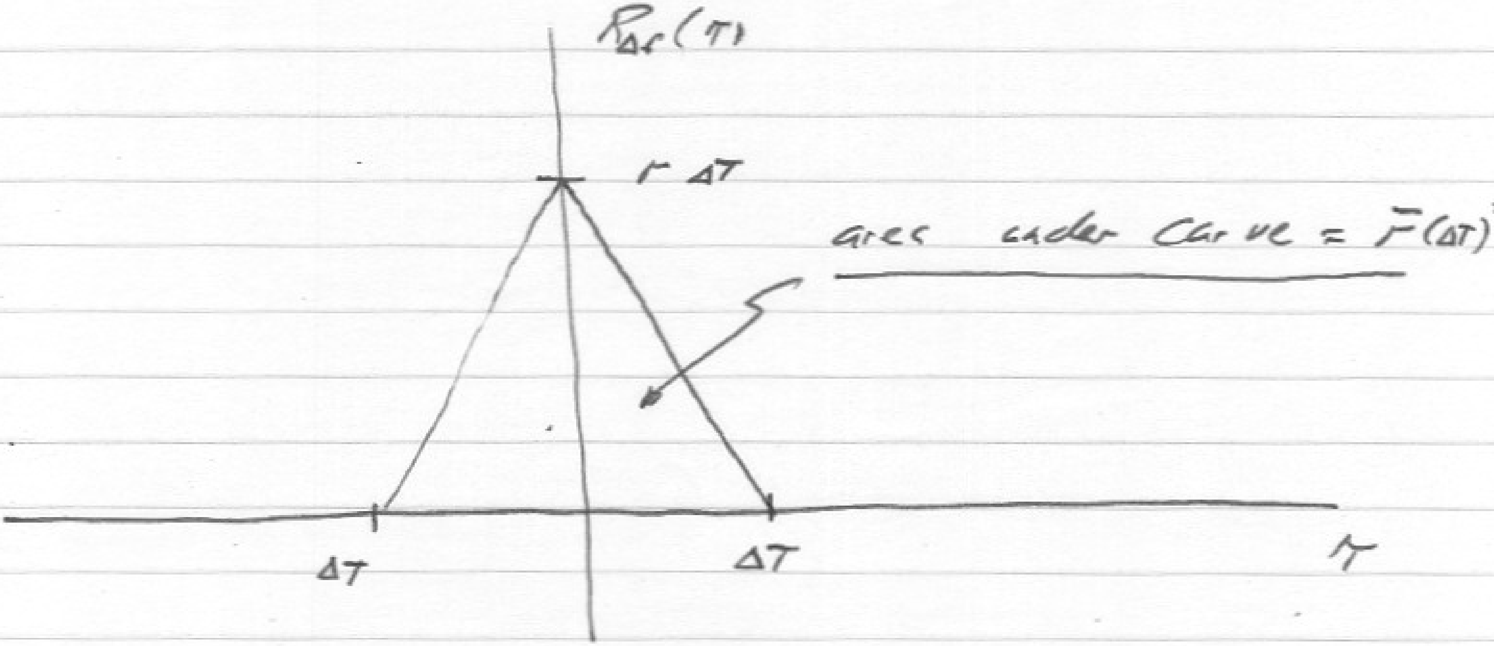
Now, what is the autocorrellation function of $\Delta f$?

If the time offset $\tau$ of $R(\tau)$ is

$\tau > \Delta T$, then $R_{\Delta f}(\tau) = 0$

because there ~~are~~ is no overlap in the two time periods $\Delta T$.

If $\tau < \Delta T$, then $R_{\Delta f}(\tau) = (F \Delta T)\left(1 - \frac{|\tau|}{\Delta T}\right)$



And the power spectral density of $\Delta f$ is:

$$S_{\Delta f}(\omega) = \bar{F}\cdot(\Delta T)^2 \cdot \left[\frac{\sin(\omega\cdot\Delta T/2)}{(\omega\cdot\Delta T/2)}\right]^2$$

—

Now, since the input & output are related by

filter impulse response:

$$\Delta f(t) = \Delta r(t) * h(t)$$

then

$$S_{\Delta f}(\omega) = S_{\Delta r}(\omega)\cdot\left\| H(\omega)\right\|$$

$$= S_{\Delta r}(\omega)\cdot\left\|\frac{\sin(\omega\Delta T/2)}{(\omega\cdot\Delta T/2)}\cdot\Delta T\right\|^2$$

so

$$\boxed{S_{\Delta r}(\omega) = \bar{r}}$$

We have proved 2 things:

1) The shot noise process has a constant power spectrum over all frequencies, equal to the flux. Even at frequencies above the mean arrival rate.

2) If we filter shot noise with some impulse response $\Delta T$, such that $(\bar{r}\Delta t) \gg 1$ then the filtered process is also Gaussian.

for filtering with $\bar{r}\Delta t \sim 1$, the noise remains white, but is no longer gaussian.

To convert electron fluxes into currents, note that if

$$I = q\, r$$

then $$\bar{I} = q\, \bar{r}$$

and $$\sigma_I^2 = \cancel{2} q^2 \sigma_r^2$$

$$S_I(\omega) = q^2 \cdot \bar{r}$$

$$= q \cdot (q \bar{r})$$

$$= q \bar{I}$$ 2-sided spectral density

or

$$\boxed{\frac{\partial \langle \Delta I \cdot \Delta I^* \rangle}{\partial f} = \frac{\partial \langle I_n I_n^* \rangle}{\partial f} = 2 q \bar{I}}$$

