ECE 130A: Midterm Examination Problems

INSTRUCTIONS: Problems are weighted as shown. Show your work. *No credit without proper justification, even if your answers are correct.* The exam is closed book, closed notes, except for the sheet of formulas provided separately and your two sides of handwritten notes.

Problem 1 (10 points) A system with input $x(t)$ has output

$$y(t) = \int_{-\infty}^{t} \tau x(\tau)\, d\tau$$

1(a) 3 points Is the system causal?
1(b) 3 points Is the system stable?
1(c) 4 points Find and sketch $y(t)$ when the input $x(t) = \delta(t-1)$.

Problem 2 (10 points) Sketch the following signals as a function of time, carefully labeling representative points on the axes.

$$x(t) = \sin(\pi t) I_{[0,1]}(t)$$
$$x_1(t) = x(2-t) - x(t)$$
$$x_2(t) = x(1 - \tfrac{t}{2})$$

Problem 3 (10 points) Let $x(t) = 3I_{[-1,1]}(t) - 2I_{[0,2]}(t)$. Find and sketch the output $y(t)$ when $x(t)$ is passed through an LTI system with impulse response $h(t) = I_{[0,1]}(t)$.

Problem 4 (20 points) Let $x(t)$ denote a periodic function with fundamental period 6, specified as follows:

$$x(t) = \begin{cases} 2, & 0 < t < 2 \\ -1, & 2 < t < 6 \end{cases}$$

4(a) 10 points Find the complex exponential Fourier series $\{a_k\}$ for $x(t)$ and specify the value of the fundamental frequency ω_0 in radians/sec. Simplify your expression for the Fourier series coefficients as much as possible.
4(b) 5 points Set $x_1(t) = x(2t+4)$. What is the fundamental period T_1 of $x_1(t)$? Sketch $x_1(t)$ over the interval $[-T_1, T_1]$, carefully labeling the axes.
4(c) 5 points Find the complex exponential Fourier series for $x_1(t)$, specifying the fundamental frequency ω_1.