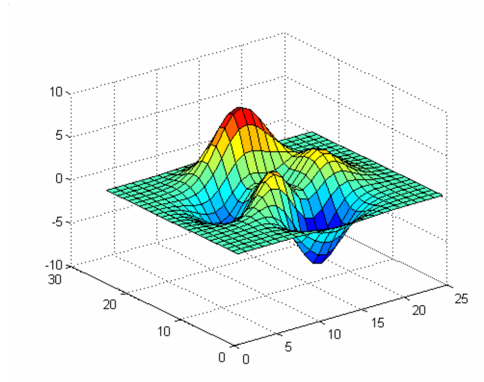
NUMERICAL OPTIMIZATION METHODS
ECE271B — WINTER 2003

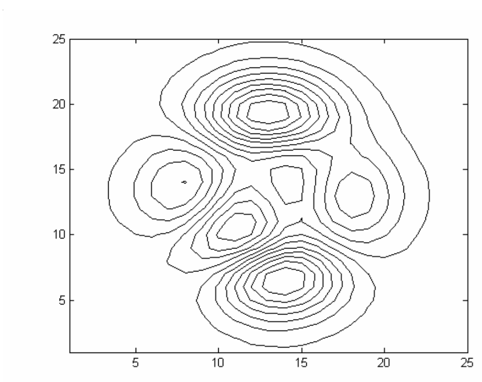
Abstract

Optimization is one of the fundamental tools that every engineer keeps in her "bag-of-tricks." Linear optimization (i.e., minimization/maximization linear functions subject to linear constrains) already reached the point where excellent software packages are available to solve very large problems (with literally millions of variables to be optimized). Unfortunately, most problems are nonlinear.



The purpose of this course is to provide students with the basic tools needed to solve numerically nonlinear optimization problem. The course covers:

- unconstrained optimization
- optimization of convex functions over convex sets,
- optimization of general nonlinear functions under equality and inequality constraints using both Langrage Multiplier theory and Duality theory, and
- an introduction to multi-stage optimization through dynamic programming.



The classes will focus on the computational algorithms and their performance.

The intended audience for this course includes (but is not restricted to) students in circuits, communications, control, and signal processing.

The class will be project oriented (no final!) and the students are strongly encouraged to choose a project that is relevant for their own area of research. Typical projects will consist of implementing a specific optimization algorithms or adapting an existing algorithm to a specific research problem.

A detailed syllabus is available on the web at http://www.ece.ucsb.edu/~hespanha/ece271b/.

Instructor

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Office hours: Please email or phone me in advance to schedule for an appointment. Preferred times are MoWe 2:30-4:00pm

Prerequisites

ECE 210A Matrix Analysis and Computation

Graduate level-matrix theory with introduction to matrix analysis and computations: SVD's, pseudo-inverses, variational characterization of eigenvalues, perturbation theory (e.g., ECE 210A).

Course's Web Page

The syllabus, homework, solutions to homework, and all other information relevant to the course will be continuously posted at the course's web page. The URL is

http://www.ece.ucsb.edu/~hespanha/ece271b/

Textbook

The main textbooks are:

[1] D. Bertsekas. *Nonlinear programming, 2nd ed., Athena Scientific*. Athena Scientific, 1999. ISBN 1-886529-00-0.

[2] D. Bertsekas. *Dynamic programming and optimal control, vol I, Athena Scientific*. Athena Scientific, 1995. ISBN 1-886529-12-4.

[3] *Convex Optimization*, S. Boyd and L. Vandenberghe. A draft of a textbook that will be published in 2003. Available at http://www.stanford.edu/~boyd/cvxbook.html

[4] A. Peressini, F. Sullivan, and J. Uhl, Jr. *The Mathematics of Nonlinear Programming*. Springer, 1988. ISBN 3-540-96614-5

The classes will follow closely chapters 1-5 of Bertsekas' book [1] and selected portions of chapter 1 of Bertsekas book [2] (see syllabus below).

Detailed Syllabus

The following is a tentative schedule for the course. As revisions are needed, they will be posted on the course's web page. The rightmost column of the schedule contains the recommended reading for the topics covered on each class. ***Students are strongly encouraged to read these materials prior to the class.***

Class	Content	Reference
#1 Jan 6	**Introduction and course overview** **Unconstrained optimization via Calculus** 1. Definitions of (unconstrained) local and global minima (and maxima) 2. Necessary conditions: first and second order (Prop 1.1.1) 3. Sufficient condition: second order (Prop 1.1.3) 4. Existence results (Prop A.8)	Sec 1.1
#2 Jan 8	**Computational methods for unconstrained optimization: gradient methods** 1. Gradient method: definition; iterative descent 2. Selection of descent directions: Steepest, Diagonally scaled steepest, Newton's, Modified Newton, Discretized Newton, Gauss-Newton 3. Selection of step-size: (limited) minimization and Armijo rules 4. Termination condition	Sec 1.2
#3 Jan 13	**Convergence of gradient methods** 1. Convergence Theorem: Limit points (not necessarily unique) for Armijo and (limited) minimization (Prop 1.2.1) 2. Capture Theorem (Prop 1.2.5)	Secs 1.2