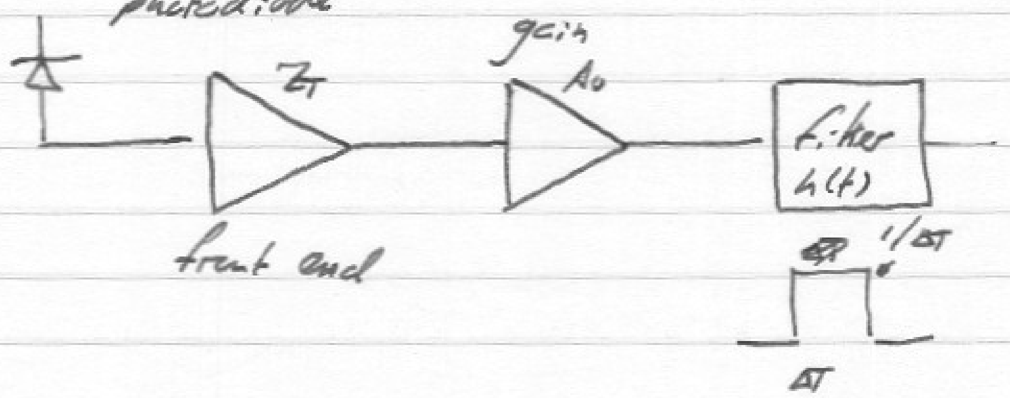
Note that this assumes that electron events are $\perp\!\!\!\perp$.

Specifically, DC current in a resistor **need** **not** show shot noise.

The **argument** is that motion of each electron produces fields which alter electron flow for all other electrons. Can be proved formally by adding thermal velocity distribution to drift equation. I won't do this year, but will add to next years' notes.

**Conclusion:** Linear resistors should not show shot noise.

Example




Here is an optical receiver. Photocurrent is amplified and then filtered ($h(t)$) over a time $\Delta T \sim 1/\text{Bit rate}$. The bit rate is 40 Gb/sec, and the photodiode has ~~...~~ 100 nA leakage.

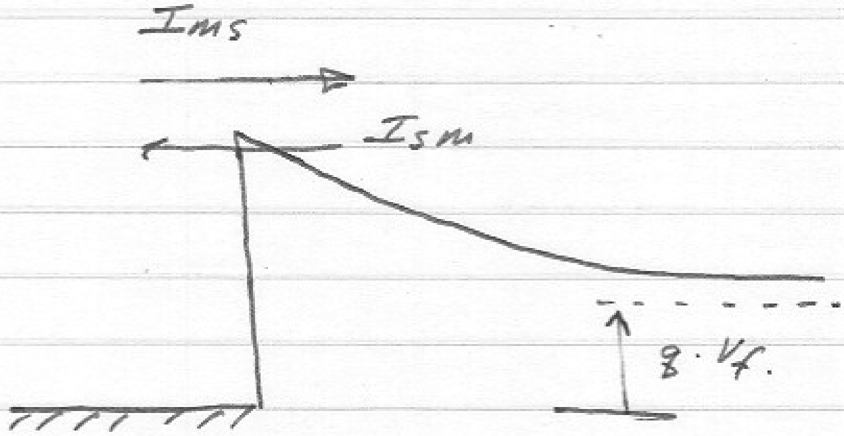
→ input referred spectral density

$$\frac{d}{df}\langle I_n I_n^* \rangle = 2q\,(100\,nA) = 3.2\,(10^{-26})\ A^2/Hz.$$

But note over the bit period that

$$\Delta T \cdot \bar{F} = \frac{100\,nA}{q} \cdot \frac{1}{40\ Gb/sec} = 16 \text{ electrons.}$$

So the noise is not gaussian over this time period shot noise in a Schottky diode




$I_{ms} = I_0$

$I_{sm} = I_0 \cdot \exp(qV_f / kT)$

these are # currents, in opposite directions, each of which arises from an electron being randomly lifted above the barrier energy by the thermal energy distribution $\rho(E_1) \sim \exp(-E_1/kT)$ from statistical thermodynamics.

The total current is therefore

$$I = I_{sm} - I_{ms} = I_0 \left[ \exp(qV_f/(kT)) - 1 \right]$$

both $I_{sm}$ & $I_{ms}$ have full shot noise

so that the shot noise of $I$ is

$$\boxed{\frac{\partial}{\partial f} \langle I_n I_n^* \rangle = 2q I_{sm} + 2q I_{ms}}$$

Lets look at 3 cases:

Zero bias: $g \triangleq \partial I / \partial V = \dfrac{q I_0}{kT}$

$$\frac{\partial \langle I_n I_n^* \rangle}{\partial f} = 4 \cdot q \cdot I_0$$

$$\frac{\partial}{\partial f} \langle P_{AV} \rangle = \frac{1}{4g} \cdot \frac{\partial \langle I_n \cdot I_n^* \rangle}{\partial f} = kT$$

as required, at zero bias [equilibrium]

the available noise power is $(kT \cdot \Delta f)$

Subtle note

The available noise power should not be

$kT$, but

$$\frac{\partial \langle P_{av} \rangle}{\partial f} = \left[ \frac{1}{2} hf + \frac{hf}{\exp(hf/kT) - 1} \right]$$

... This means that the shot noise must really be

$$\frac{\partial}{\partial f} \langle I_n I_n^* \rangle = 2 \cdot q \cdot I_0 \cdot \frac{1}{kT} \left[ \frac{1}{2} hf + \frac{hf}{\exp(hf/kT) - 1} \right]$$

$$\frac{\partial}{\partial f} \langle I_n I_n^* \rangle = 2 q I_0 \left[ \frac{hf/kT}{\exp(hf/kT) - 1} + \frac{1}{2} \frac{hf}{kT} \right]$$

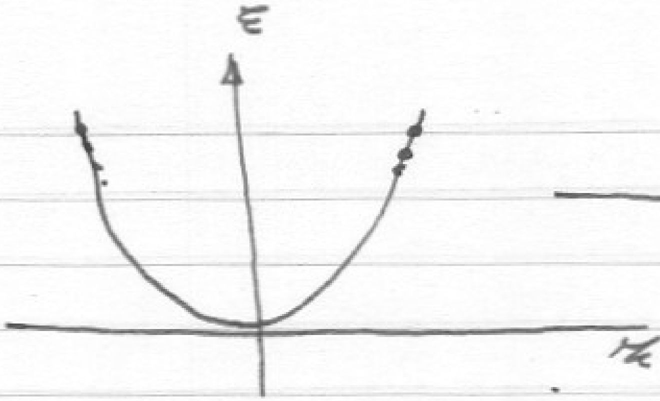
which ~~drops below~~ diverges from $2qI_0$ for $hf > kT$

I am troubled by this discrepancy, as I have never seen this latter expression.

