Mixed-Signal IC Design Notes set 2:
Fundamentals of Analog IC Design

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This note set

- reviews the basics
- starts at the level of a first IC design course
- moves very quickly

This will

- establish a common terminology
- accommodate capable students having minimal background in ICs.
DC models
DC bias analysis
Provided that $V_{ce} > 0$,

$$I_c = I_s \exp\left(\frac{V_{be}}{V_T}\right) \text{ and } I_b = I_c / \beta, \text{ where } V_T = kT / q$$

...note that $V_{be}$ is specified internal to the emitter resistance $R_{ex}$

The $I_e R_{ex}$ drop is significant for HBTs operating at current densities near that required for peak transistor bandwidth.
We have \( V_{be1} + I_{el} (R_{ex1} + R_{ee1}) = V_{be2} + I_{el} (R_{ex2} + R_{ee2}) \)
and \( V_{be1} = V_t \ln(I_{c1} / I_{s1}) \), \( V_{be2} = V_t \ln(I_{c2} / I_{s2}) \)

Assume that \( \beta >> 1 \), \( R_{ee2} = 2R_{ee1} \)
& assume that \( A_{E1} = A_{E2} \) (\( A_E \) is the emitter area).

This implies \( R_{ex1} = R_{ex2} / 2 \), and \( I_{s1} = 2I_{s2} \),
from which we find \( I_{c2} = I_{c1} / 2 \)
Simpler DC Model for Bias Analysis

It is often sufficient in bias analysis to ignore the variation of $V_{be}$ with $I_c$ and instead take $V_{be} = V_{be, on} = \phi$.

$V_{be, on}$ depends upon current density and technology.

Biased at current densities within $\sim 10\%$ of peak bandwidth bias,

$$V_{be, on} = \phi \sim \begin{cases} 0.9 \text{ V Modern Si/SiGe HBTs} \\ 0.7 \text{ or } 0.9 \text{ V InGaAs/InP HBTs} \\ 1.4 \text{ V GaAs/GaInP HBTs} \end{cases}$$
If we neglect the $I_b R_b$ drops, then $V_{b1} = V_{b2} = 0$ Volts.

Approximate $V_{e1} = V_{e2} = -\phi \approx -0.9$ V (SiGe).

$I_{c1} + I_{c2} = 2I_{c1} = (-V_{ee} - 0.9V) / R_{ee}$

$I_{c1} = I_{c2} = (-V_{ee} - 0.9V) / 2R_{ee}$
If $I_b R_b$ drop is significant, one can solve simultaneous equations:

$$I_{c1} + I_{c2} = 2I_{c1} = (-V_{ee} - \phi - I_{b1} R_b) / R_{ee}$$

where $I_{b1} = I_{c1} / \beta$,

Quicker: find by iteration:

1) solve $I_{c1} = (-V_{ee} - \phi) / 2R_{ee}$
2) solve $I_{b1} \approx I_{c1} / \beta$
3) use this value of $I_b$ to solve $I_{c1} = (-V_{ee} - \phi - I_{b1} R_b) / 2R_{ee}$

Works because any well-designed circuit has DC bias only weakly dependent upon $\beta$. 
Typical Circuit Biasing in Mixed-Signal ICs

**Biasing with current mirrors**
- more precise---
- often more compact
- but loads emitter node with $C_{cb}$ --- or worse
- and provides path for stage-stage coupling

**Resistive biasing**--- resistively loads emitter node---lower DC precision
Problems with Current Mirror Biasing

stage-stage coupling through $C_{cb}$ and the shared current mirror reference...
Problems with Current Mirror Biasing

Because of $C_{cb}$ and $C_{be}$, the current source output impedance $Z_{cs}(j\omega)$ varies with frequency.

We will derive this in later lectures.

