ECE 130A: Midterm Examination Problems

INSTRUCTIONS: Problems are weighted as shown. Show your work. *No credit without proper justification, even if your answers are correct.* The exam is closed book, closed notes, except for the sheet of formulas provided separately.

Problem 1 (20 points) This question has four unrelated parts.
1(a) 4 points A system with input $x(t)$ has output

$$y(t) = \sin(\pi t) e^{j|x(t)|}$$

Is the system stable? Is it time-invariant?
1(b) 4 points Evaluate the integral

$$\int_{-1}^{1} \left\{ \delta(t+2)e^t + \delta(t-\frac{1}{2})\sin(\pi t) \right\} \, dt$$

1(c) 7 points Sketch the following three signals as a function of time, carefully labeling the important points.

$$x(t) = (1-t)(u(t) - u(t-1))$$
$$x_1(t) = x(t-2) + x(2-t)$$
$$x_2(t) = x_1(5+2t)$$

1(d) 5 points Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\sin^2 5t}{t^2} \, dt$$

Problem 2 (10 points) Let $x(t) = e^{-2t}u(t)$. Find and sketch the output $y(t) = (x * h)(t)$ when $x(t)$ is passed through a filter with impulse response $h(t) = x(-t)$.

Problem 3 (10 points) Find the Fourier transform $X(j\omega)$ or $X(j2\pi f)$ of the signal

$$x(t) = u(t+2) - 2u(t) + u(t-1)$$

Evaluate the magnitude $|X(j\omega)|$ and the phase $\arg(X(j\omega))$ at $\omega = \pi$.

Problem 4 (25 points) A periodic waveform $x(t)$ has period 4. We know that the values taken by $x(t)$ over the interval $100 \le t \le 104$ are as follows:

$$x(t) = \begin{cases} 1 & 100 \le t < 103 \\ -1 & 103 \le t < 104 \end{cases}$$

(a) 5 points Sketch $x(t)$ over the interval $-4 \le t \le 4$, labeling both axes carefully.
(b) 10 points Find the complex exponential Fourier series $\{a_k\}$ for $x(t)$ and specify the value of the fundamental frequency ω_0 in radians/sec. Simplify your expression for the Fourier series coefficients as much as possible.
(c) 10 points Find the output $y(t)$ when $x(t)$ is passed through a filter with impulse response $h(t) = \text{sinc}(\frac{t}{6})$.