To reconcile with our shot noise derivation, note we have taken all states to be filled & re-filled at essentially instantaneously.

this may not be reasonable: If an electron with low probability $e^{-0.2V/0.026V} \sim e^{-7.7}$ is driven 0.2 V above the fermi energy, and thus crosses the diode, is this uniform with time?

unresolved: (Try to solve for next time)




Note, we have assumed transport of electrons over barrier if $E > E_b$, and used equilibrium thermodynamics to calculate this probability in order to get a DC I-V curve. We have further assumed ∥ electron emissions. Yet, removing eletrons from states above $E_b$ disturbs this equilibrium. We would expect, from $\Delta E \cdot \Delta t \sim \hbar$, that states be repopulated at times of $\sim \hbar / \Delta E$, and the simple shot noise model breaks down breaks down when $f^{-1} \sim h / \Delta E \doteq h / kT$

This is a hand-waving approach to the shot noise vs Plank distribution discrepancy.

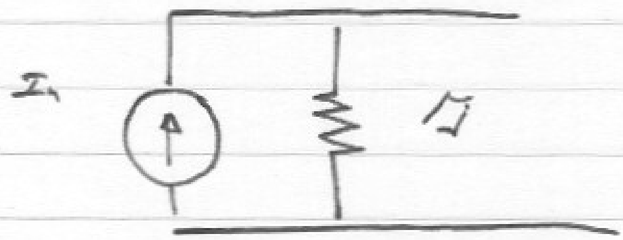
<u>Case 2:</u> strong forward bias

then $I \sim I_0 \exp(qV/kT)$ and

$$\frac{\partial \langle I I_n^* \rangle}{\partial f} = 2qI$$

and $G \triangleq \frac{\partial I}{\partial V} = \cdots = \frac{qI}{kT}$

So ~~the~~ diode is modelled thus



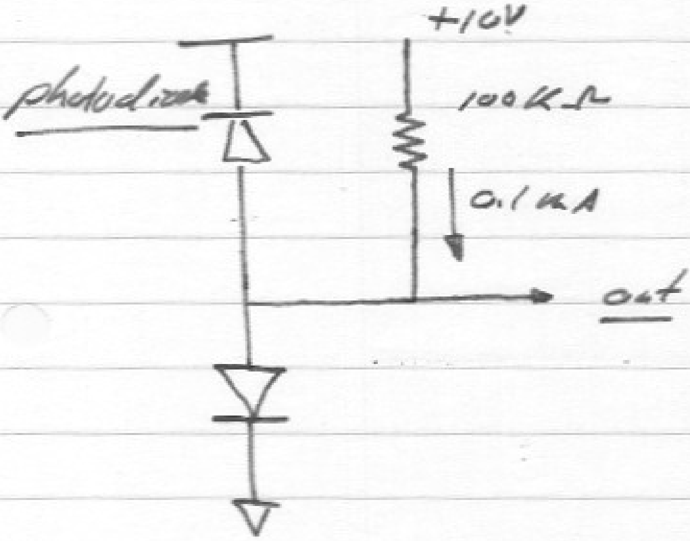

$$\frac{\partial}{\partial f}\langle I_n I_n^* \rangle = 2qI, \quad r_j = \frac{kT}{qI}$$

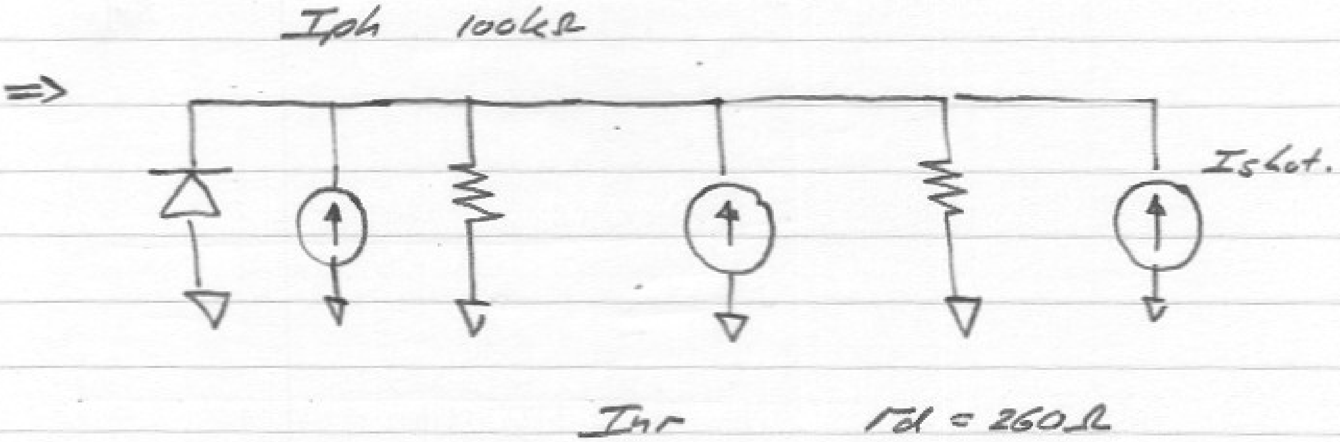
So $$\boxed{\frac{\partial \langle P_{av} \rangle}{\partial f} = \frac{kT}{2}}$$

This can be modelled thus:

A forward-biased diode is like a cold resistor.

This could be useful:



<u>the total noise current is therefore</u>

$$\frac{\partial}{\partial f}\langle I_n I_n^* \rangle = \frac{4kT}{100k\Omega} + 2q\,(100\mu A)$$

$$= \frac{4kT}{100k\Omega} + 2kT \cdot \frac{q\,(100\mu A)}{kT}$$

$$\simeq 2 \cdot kT \,\frac{(100\mu A) \cdot q}{kT}$$

$$= 2 \cdot kT \cdot \frac{1}{260\Omega}.$$

we have loaded the photodiode in $260\Omega$ small-signal impedance while introducting only 1/2 the mean-squared noise current.