Implications:
increased common-mode gain at high frequencies
common-mode instability
Avoiding Thermal Instability

Given that $V_{be} = V_t \ln(I_{c2} / I_{s2}) = (kT / q) \ln(I_{c2} / I_{s2})$

....note that $I_s \propto T^{3/2}$, so at constant $I_c$,

$$dV_{be} / dT = (k / q) \ln(I_{c2} / I_{s2}) - (kT / q)(dI_{s2} /dT)^{-1}$$

The device has some thermal resistance $dT / dP = \theta$ (Kelvin/Watt). Current mirror is thermally unstable if

$$1 < \frac{dI_c}{dV_{be} \cdot dT \cdot dP \cdot dI_c}$$

$$1 < \frac{1}{R_{ex} + kT / qI_e + R_{ee}} \cdot \frac{dV_{be} \cdot \theta \cdot V_{ce}}{dT}$$

Fast Bipolar ICs normally use $I_e * R_{ee} \sim 300mV$ to avoid thermal runaway. Thermal runaway can also result from internal device temperature gradients. Large area devices must therefore be avoided...use array of smaller devices with ballasting. (process dependent)...UCSB HBT process puts safe maximum device area $\sim 10$ square microns at $10^5 A / cm^2$ biasing
small-signal baseband analysis
Hybrid-π Bipolar Transistor Model

\[ R_{be} = \beta / g_m \]
\[ \tau_f = \tau_b + \tau_c \]

Accurate model, but too detailed for quick hand analysis
In most high-frequency circuits, the node impedance is low and $R_{ce}$ is therefore negligible.

Neglecting $R_{bb}$ in high-frequency analysis is a poor approximation but is nevertheless common in introductory treatments.

The "textbook" analyses which follow use this oversimplified model. These introductory treatments will later be refined.
Common Emitter Stage: Basics

7) $R_{in,T} = \frac{\delta V_b}{\delta I_b} = \beta(r_e + R_e)$

2) $\delta V_C = -R_{Leq} \cdot \delta I_C$

5) $\delta I_b = \delta I_c / \beta$

4) $\delta V_{be} = \delta I_e / g_m$

3) $\delta V_e = R_E \cdot \delta I_e$

6) $\delta V_b = \delta I_e \cdot (r_e + R_e) = \delta I_b \cdot \beta(r_e + R_e)$

8) $\frac{\delta V_{out}}{\delta V_{in}} = \frac{\delta V_{out}}{\delta V_B} = -R_{Leq} / (r_e + R_e) = -R_{Leq} / (R_e + 1 / g_m)$

Gain is $-R_{Leq} / (R_e + 1 / g_m)$; Transistor $R_{in}$ is $\beta(r_e + R_E)$
6) \( R_{in,T} = \frac{\delta V_b}{\delta I_b} = \beta(r_e + R_{Leq}) \)

4) \( \delta I_b = \frac{\delta I_c}{\beta} \)

3) \( \delta V_{be} = \frac{\delta I_e}{g_{m-}} \)

2) \( \delta V_e = R_{Leq} \cdot \delta I_e \)

5) \( \delta V_b = \delta I_e \cdot (r_e + R_{Leq}) = \delta I_b \cdot \beta(r_e + R_{Leq}) \)

7) \( \frac{\delta V_{out}}{\delta V_{in}} = \frac{\delta V_{out}}{\delta V_E} = \frac{R_{Leq}}{(r_e + R_{Leq})} = \frac{R_{Leq}}{(R_{Leq} + 1/g_m)} \)

Gain is \( R_{Leq}/(R_{Leq} + 1/g_m) \); Transistor \( R_{in} \) is \( \beta(r_e + R_E) \)
Common-Base Stage: Basics

6) $\delta V_{in} = \delta I_e \cdot (r_e + R_b / \beta)$

7) $R_{in,T} = \delta V_e / \delta I_e = r_e + R_B / \beta$

5) $\delta V_{be} = \delta I_e / g_m$

4) $\delta V_b = \delta I_c \cdot R_b / \beta$

1) $\delta I_e \approx \delta I_c$

2) $\delta V_{out} = R_{Leq} \cdot \delta I_c$

3) $\delta I_b = \delta I_c / \beta$

7) $\delta V_{out} / \delta V_{in} = R_{Leq} / (r_e + R_b / \beta)$

Gain is $R_{Leq} / (r_e + R_b / \beta)$; Transistor $R_{in}$ is $r_e + R_b / \beta$
Emitter Follower Output Impedance

E.F. output impedance is same problem as C.B. input impedance

\[ R_{out, \text{emitter}} = r_e + R_B / \beta = 1 / g_m + R_B / \beta \]

\[ R_{out, \text{amp}} = R_{out, \text{emitter}} || R_{EE} \]
Including Bias Circuit Resistances

These are (trivially) added in parallel with the transistor terminal impedances to determine the net circuit impedances.

