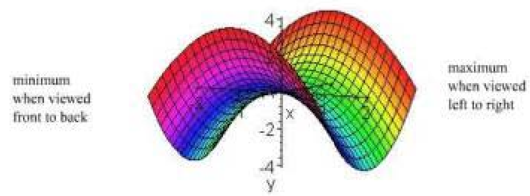
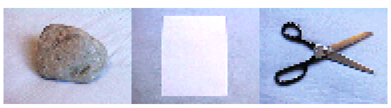
NONCOOPERATIVE GAME THEORY
ECE594D — SPRING 2003

UCSB

Abstract

In optimization, one attempts to find values for parameters that minimize a suitably defined criterion (such as monetary cost, energy consumption, heat generated, etc.) However, in most engineering applications there is always some uncertainty as to how the selected parameters will affect the final objective. One can then pose the problem of how to make sure that the selection will lead to acceptable performance, *even in the presence of some degree of uncertainty.* This question is at the heart of most zero-sum games that appear in engineering applications. In fact, game theory provides the mathematical framework for robust design in engineering.





Modern game theory was born in the 30's, mostly propelled by the work of John von Neumann, further refined by Morgenstern, Kuhn, Nash, Shappley and others. Throughout most of the 40's and 50's, Economics was its main application, eventually leading to the 1994 Nobel prize in Economic Science awarded to John Nash, John C. Harsanyi, and Reinhard Selten for their contributions to Game Theory. It was not until the 70s that it started to have a significant impact on engineering and in the late 80's it made a major splash in control theory and robust filtering.

The purpose of this course is to teach students to formulate problems as mathematical games and provide the basic tools to solve them. The course covers:

- Static games, starting with two-player zero-sum games and eventually building up to n-player non-zero sum games. Saddle-points, Nash equilibria, and Stackelberg solutions will be covered.
- Dynamic optimization (dynamic programming) for discrete and continuous time.
- Dynamic games, both open and closed-loop policies. Saddle-points and Nash equilibria will be covered.

The intended audience includes (but is not restricted to) students in communications, controls, signal processing, and computer science. The class will be entirely project-oriented (no exams!) and the students are strongly encouraged to choose a project that is relevant to their own area of research.

Further information (including a detailed syllabus) is available on the web at http://www.ece.ucsb.edu/~hespanha/ece594d/.

Instructor

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Office hours: Please email or phone me in advance to schedule for an appointment. Preferred times are MoWe 2:30-4:00pm

Prerequisites

ECE 210A Matrix Analysis and Computation

Graduate level-matrix theory with introduction to matrix analysis and computations: SVD's, pseudo-inverses, variational characterization of eigenvalues, perturbation theory (e.g., ECE 210A).

A review of basic probability theory is very much encouraged!

Course's Web Page

The syllabus, homework, solutions to homework, and all other information relevant to the course will be continuously posted at the course's web page. The URL is

http://www.ece.ucsb.edu/~hespanha/ece594d/

Textbook

The main textbooks are:

[1] Tamer Başar and Geert Olsder. *Dynamic Noncooperative Game Theory, 2nd ed*. SIAM Classics in Applied Mathematics, 1999. ISBN 0-89871-429-X.

[2] Drew Fudenberg and Jean Tirole. *Game Theory.* MIT Press, 1991. ISBN 0-262-06141-4

[3] D. Bertsekas. *Dynamic programming and optimal control, vol I, Athena Scientific*. Athena Scientific, 1995. ISBN 1-886529-12-4.

The classes will follow closely chapters 1-6 of Başar' book [1].

Projects

The following two types of projects are possible in this course:

1. Solution of a research problem relevant to the student's area of research
2. Independent study of a topic not covered in class (e.g., reading a paper or book chapter). Possible topics include:
    a. N-person games in extensive form (Sec 3.5 of [1])
    b. Static infinite games (Ch. 4 of [1])
    c. Learning and evolution in repeated games (pp. 23-29 of [2])
    d. Markov games
    e. Computer network games (security, resource management)
    f. Pursuit-evasion (Ch. 8 of [1])
    g. Cooperative games

A one-page project proposal is due on May 5th.

The project evaluations will be based on your class presentations, according to the following criteria:

Scoring rubric	**4**	**3**	**2**	**1**
Choice of content	Correct level of sophistication for the time allowed	Appropriate level of sophistication overall.	Too simple/too difficult for the audience.	Not understandable or unrelated to the course.
Delivery	Spoke clearly, notes were organized and only used as a reference	Spoke clearly, speech pattern inexact (pauses, repetition of words, etc.), read from notes occasionally	Spoke clearly, read directly from notes, speech patterns inexact using "like," "you know," more than one time	Unclear speech, read directly from notes, distracting speech patterns
Use of communication aids	Blackboard/slides clearly illustrated the most important material	Blackboard/slides were not completely clear	Blackboard/slides contained too much/too little information or were confusing	No use of presentation aids
Mastery of content	Sophisticated, elaborate, quoted	Average level of sophistication and	Simple, unable to answer questions	Incomplete, unable to answer questions

	sources, correctly answered questions	elaboration, answered questions		
Interest and participation	The whole class was involved in the presentation throughout. Many of the group spoke/got the audience to respond in some way	There was a lot of interest from the audience. Some of the group spoke. Only one or two students were not paying attention	There was little interest from the audience. Few of the group spoke. Audience's interest was sparked once or twice but not sustained.	There was no interest from the audience. Audience was waiting for presentation to end
Timing	Finished on time, allowing questions during presentation	Finished on time, allowing questions only at the end of presentation	Finished on time but without adequate time for questions	Not able to complete talk on time