In strong reverse bias:

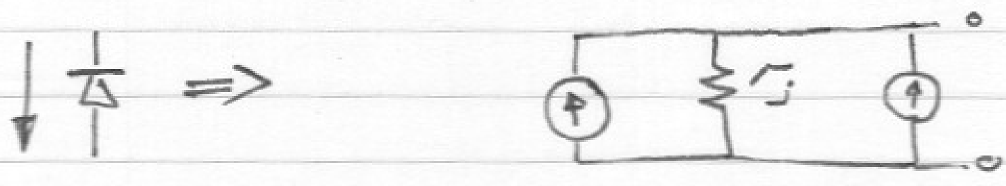
$$\frac{\partial}{\partial f} \langle I_n I_n^* \rangle \simeq 2q I_0 \cong 2q I_{dc}$$

but $g = \frac{\partial}{\partial V}\left[ I_0 e^{qV/kT} - I_0 \right] = \frac{1}{r_j}$

$$= \frac{q I_0}{kT} e^{qV/kT}$$

so the model is still:

$I_{dc}$ reverse current.




$$\frac{\partial}{\partial f} \langle I_n I_n^* \rangle = 2q I_{dc}$$

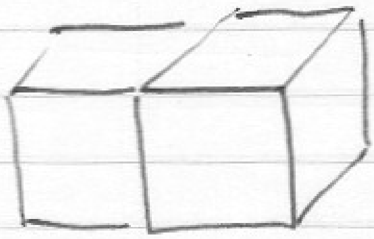
because $r_j$ is now very large, the available noise power is now $\gg kT \cdot \Delta f$.

Diffusion Noise after Van Der Ziel.

Ironically, the preceding analysis did not apply (at least not proven) for a p-N junction diode. We need to work the diffusion problem.

Consider a lump of semiconductor divided into small boxes $\Delta x \Delta y \Delta z$. Index them by $(k, l, m)$

We could work out the fluctuations in electron flux between boxes from the thermal velocity distribution we had derived earlier, but that is not really necessary.



2 boxes $(k, l, m)$ & $(k+1, l, m)$

have electron concentrations $n(k, l, m)$ & $n(k+1, l, m)$

The flux of electrons ~~#~~ from $1 \rightarrow 2$ is:

$$W_{k \rightarrow k+1} = a\, n(k, l, m) \cdot \Delta x \cdot \Delta y \cdot \Delta z$$

The flux from $2 \rightarrow 1$ is

$$W_{k+1 \rightarrow k} = a\, n(k+1, l, m) \cdot \Delta x \cdot \Delta y \cdot \Delta z$$

Both these fluxes have full shot noise.

The net flux is

$$W = -W_{k \rightarrow k+1} + W_{k+1, k}$$

$$W = -a \cdot \frac{\partial n}{\partial x} \cdot (\Delta x)^2 \cdot \Delta y \cdot \Delta z$$

From this we can relate $\underline{a}$ to the diffusion coefficient:

$$w = -D_n \frac{\partial n}{\partial x} \cdot (\Delta y \Delta z)$$

$$\Rightarrow \quad a = \frac{D_n}{(\Delta x)^2}$$

Hence the fluxes are

$$w_{k \to k+1} = D_n \, n(k, l, m) \, \frac{\Delta y \cdot \Delta z}{\Delta x}$$

... and similarly for $w_{k+1 \to k}$

the net flux $w$ from $1 \to 2$ therefore has shot noise with spectral density:

$$\frac{\partial}{\partial f} \langle w w^* \rangle = 4 D_n \cdot n \cdot \frac{\Delta y \cdot \Delta z}{\Delta x}$$

The current $I$ between boxes $1 \to 2$ is $I = q w$,

so the diffusion has shot noise:

$$\frac{\partial}{\partial f} \langle I I^* \rangle = 4 q^2 \cdot D_n \cdot n(x) \, \frac{\Delta y \cdot \Delta z}{\Delta x}$$

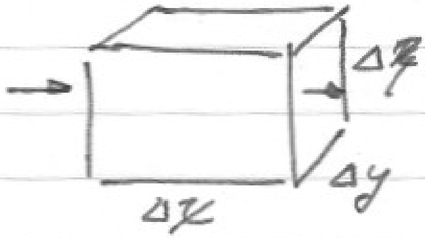
Since $\frac{D}{\mu} = \frac{kT}{q}$ near equilibrium

$$\boxed{\frac{\partial}{\partial f}\langle I I^{*}\rangle = 4 \cdot k \cdot T \left[ q \mu_n \cdot n(x) \right] \frac{\Delta y \cdot \Delta z}{\Delta x}}$$

so far we have not said if the carriers we are treating are majority or minority carriers. Lets suppose they are majority carriers, and that minority carriers are negligible.

In this case $\Delta R = \frac{1}{q \mu_n n(x)} \cdot \frac{\Delta x}{\Delta y \cdot \Delta z}$