Class	Content	Reference
#4 Jan 15	**Rate of convergence for gradient methods** 1. Steepest descent with quadratic cost: a. exponential convergence in *x* and $f^*(x)$ (Prop 1.3.1); condition number; b. scaling; c. nonquadratic case 2. Newton-like Methods: superlinear convergence (Prop 1.3.2) 3. Singular and difficult problems	Secs 1.3
Jan 20	*Martin Luther King Day*	
#5 Jan 22	**Conjugate direction methods** 1. Quadratic case 2. Nonquadratic case	Sec 1.6
#6 Jan 27	**Quasi-Newton methods** 1. DFP & BFGS methods 2. Quasi-Newton and descent direction (Prop 1.7.1) 3. Quasi-Newton vs. Newton for quadratic case (Prop 1.7.2) **Nonderivative methods** 1. Coordinate descent (Sec. 1.8.1) 2. Direct Search (Sec. 1.8.2)	Secs 1.7 and 1.8
#7 Jan 29	**Constrained optimization of a convex function over a convex set** 1. Definitions: convex set, convex functiondifferentiable condition; checking convexity 2. Minimization of convex functions: local vs. global minima 3. Optimality conditions (Props. 2.1.1 and 2.1.2) 4. Projection Theorem (Prop 2.1.3)	Sec 2.1
#8 Feb 3	**Computational methods for convex constrained optimization** 1. Feasible direction methods: definition, convergence (Sec. 2.2.1), the conditional gradient method (Sec. 2.2.2) 2. Gradient projection methods (Sec. 2.3)	Secs 2.2 and 2.3
Feb 5	*No lecture*	
#9 Feb 10	**Lagrange multiplier theory: optimization with equality constraints** 1. Necessary conditions (Prop 3.1.1): proofs through penalty (Sec 3.1.1) and elimination approaches (Sec 3.1.2) 2. Lagrangian function (Sec. 3.1.3)	Sec 3.1
#10 Feb 12	**Lagrange multiplier theory: optimization with equality constraints (cont.)** 3. Sufficient conditions (Prop 3.2.1): proof through the augmented Lagrangian (Sec. 3.2.1) and feasible direction (Sec. 3.2.2) approaches. 4. Sensitivity Theorem (Sec 3.2.3): linear constraints and nonlinear case (Prop 3.2.2)	Sec 3.2
Feb 17	*President's Day*	

Class	Content	Reference
#11 Feb 19	**Lagrange multiplier theory: optimization with equality and inequality constraints** 1. Karush-Kuhn-Tucker necessary conditions (Prop 3.3.1): proof through penalty (pp 310-311) and conversion to equality case (Sec 3.3.2). 2. Second order sufficiency conditions (Prop 3.3.2) 3. Sensitivity Theorem (Prop 3.3.3) 4. Nondifferentiable sufficiency conditions (Prop 3.3.4) (*)	Sec 3.3
#12 Feb 24	**Lagrange multipliers algorithms: Augmented Lagrangian Methods** 1. Quadratic penalty function methods (Sec 4.2.1): convergence (Prop 4.2.1), implementation issues. 2. Methods of multipliers (pp. 399-400): convergence rates.	Sec 4.2
#13 Feb 26	**Duality: (nondifferentiable) global optimization** 1. Lagrange multiplier (Sec 5.1.1): definition, necessary and sufficient condition for optimality (Prop 5.1.1) 2. Dual problem (Sec. 5.1.2): definition, concavity (Prop 5.1.2), Weak Duality Theorem (Prop 5.1.3), duality gap (Prop 5.1.4) 3. Optimality conditions (*): primal-dual problem (Prop 5.1.5), Saddle-point Theorem (Prop 5.1.6) 4. Equality constraints (Sec 5.1.5)	Sec 5.1
#14 Mar 3	**Barrier methods/Interior-point methods** 1. Barrier functions, central path, convergence (Prop 4.1.1) 2. Implementation issues: per-step optimization, feasibility optimization problem, short/long-step. 3. Convex case (Chap 11 of [3]): dual points from central path 4. Strong duality theorem (Prop 5.3.1)	Secs 4.1, Chap 11 of [3]
#15 Mar 5	**Exploring duality (*)** 1. Monotropic programming duality (Sec 5.4.1) 2. Network optimization (Sec. 5.4.2) 3. Games and minimax Theorem (Sec. 5.4.3) 4. Discrete optimization (Sec 5.5)	Secs 5.4-5.5
#16 Mar 10	**Multi-stage optimization: dynamic programming** (*) 1. multi-stage optimization: discrete-time system, additive cost-function, examples (Sec 1.1) 2. dynamic programming: principle of optimality, the DP algorithm(Sec 1.3)	Secs 1.1 and 1.3 of [2]
#17 Mar 12	**Multi-stage optimization: dynamic programming (cont.)**	Secs 1.1 and 1.3 of [2]