Presentation schedule

Time	**Student**	**Topic**
June 2nd		
12:00	Maria	Auctions
12:15	Anuroop	Strategic behavior, monopoly and Oligopoly
12:30	Eric	Negotiations and Bargaining in cooperative games
12:45	Bharath & Kostas	Bargaining Theory
1:10	Ted	Reputation
1:25	Seung-Jun	Power control
1:40	Sunwoo	Parameter estimation
1:55	John	N-person games in extensive form
June 4th		
10:30	Sara	Robust controller design with parameter uncertainty
10:45	Prabir	H-infinity control
11:00	Aundrea	Pursuit-evasion
11:15	James	Pursuit-evasion in sports
11:30	Max	Pursuit-evasion in robotics
11:45	Payam	Games on Markov chains
12:00	Robert	Minimax lookahead training for Chess
12:15	Dragan & Gwen	Board games

Detailed Syllabus

The following is a tentative schedule for the course. As revisions are needed, they will be posted on the course's web page. The rightmost column of the schedule contains the recommended reading for the topics covered on each class. Students are strongly encouraged to read these materials prior to the class.

Class	Content	References
#1 Mar 31	**Course overview** 1. Logistics a. Web site b. Textbook c. Homework d. Projects 2. Syllabus a. Two-player, zero-sum, single-act games (Matrix and Extensive forms) b. Extension to multi-act games c. Extension to nonzero-sum case d. Extension to multi-player games e. Stackelberg games (leader/follower, nonsymmetric) f. Dynamic games (preceded by recall of dynamic optimization)	
#2 Apr 2	**Noncooperative games** 1. Players, actions, and strategies 2. Information structure 3. Optimality vs. equilibrium 4. Static vs. Dynamic games	Ch 1 of [1]
#3 Apr 7	**Zero-sum games: Matrix form** 1. Matrix/normal/strategic form 2. Security strategies 3. Saddle-point strategies	Secs 2.1-2.2 of [1]
#4 Apr 9	4. Mixed strategies	Sec 2.2 of [1]
#5 Apr 14	5. Minimax Theorem 6. Computation of mixed strategies: graphical method	Secs 2.2-2.3 of [1] and these notes
#6 Apr 16	7. Computation of mixed strategies: LP 8. Dominating strategies **Zero-sum single-act games: Extensive form** 1. Extensive/tree form	Secs 2.3-2.4 of [1]
#7 Apr 21	2. Saddle-point strategies 3. Mixed vs. behavioral strategies	Sec 2.4 of [1]
Apr 23	*no class*	
# 8 Apr 28	**Zero-sum multi-act games: Extensive form** 1. Feedback games 2. Saddle-point strategies 3. Mixed vs. behavioral strategies 4.	Sec 2.5 of [1]
#9 Apr 30	**Bimatrix games** 1. Nash equilibrium 2. Mixed strategies 3. Existence of Nash equilibrium in mixed strategies 4.	Secs 3.1-3.2 of [1]

#10 May 5	**N-person games: Matrix form** 1. Nash equilibrium 2. Mixed strategies 3. Existence of Nash equilibrium in mixed strategies 4. Computation of mixed strategies: QP 5. Perfect and proper equilibria N-person games: Extensive form* 1. Nash strategies 2. Pure Nash strategies for single-act games 3. Behavioral/mixed Nash strategies for single-act games 4. Pure Nash strategies for single-act games 5. Mixed/behavioral Nash strategies for single-act games	Secs 3.3-3.5 of [1]
# 11 May 7	Static infinite games 1. ε-Nash equilibrium 2. Pure vs. mixed strategies 3. Semi-infinite bimatrix games 4. Continuous kernel games 7.	Secs 4.1-4.4
#12 May 12	**Infinite dynamic games** 1. Discrete-time dynamic games 2. Continuous-time dynamic games	Secs 5.1-5.3
#13 May 14	**Dynamic (one player) optimization (discrete- and continuous-time)** 1. Dynamic programming	Secs 5.5-5.6
#14 May 19	*no class*	
#15 May 21	2. Minimum principle 3. **Dynamic zero-sum games (discrete- and continuous-time)** 1. Necessary conditions for open-loop equilibrium	Secs 5.5-5.6, 6.1-6.2, 6.5
May 26	*Memorial day*	
#16 Mar 28	2. Sufficient conditions for open-loop equilibrium 3. Sufficient conditions for feedback equilibrium 4. Linear quadratic games (H-infinity control) 5.	Secs 6.1-6.2, 6.5
# 17 Jun 2	Project presentations @ Leadbetter (Phelps 1530) from 12-2:10pm	
# 18 Jun 4	Project presentations @ Leadbetter (Phelps 1530) from 10:30-12:40pm	