is the resistance

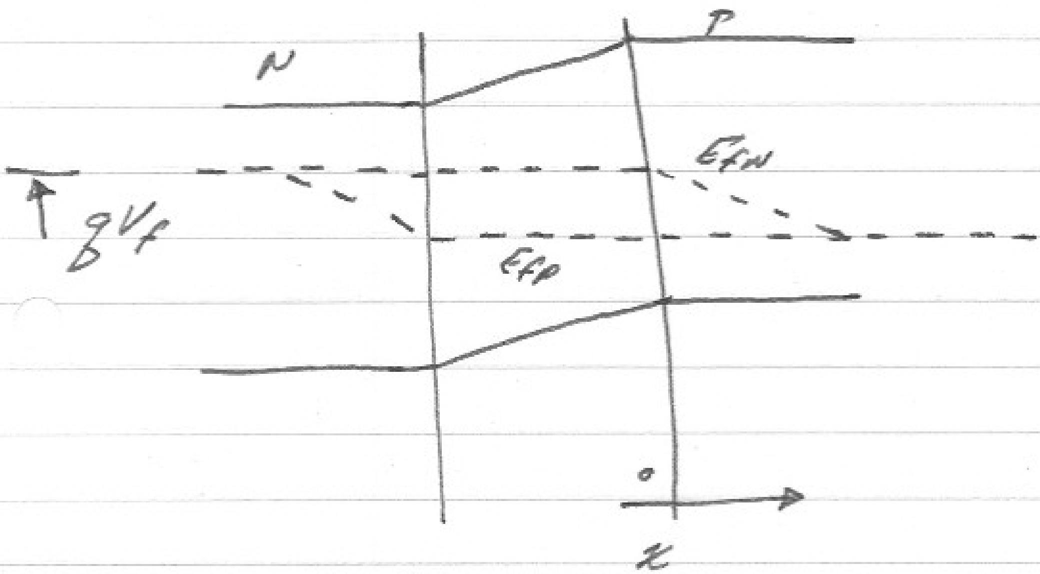



and

$$\frac{\partial}{\partial f}\langle I I_n^{*}\rangle = \frac{4kT}{\Delta R} \quad \text{as expected.}$$

Now we can compute shot noise in p-N junctions and bipolar transistors:

Recall the law of the junction:




This states (assumes) that the carriers are in equilibrium across the depletion region & therefore:

$$\underbrace{n_p(0)}_{} = n_{po} \cdot e^{qV_f/kT}$$

minority carrier concentration at depletion edge

$n_{po}$ = equilibrium minority carrier concentration.

Consider an $N^+$-$p^-$ diode, so that we can concentrate on one carrier species (electrons).

In the quasi-neutral $p$-region.

$$n' = n(x) - n_{p0}$$

$$\begin{cases} i_n = q D_n \cdot A \cdot \dfrac{\partial n'}{\partial x} \\ \dfrac{\partial n'}{\partial t} = -\dfrac{n'}{\tau_r} - \dfrac{1}{A}\dfrac{1}{q}\dfrac{\partial i_n}{\partial x} \end{cases}$$

$A$ is the junction area, $\tau_r$ is the recombination lifetime.

We must now add <u>noise</u> <u>terms</u>.

Generation -
Recombination Noise

In a volume $\Delta V = A \cdot \Delta x$ an electron current disappears by recombination:

$$I_r = \frac{q \cdot n \cdot \Delta V}{\tau_R}$$

...And an electron current appears by ~~recombination~~ generation.

$$I_G = \frac{q \cdot n_{p0} \Delta V}{\tau_R}$$

Both of these have full shot noise.

So there is a net noise current of

$$\frac{\partial}{\partial f}\langle I_{GR} I_{GR}^{*} \rangle = 2q \left[ \frac{n + n_{p0}}{\tau_n} \right] \cdot A \, \Delta x$$

$$= 2q \left[ \frac{n' + 2n_{p0}}{\tau_n} \right] \cdot A \, \Delta x$$

Diffusion Noise

Earlier we had shown that diffusion has a noise current

$$\frac{\partial}{\partial f}\langle I_d I_d^* \rangle = 4 q^2 \cdot D_n \cdot n \cdot \overbrace{\Delta y \cdot \Delta z}^{A} \frac{1}{\Delta x}$$

This can be equivalently represented as an electron density fluctuation as thus:

$$\frac{\partial}{\partial f}\langle (\Delta n)(\Delta n)^* \rangle = \frac{4 n}{D_n A} \Delta x$$

(justification)

$$\left\{ \begin{aligned} &= \left\| \left( \frac{\Delta n}{I_d} \right) \right\|^2 \cdot \frac{\partial}{\partial f} \cdot \langle I_d \cdot I_d^* \rangle \\ &= \left[ \frac{\Delta n}{q D_n \frac{\Delta n}{\Delta x} \cdot A} \right]^2 \cdot 4 q^2 D_n \frac{n A}{\Delta x} \end{aligned} \right\}$$

The noise-free equations can be written:

$$\begin{cases} \dfrac{\partial n'}{\partial x} = \dfrac{1}{q A D_n} i_n \\ \\ \dfrac{\partial i_n}{\partial x} = -\dfrac{q A}{\tau_n} \cdot n' - q A \dfrac{\partial n'}{\partial t} \end{cases}$$

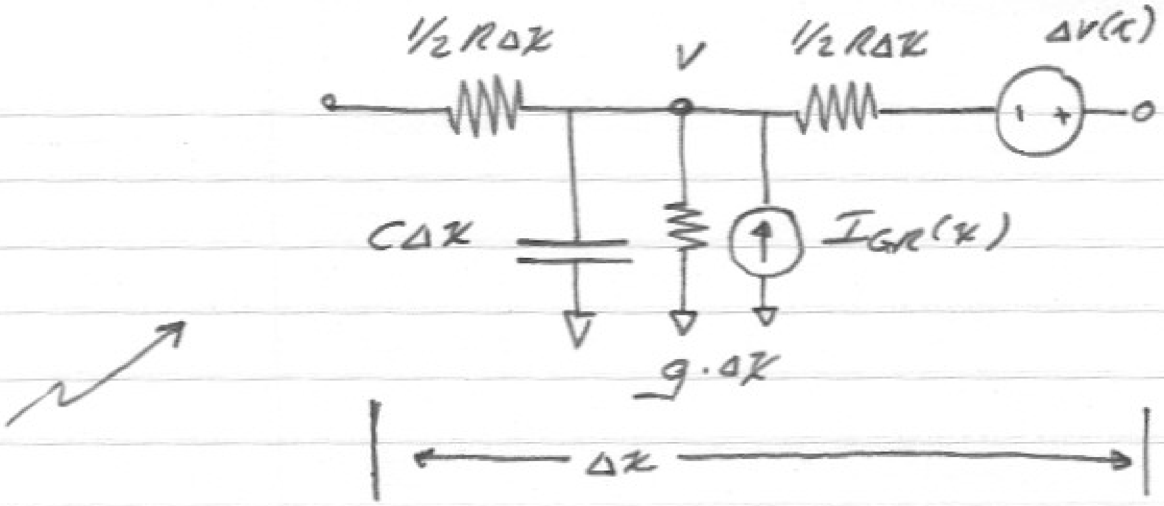
to which we can now add noise terms:

$$\frac{\partial n'}{\partial x} = \frac{1}{q A D_n} \cdot i_n + \Delta n(x)$$

$$\frac{\partial i_n}{\partial x} = -\frac{q A}{\tau_n} n' - q A \frac{\partial n'}{\partial t} + I_{GR}(x)$$

the last 2 terms have the spectral densities

we have earlier computed...