From which, \( \frac{V_{in}}{V_{gen}} = \frac{R_{in,amp}}{(R_{in,amp} + R_{gen})} \), etc.
Baseband Analysis Of Multistage Circuits

For baseband analysis of multi-stage circuits, simply break into individual stages.

Load impedance of the Nth stage includes the input impedance of the \((N+1)^{th}\) stage

Analysis is then trivial...
small-signal baseband analysis
Analyzing frequency response is difficult: cannot separate stage-by-stage

Method #1: nodal analysis: accurate, general, tedious.

Method #2: method of time constants: accurate, limited applicability, quick & intuitive
Nodal Analysis
Simple & very familiar example: common-emitter amplifier.
Reduced circuit:

\[ I_i = \frac{V_{\text{gen}}}{R_{\text{gen}}} C_{be} R_i \quad g_m V_{be} C_L R_L \]

Step 1: Write Nodal Equations from KCL

\[
\begin{bmatrix}
G_i + sC_{be} + sC_{cb} & -sC_{cb} \\
g_m - sC_{cb} & G_L + sC_L + sC_{cb}
\end{bmatrix}
\begin{bmatrix}
V_{\text{in}} \\
V_{\text{out}}
\end{bmatrix}
= \begin{bmatrix}
I_i \\
0
\end{bmatrix}
\]

\[ (R_i = R_{\text{gen}} \parallel R_{be} \parallel R_b) \]
Tutorial: Transfer Function Analysis: Nodal Analysis III

Step 2: Solve Nodal Equations:

\[ \frac{V_{out}}{I_{in}} = \frac{N(s)}{D(s)} \]

\[ N(s) = \begin{vmatrix} G_i + sC_{be} + sC_{cb} & 1 \\ g_m - sC_{cb} & 0 \end{vmatrix} = -(g_m - sC_{cb}) \]

\[ D(s) = \begin{vmatrix} G_i + sC_{be} + sC_{cb} & -sC_{cb} \\ g_m - sC_{cb} & G_L + sC_L + sC_{cb} \end{vmatrix} \]

\[ D(s) = (G_i + sC_{be} + sC_{cb})(G_L + sC_L + sC_{cb}) - (g_m - sC_{cb})(-sC_{cb}) \]

Step 3: Organize in powers of \( s \)

\[ D(s) = G_i G_L \]

\[ + s(G_i C_L + G_i C_{cb} + G_L C_{be} + G_L C_{cb} + g_m C_{cb}) \]

\[ + s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L + C_{cb} C_{cb} - C_{cb} C_{cb}) \]
Step 4: Separate into dimensionless ratio-of-polynomials form, separating constants and gains from the transfer function...

\[
\frac{V_{out}}{I_{in}} = \frac{V_{out}}{V_{gen} / R_{gen}} = \frac{N(s)}{D(s)} = \frac{-(g_m - sC_{cb})}{G_i G_L + s(G_i C_L + G_i C_{cb} + G_L C_{be} + G_L C_{cb} + g_m C_{cb}) + s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L)}
\]

\[
\frac{V_{out}}{V_{gen}} = \frac{-(g_m - sC_{cb})R_i R_L / R_{gen}}{1 + s(R_L C_L + R_L C_{cb} + R_i C_{be} + R_i C_{cb} + g_m R_i R_L C_{cb}) + s^2(C_{be} C_L + C_{be} C_{cb} + C_{cb} C_L)R_i R_L}
\]
note that \[
\frac{R_i}{R_{gen}} = \frac{(R_{be} \parallel R_b \parallel R_{gen})}{R_{gen}} = \frac{(R_{be} \parallel R_b)}{(R_{be} \parallel R_b) + R_{gen}} = \frac{R_{in,Amp}}{R_{in,Amp} + R_{gen}}
\]

so...

