UCSB Fall 2006

NAME: **PERM NO.:**

ECE 130A: Final Examination Problems

INSTRUCTIONS: Problems are weighted as shown. Answers should be given on these sheets. Show your work. **No credit without proper justification, even if your answers are correct.** The exam is closed book, closed notes, except for the sheet of formulas provided separately, and the two sides of handwritten notes that you are allowed to bring.

Problem 1 (20 points): The output $y(t)$ of a system with input $x(t)$ is given by

$$y(t) = \int_{-\infty}^{t+2} x^2(\tau)\, d\tau$$

1(a) (10 points) True or False The system is causal. **(no credit without proper justification)**

1(b) (10 points) True or False The system is time-invariant. **(no credit without proper justification)**

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Problem 2 (10 points): Let $x_1(t) = 2I_{[0,1]}(t) - I_{[1,2]}(t)$, and $x_2(t) = 3I_{[2,3]}(t)$. Find and sketch the convolution $y(t) = (x_1 * x_2)(t)$.

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Problem 3 (40 points): Consider the signal $x(t) = \text{sinc}(8(t+2))$.
3(a) 10 points Find and sketch the magnitude and phase of the Fourier transform $X(j2\pi f)$. That is, sketch $|X(j2\pi f)|$ versus f, and sketch $\arg(X(j2\pi f))$ versus f.

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Problem 3 (restated for convenience): Consider the signal $x(t) = \text{sinc}(8(t+2))$.

3(b) 10 points Find the energy $\int_{-\infty}^{\infty} x^2(t)\ dt$.

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Problem 3 (restated for convenience): Consider the signal $x(t) = \text{sinc}(8(t+2))$.
3(c) 10 points Find a time domain expression (simplify as much as possible) for the output $y(t)$ when $x(t)$ is passed through an LTI system with impulse response $h(t) = \text{sinc}(2(t-5))$.

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Problem 3 (restated for convenience): Consider the signal $x(t) = \text{sinc}(8(t+2))$.
3(d) 10 points For $z(t) = x(t)\sin 20\pi t$, find and sketch the magnitude of the Fourier transform $|Z(j2\pi f)|$.

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Problem 4 (10 points): A causal LTI system with transfer function

$$H(s) = \frac{s - s_3}{(s - s_1)(s - s_2)}$$

has pole-zero plot as shown in Figure 1, where $s_2 = s_1^*$. Lengths and angles relevant for geometric evaluation of the frequency response at frequency ω are shown in the figure. The input to the system

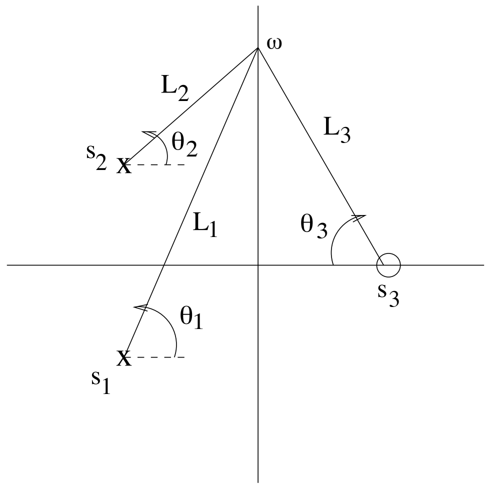

Figure 1: Geometric evaluation via pole-zero plot

is $x(t) = 2\cos(\omega t)$.

State why the output must be of the form $y(t) = A\cos(\omega t + \phi)$, and specify A and ϕ in terms of the lengths and angles shown in the figure.

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Problem 5 (20 points): A *stable* LTI system has transfer function

$$H(s) = \frac{2s+1}{s^2+s-6}$$

5(a) 10 points Sketch the pole-zero plot, and clearly mark the ROC. Is the system causal?

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Problem 5 (restated for convenience): A *stable* LTI system has transfer function

$$H(s) = \frac{2s + 1}{s^2 + s - 6}$$

5(b) 10 points Find and sketch the system impulse response $h(t)$.

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Problem 6 (20 points): A *causal* system with input $x(t)$ and output $y(t)$ obeys the differential equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y(t) = \frac{dx}{dt} + x(t)$$

6(a) 10 points Starting from initial rest, find the response of the system corresponding to the input $x(t) = e^{-t}u(t)$.

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Problem 6 (restated for convenience): A *causal* system with input $x(t)$ and output $y(t)$ obeys the differential equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y(t) = \frac{dx}{dt} + x(t)$$

6(b) 10 points Now, assume that the output obeys the initial conditions $y(0^-) = A$ and $y'(0^-) = B$. Find values of A and B such that the response to the input $x(t) = e^{-t}u(t)$ is identically zero.

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Problem 7 (15 points): Consider a *periodic* signal $x(t)$ with fundamental period 3, specified by

$$x(t) = \begin{cases} t+1, & -1 < t < 0 \\ 2-t, & 0 < t < 2 \end{cases}$$

Find an explicit expression (simplify as much as possible) for the output $y(t)$ when $x(t)$ is passed through an LTI system with impulse response $h(t) = \text{sinc}(t)$.