This is an analog of the equations if we set:

| real | model |
| --- | --- |
| $n$ | $V$ |
| $I_n$ | $I$ |
| $(qAD_n)^{-1}$ | $R$ |
| $qA/\tau_n$ | $g$ |
| $qA$ | $C$ |

This may help us in seeing how to solve the differential equations...

The transmission line has

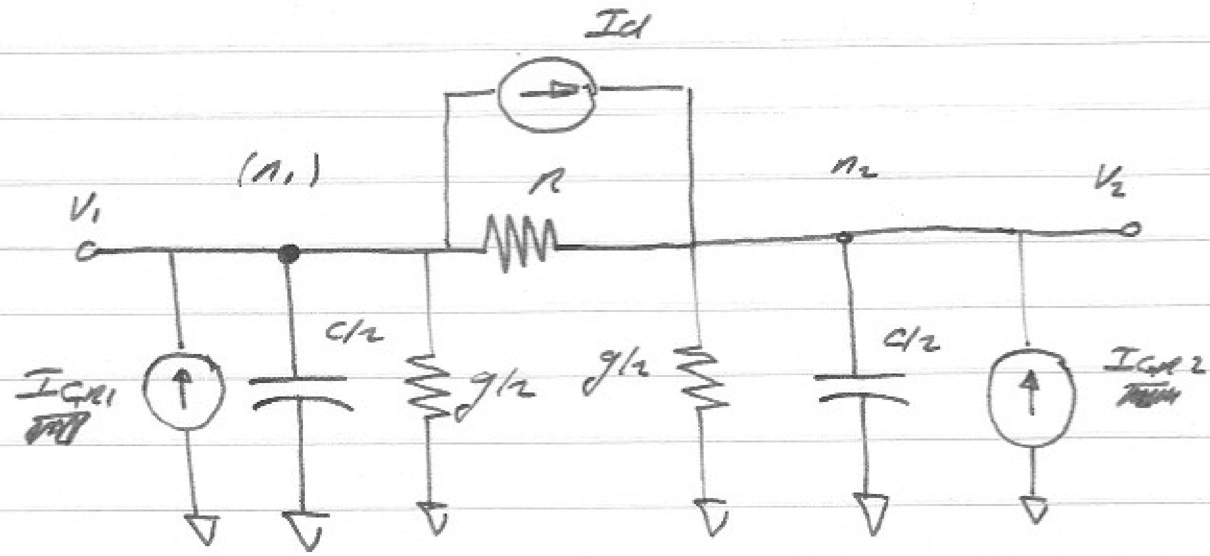
Impedance

$$Z_0 = \left( \frac{R}{G + j\omega C} \right)^{1/2} = \frac{\tau^{1/2}}{qA\sqrt{D_n(1 + j\omega\tau)}}$$

and ~~velocity~~ propagation constant $(e^{\pm\gamma z})$

$$\gamma = R^{1/2}(G + j\omega C)^{1/2} = \frac{(1 + j\omega\tau)^{1/2}}{(D_n\tau)^{1/2}}$$

Note, we can also model an incremental width of the base thus:




$G \Rightarrow \quad qA\,\Delta x / \tau$

$C \Rightarrow \quad q \cdot A \cdot \Delta x$

$R \Rightarrow \quad (1 / q\mu_n A) \cdot \Delta x$

$I \Rightarrow \quad I_n$

$V \Rightarrow \quad n' = n - n_{po}$

$I_{gR1,2}$: spectral density $= 2q \dfrac{(n' + 2n_{po})}{\tau} \left(\dfrac{A \cdot \Delta x}{2}\right)$

Id: spectral density

Again, between points 1 & 2 having electron densities $n_1$ & $n_2$ the electron current ~~flow~~ from $1 \to 2$ is

$$I_{1-2} = q\frac{A D_n}{\Delta x} n_1$$

while the current from 2-1 is

$$I_{2-1} = q\frac{A D_n}{\Delta x} n_2$$

The total current is

$$I = I_{12} - I_{21} = q A D_n \frac{n_1 - n_2}{\Delta x}$$

but as both currents have full shot noise:

$$\frac{\partial}{\partial t}\langle I_d I_d^* \rangle = 2q I_{12} + 2q I_{21}$$

$$= 2q^2 \frac{A D_n}{\Delta x}(n_1 + n_2)$$

$$\frac{\partial}{\partial t} \langle I_d I_d^* \rangle = \frac{2q^2 A D_n}{\Delta x} (n_1' + n_2' + 2n_{po})$$

where, as before $n_1' = n_1 - n_{po}$
$n_2' = n_2 - n_{po}$

and for the g-R current, the fluxes (currents) are:

$$I_R = q \cdot \frac{A \cdot (\Delta x/2) n_1}{\tau}$$

$$I_G = q \cdot \frac{A \cdot (\Delta x/2) n_{po}}{\tau}$$

$\left\{ \begin{array}{l} (\Delta x/2) \text{ because we} \\ \text{are separating into} \\ \text{2 equal pieces.} \\ \text{of length } (\Delta x/2) \end{array} \right\}$