\[
\frac{V_{out}}{V_{gen}} = \left(\frac{R_{in,Amp}}{R_{in,Amp} + R_{gen}}\right) \left(-g_m R_L\right) \cdot \left(1 - \frac{sC_{cb}}{g_m}\right) \times \left(1 + s\left(R_L C_L + R_L C_{cb} + R_i C_{be} + R_i C_{cb} + g_m R_i R_L C_{cb}\right)\right)
\]

\[
= \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}
\]

\[
\Rightarrow \frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen}} \bigg|_{mid-band} = \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}
\]
Step 5: Find the roots (poles & zeros) of the polynomial

\[
\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen}} \mid_{\text{mid-band}} \frac{1 + b_1s}{1 + a_1s + a_2s^2} = \frac{V_{out}}{V_{gen}} \mid_{\text{mid-band}} \frac{1 + b_1s}{(1 - s / s_{p1})(1 - s / s_{p2})}
\]

What are efficient methods of finding the poles?
Finding Poles from Transfer Functions
Finding Poles and Zeros

Ratio - of - Polynomial Form :

\[
\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen}} \quad \star \quad S^m \quad \frac{1 + b_1 s + b_2 s^2 + ...}{1 + a_1 s + a_2 s^2 + ...}
\]

at mid-band

Poles and Zeros :

\[
\frac{V_{out}(s)}{V_{gen}(s)} = \frac{V_{out}}{V_{gen}} \quad \star \quad S^m \quad \frac{(1 - s / s_{z1})(1 - s / s_{z1})(1 - s / s_{z1})...}{(1 - s / s_{p1})(1 - s / s_{p1})(1 - s / s_{p1})...}
\]

at mid-band
Finding Poles: Complex Poles

\[
\frac{V_{out}}{V_{gen}} = k \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2 + a_3 s^3}
\]

If \( a_3 / a_2 \ll a_2 \) then we can ignore the \( s^3 \) at moderate frequencies and

\[
\frac{V_{out}}{V_{gen}} \approx k \left( \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2} \right)
\]

If the roots of this are complex, then

\[
\frac{V_{out}}{V_{gen}} = k \left( \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2} \right) = k \left( \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + (2 \zeta / \omega_n) s + s^2 / \omega_n^2} \right)
\]

\[
\frac{V_{out}}{V_{gen}} = k \left( \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 - \frac{s}{\zeta \omega_n + j \omega_d}} \right) \left( 1 - \frac{s}{\zeta \omega_n - j \omega_d} \right)
\]

\[\omega_d^2 = \omega_n^2 (1 - \zeta^2)\]
Finding Poles: Separated Pole Approximation

If the roots are widely separated
e.g. \((a_2/a_1) << a_1\), then

\[
\frac{V_{out}}{I_{in}} = k \left( \frac{1 + b_1 s + b_2 s^2 + \ldots}{1 + a_1 s + a_2 s^2} \right)
\]

\[
\frac{V_{out}}{I_{in}} \approx k \frac{1 + b_1 s + b_2 s^2 + \ldots}{(1 + a_1 s) \left( 1 + \left( \frac{a_2}{a_1} \right) s \right)}
\]

\(a_1\) is the dominant pole.
Introductory Circuit Design: summary
Gain Stages: Elementary Bandwidth Analysis

Using the oversimplified device model below, with $C_{pi}$ denoting the sum of base-emitter depletion and diffusion capacitances, bandwidth of CE/CB/CC stages can be found....
CE Stage: Elementary Bandwidth Analysis