so that the net recombination current is

$$I_G - I_R = qA \frac{(\Delta x/2)}{\tau} (n_1 - n_{po})$$

but each one of these currents has full shot noise, so

$$\frac{\partial}{\partial t} \langle I_{gr} I_{gr}^* \rangle = 2q^2 \frac{A(\Delta x/2)}{\tau} (n_1 + n_{po})$$

$$= 2q^2 \frac{A(\Delta x/2)}{\tau} (n_1' + 2n_{po})$$

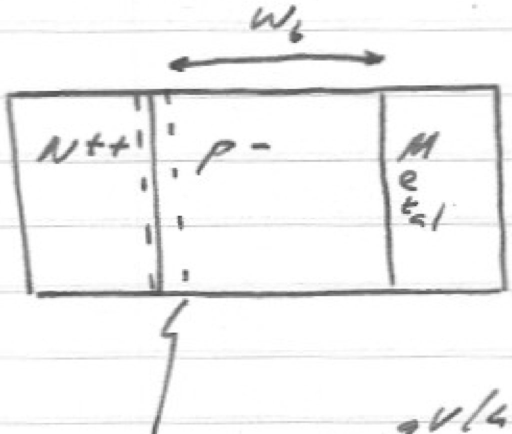
where again $n_1' = n_1 - n_{po}$

now we should be able to do a

clean analysis of pn diode and Bipolar

transistor noise.

<u>short-base diode:</u>




$n'(0) = n_{po}(e^{qV/kT} - 1)$ at depletion edge.

$n'(W_b) = 0$ because we will assume

$\infty$ recombination velocity at metal contact.

<u>Note</u> If $V = V_o + \delta V$ (bias + small signal)

then $n' = n_o' + \delta n'$

where $n_o' = n_{po}(e^{qV/kT} - 1) \cong n_{po} e^{qV/kT}$

& $\delta n' = n_o' \frac{q}{kT} \cdot \delta V$

Note,

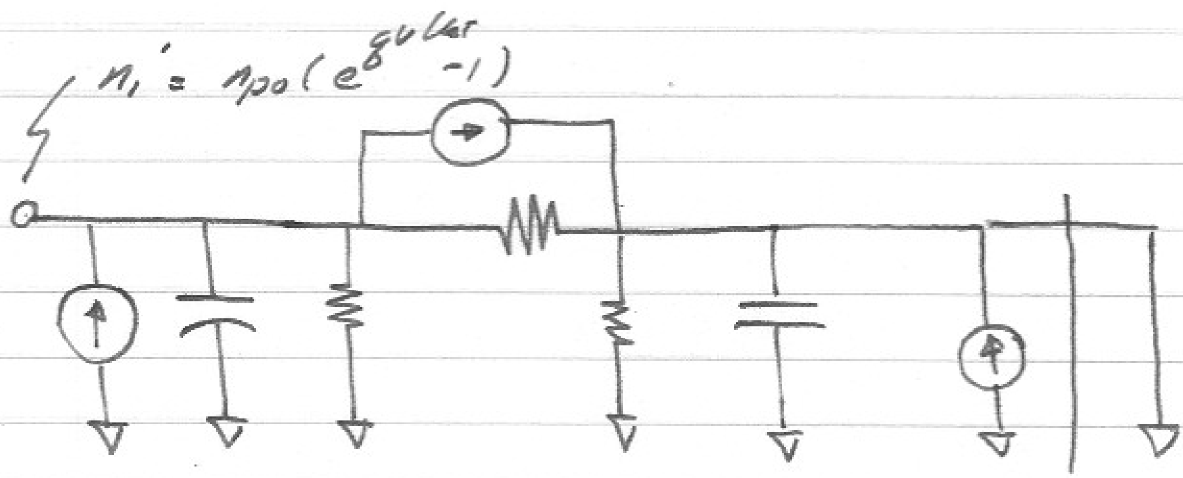
* Before, with the schottky diode, we were able to build a noise model which applied over all biases, whether forward bias, zero bias, or reverse bias.

* This required not approximating $(e^{qV/kT} - 1)$ by $e^{qV/kT}$.

* It is convenient in the p-n junction analysis to make this approximation

* We will therefore end up with models applicable in strong forward bias only.

again, with the short-base diode, we can model the diffusion region with a single lump, thus:




... with values as given on page 36.

First lets calculate the small-signal diode parameters ...

"admittance:"

$$\cancel{\partial f} \quad \text{"}Y\text{"} \triangleq \frac{\partial i}{\partial n} = \frac{G W_b}{2} + \frac{1}{R_s} + \frac{j\omega C W_b}{2}$$

$$\text{"}Y\text{"} = \frac{q \cdot A \cdot W_b}{2\tau} + \cancel{W} \frac{q D_n \cdot A}{W_b} + \frac{j\omega\, q A W_b}{2}$$

$$Y = \frac{\partial I}{\partial V} = \frac{\partial I}{\partial n} \cdot \frac{\partial n}{\partial V}$$

$$= \frac{\partial I}{\partial n} \frac{q}{kT} \cdot n_0' = \frac{\partial I}{\partial n} \cdot \frac{q}{kT} n_{po} (e^{qV/kT} - 1)$$

$$= \frac{\partial I}{\partial n} \cdot \frac{q}{kT} \cdot \frac{I_{DC}}{\left[ \left. \frac{\partial I}{\partial n} \right|_{DC} \right]}$$