Ri is the parallel combination of Rgen, Rin, and Rpi

RLeq is the parallel combination of RL, Rc, and Ro

Note in the dominant pole (a1) the miller-multiplication of the collector base capacitance

\[ V_{out}/V_{gen} = \left( V_{out}/V_{gen} \right)_{MB} \frac{1 + s\tau_{zero}}{1 + a_1s + a_2s^2} \]

\[ a_1 = R_i \left( C_\pi + C_\mu \left( 1 + g_mR_{Leq} \right) \right) + R_{Leq} \left( C_\mu + C_L \right) \]

\[ a_2 = R_i R_{Leq} \left( C_\mu C_L + C_\mu C_\pi + C_\pi C_L \right) \]

\[ \tau_{zero} = -C_\mu/g_m \]
CC Stage: Elementary Bandwidth Analysis

\[ \frac{V_{out}}{V_{gen}} = \left( \frac{V_{out}}{V_{gen}} \right)_{MB} \frac{1 + s\tau_{zero}}{1 + a_1 s + a_2 s^2} \]

given that \( A_{ymb} = \left( r_e \left/ \left( r_e + R_{Leq} \right) \right. \right) \):

- \( a_1 = C_\pi \left( R_\pi \left/ \left( r_e \left| R_{Leq} + R_i \left( 1 - A_{ymb} \right) \right. \right. \right) \right) \)
- \( + C_\mu \left( R_i \left| \text{transistor input resistance} \right. \right) \)
- \( + C_L \left( R_{Leq} \left| \text{transistor output resistance} \right. \right) \)

- \( a_2 = \left( R_i \left| \text{transistor input resistance} \right. \right) \left( R_{Leq} \left| r_e \right. \right) \)
- \( \times \left( C_\mu C_\pi + C_\mu C_L + C_L C_\pi \right) \)

- \( \tau_{zero} = g_m / C_\pi \)

\( R_i \) is the parallel combination of \( R_{gen} \), and \( R_{in} \),

\( R_{Leq} \) is the parallel combination of \( R_{ee} \) and \( R_L \)

Note that the frequency response is a mess. Given \( CL \), the transfer function very often has complex poles, and may show strong gain peaking, hence ringing in the pulse response.
CB Stage: Elementary Bandwidth Analysis

Here we have a problem. To the extent that the CB stage is modeled by a very very simple hybrid-pi model (explicitly, with zero Rbb), we find (by very simple analysis) very high bandwidth, with poles having time constants equal to \( \tau_b \), to \( \tau_c \), and to the product of the load resistance times \((Ccb+CL)\).

**Note that**

1) **Input capacitance is indeed as noted. Does not include effect of \( \tau_c \)**

2) **Ignoring Rbb in CB stage analysis, while appealing for simplicity (e.g. undergrad classes) is quite unreasonable, as CcbRbb often dominates high frequency rolloff.** More regarding this later.
Method of
Time Constants
make revision for 2008----first before MOTC, give by summary without derivation the standard stage expressions.

then define MOTC, first and second order
then show a 1-stage Darlington diff amp, and say caps to ground, caps between inputs and outputs.

Give expression for caps to ground
Give expression for caps between in and out of general block

then use this for CD stage  Cgs only
then use this for CC stage Cbe only
then do for CE stage  Ccb only

then work the full Darlington diff amp

then show how CE (with degen) CB CC are same problem

then re-show stage relationship
Finding Bandwidth: Method of Time Constants

take a general RC network (no inductors or delays tau), and separate into 2 parts, network without capacitors, and the capacitors:
The internal capacitor-free network is now frequency-independent. The MOTC method (not proven here) relies on results from n-port network theory.
$R_{11}^0$ is the small signal resistance measured at port one with all other ports open-circuited. This is determined by applying a test voltage (or current) at the port and computing from this the resulting current (or voltage).
MOTC: the Dominant Time Constant

\[
\frac{V_{out}}{V_{gen}} = \left( \frac{V_{out}}{V_{gen}} \right)_{mb} \frac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}
\]

\[a_1 = R_{11}^0 C_1 + R_{22}^0 C_2 + R_{33}^0 C_3 + R_{44}^0 C_4\]

The MOTC first-order time constants directly give us the dominant time constant \(a_1\) of the circuit. If (and only if) the secondary time constant \(a_2\) is negligible, the 3-dB bandwidth is \(1/2\pi a_1\). We must use the second-order (short-circuit) time constants to determine \(a_2\).
MOTC: Are We Saving Any Work?

Are we saving work relative to brute-force nodal analysis: MOTC would be of only moderate value if we had to calculate all the Ri's each time. Fortunately, most terms involve quantities already found in midband stage analysis: input and output impedances, load impedances, etc.

work examples to illustrate this
$R^2_{11}$ is the small signal resistance measured at port one with all other ports open - circuited, except for port 2, which is shorted.
MOTC: The Second-Order Time-Constant

\[
\frac{V_{\text{out}}}{V_{\text{gen}}} = \left( \frac{V_{\text{out}}}{V_{\text{gen}}} \right)_{mb} \frac{1 + b_1 s + b_2 s^2 + \cdots}{1 + a_1 s + a_2 s^2 + \cdots}
\]

\[
a_2 = R^{0}_{11} R^{1}_{22} C_1 C_2 + R^{0}_{11} R^{1}_{33} C_1 C_3 + R^{0}_{11} R^{1}_{44} C_1 C_4 + R^{0}_{22} R^{2}_{33} C_2 C_3 + R^{0}_{22} R^{2}_{44} C_2 C_4 + R^{0}_{33} R^{3}_{44} C_3 C_4
\]

notethat \( R^0_{xx} R^x_{yy} = R^y_{xx} R^0_{yy} \)
MOTC: An Example

work example…second order terms in either CE stage or CC stage....
Because \( R^y_{xx} R^0_{yy} = R^0_{xx} R^x_{yy} \), we always have 2 choices in finding each term in the MOTC. The trick is to work the problem so that as much as possible:

1) terms are related to input, output, load impedances
2) terms are ones found earlier, in \( a_1 \) analysis.

There are 2 "funny" cases which arise so often that I will give them on the next 2 pages (note these are intimately related to the well-known Miller effect)
MOTC and the Miller Effect

\[ R_{xx}^y = R_i (1 + A_v) + R_{out} \]

\[ \alpha_1 = \tau = \left[ R_i (1 + A_v) + R_{out} \right] \bullet C \]
MOTC: port impedances between collector and base

derive

If we decide explicitly that $R_x$ is to denote the parallel combination of any external circuit resistances and $R_{be}$, and that $R_{Leq}$ similarly denotes the combined effect of external resistors and $R_{ce}$, then

$$R_{yy}^0 = R_x (1 + g_m R_{Leq}) + R_{Leq}$$
\[ R_{xx}^0 = R_\pi \left( \frac{r_e}{1 + A_{vmb}} \right) \left( R_{Leq} + R_i \left( 1 - A_{vmb} \right) \right) R_i \]

\[ A_{vmb} = \left( R_{Leq} \right) \left( \frac{1}{r_e + R_{Leq}} \right) \]
MOTC: Multistage Example

work on the board...
Common-base stage by MOTC

\[ a_1 = C_{be} \left( R_{bb} \left( 1 - \frac{R_{gen}}{R_{gen} + r_e} \right) + r_e \parallel r_{gen} \right) r_{be} \]

\[ + C_{cb} \left( R_{bb} \left( 1 + \frac{R_L}{r_e + R_{gen}} \right) + R_L \right) + C_L R_L \]

\[ a_2 = \ldots \]

note that because \( a_1, a_2 \) are independent of location of input and output, the form is identical to that of CE stage. Note also the Miller multiplication effect with \( R_{bb} C_{cb} \)
What's in next lecture?

Relating amplifier gains to S-parameters
Cascaded common-emitter amplifiers,
    gain-bandwidth and ft limits
Resistive feedback amplifiers for higher bandwidths
The Cherry-Hooper (transconductand-transimpedance) design
Darlington stages, benefits, limits, and headaches
Ft-doubler stages
Distributed amplifiers (breifly)

And later: input tuning for (1) improving S11, S22 and (2) improving bandwidth.

.....
End