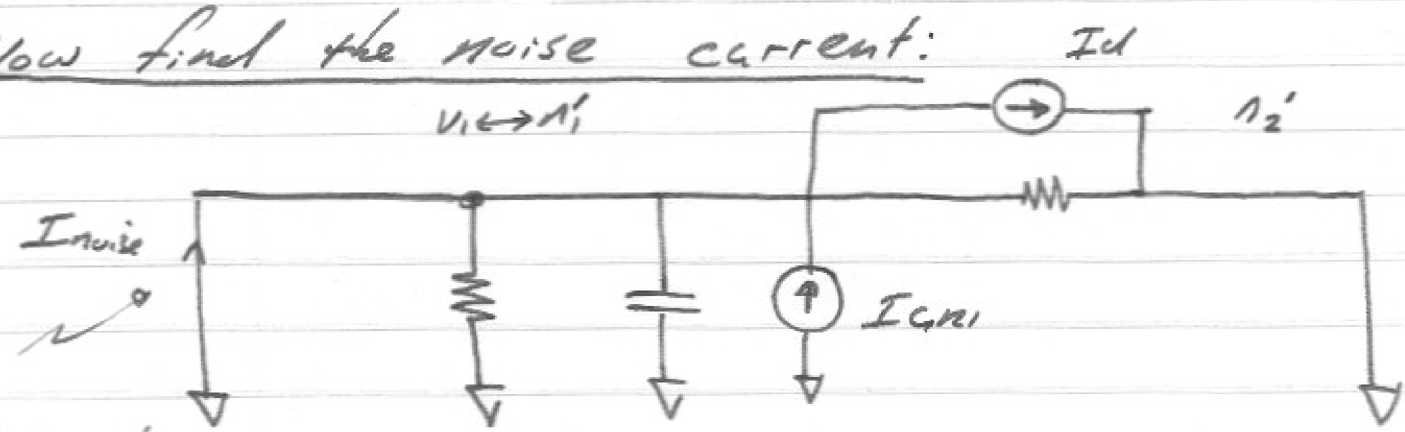
$$Y = \frac{q I_{DC}}{kT} \cdot \frac{\frac{qAW_b}{2\tau} + q\frac{AD_n}{W_b} + j\omega q\frac{AW_b}{2}}{\frac{qAW_b}{2\tau} + qAD_n \cdot W_b^{-1}}$$

still too complicated: drop $qAW_b/2\tau$ from eq.

$$\boxed{Y = \frac{q I_{DC}}{kT} \cdot \left[1 + j\omega \frac{W_B^2}{2D_n}\right]}$$

The second term is the familiar diffusion capacitance of a diode.

Now find the noise current:




ac short circuit, because at fixed dc bias, $n_1'$ is fixed.

The noise current is the sum of $I_{GR1}$ & Id.

So the spectral density of Inoise is

$$S_{I_{Noise}}(f) = 2q^2 \frac{A D_n}{\underbrace{\Delta x}_{W_b}} (n_1' + n_2' + 2n_{p0})$$

single-sided

$$+ 2q^2 \frac{A (W_b/2)}{\tau} (n_1' + 2n_{p0})$$

note $n_2' = 0$

note $n_1' >> n_{p0}$ in forward bias.

$$\frac{d}{df}\langle I_{noise} I_{noise}^{*}\rangle \cong \left(2q^2\frac{A D_n}{W_b}\cdot n_1\right) + \left(2q^2\frac{A W_b}{2\tau}\cdot n_1\right)$$

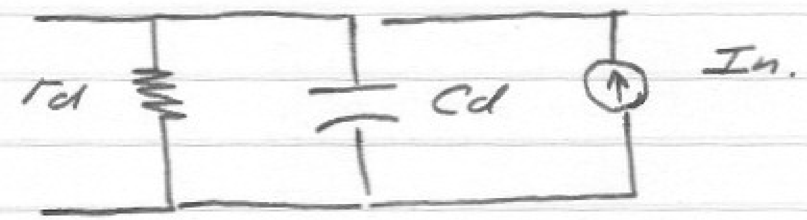
but the DC current is simply:

$$I_{dc} = q\frac{A D_n}{W_b}\cdot n_1 + qA\left(\frac{W_b}{2}\right)\frac{1}{\tau}\cdot n_1$$

so

$$\boxed{\frac{d}{df}\langle I_{noise} I_{noise}^{*}\rangle = 2q I_{dc}}$$

so our small-signal diode model is:




$$r_d = \frac{kT}{q I_{dc}}$$

$$C_d = \frac{W_b^2}{2 D_n} \cdot \frac{1}{r_d}$$

$$\frac{d}{df} \langle I_n I_n^* \rangle = 2q I_{dc}.$$

Note, we have throughout replaced

$e^{qV/kT} - 1$ with $e^{qV/kT}$

$n' + n_{po}$ with $n'$

etc.

If we had not made this approximation,

then the diode I-V curve would have been

$$I_{dc} = I_o \left( e^{qV/kT} - 1 \right)$$

rather than

$$I_{dc} \cong I_o \left( e^{qV/kT} \right)$$

and the shot noise would have been:

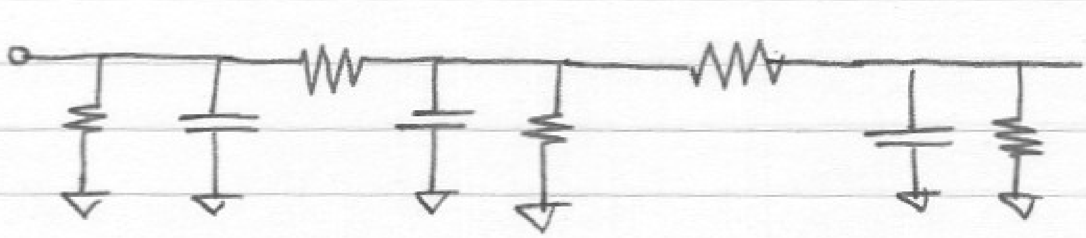
$$\frac{d}{dt} \langle I_{noise} I_{noise}^{*} \rangle = 2q \left( I_{dc} + 2 I_o \right)$$

where $I_{dc}$ is the forward current.

... I will assign this as homework...

Second note

If the diode is longer... then the diffusion must be modelled by more lumps in our finite-element analysis:



the diodes' admittance will become more complex:

$$Y(\omega) = g(\omega) + jX(\omega) \neq g_0 + j\omega C$$

$$\uparrow g_0 = g(\omega = 0)$$

Van Der Ziel is able to prove that the shot noise is

$$\left[ \frac{d}{df} \langle I_n I_n^* \rangle = 2q(I + 2I_0) + 4kT(g(f) - g_0) \right.$$
$$\left. = 4kT\, g(f) - 2q\, I_{dc} \right]$$

... this is a long and very tedious proof...