UCSB Fall 2007

ECE 130A: Problem Set 5

Assigned: Tuesday, November 6
Due: Tuesday, November 13
Reading: 4.1-4.5

Problem 1 *Fourier transform computation:* 4.21(a),(b),(f)

Problem 2 *Inverse Fourier transform computation* 4.4, 4.5

Problem 3 *Fourier transform gives system frequency response:*
(a) An LTI system has impulse response $h(t) = 10I_{[3,5]}(t)$. Find and sketch magnitude and phase of the frequency response $H(j2\pi f)$ as a function of f.
(b) Does the system amplify or attenuate a sinusoid of frequency 22 Hz? By how much?
(c) By how much does the system change the phase of the sinusoid?

Problem 4: Consider the signal $x(t) = \text{sinc}(4t)\cos(4t)$.
(a) Find and sketch the Fourier transform $X(j\omega)$ or $X(j2\pi f)$ of $x(t)$, carefully labeling the axes.
(b) Find the energy $\int_{-\infty}^{\infty} x^2(t)\ dt$.
(c) The signal $x(t)$ is passed through a filter with impulse response $h(t) = \text{sinc}(8t-4)$. Find a time-domain expression for the output $y(t) = (x * h)(t)$.

Problem 5 *Practicing use of Fourier transform properties* 4.25

Problem 6 *Matlab computation of Fourier transform using DFT*

Background for Problem 6: The Fourier transform of $h(t)$ is given by

$$H(j2\pi f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t} dt \tag{1}$$

The DFT of a discrete time sequence $\{h[n], n = 0, ..., N-1\}$ of length N is given by

$$H[m] = \sum_{n=0}^{N-1} h[n] e^{-j2\pi mn/N} \tag{2}$$

Matlab is good at doing DFTs. When N is a power of 2, the DFT can be computed very efficiently, and this procedure is called a Fast Fourier Transform (FFT). We would like this to approximately compute the continuous-time Fourier transform for a waveform $h(t)$m by sampling the waveform and approximating the integral in (1) by a sum. Assume that $h(t) = 0$ for $t < 0$ (we can shift it so that this is true). Suppose that we sample the waveform with spacing t_s to get

$$h[n] = h(nt_s)$$

Now, suppose we want to compute the Fourier transform $H(j2\pi f)$ for $f = mf_s$, where f_s is the desired frequency resolution. We can approximate the integral for the Fourier transform by a sum, using t_s-spaced time samples as follows:

$$H(j2\pi m f_s) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi m f_s t} dt \approx \sum_n h(nt_s) e^{-j2\pi m f_s n t_s} t_s$$

(dt in the integral is replaced by the sample spacing t_s.) Since $h[n] = h(nt_s)$, the approximation can be computed using the DFT formula (2) as follows:

$$H(j2\pi m f_s) \approx t_s H[m]$$

as long as $f_s t_s = \frac{1}{N}$. That is, using a DFT of length N, we can get a frequency granularity of $f_s = \frac{1}{Nt_s}$. That is, if we choose the time samples close together (in order to represent $h(t)$ accurately, then we must also use a large N to get a desired frequency granularity).

Example: We want to compute the Fourier transform of $h(t) = I_{[0,1]}(t)$ (we know we will get a phase shifted sinc). Suppose we sample with spacing $t_s = 0.1$, and want the frequency granularity to be $f_s \leq 0.05$. Then we must use a DFT with $N = \frac{1}{t_s f_s} \geq 200$. Thus, we can choose $N = 256$ (the nearest power of 2 which is larger than the N we need). Since we are sampling with spacing $t_s = 0.1$, we get 10 nonzero samples for the given $h(t)$; the 246 remaining samples are zero! This is known as zero padding, and is required to use a long enough DFT so we get the desired frequency resolution.

Statement of Problem 6: For $h(t) = I_{[0,1]}(t)$ as in the example, use an FFT to compute an approximation to $H(j2\pi f)$ and plot its magnitude against f. Comment on whether the result is in line with your expectations. Remember, the approximation is that $H(j2\pi m f_s) \approx t_s H[m]$, where $f_s = \frac{1}{N t_s}$. Use the following two settings.
(a) Time samples spaced by 0.1, and a desired frequency resolution of 0.05.
(b) Time samples spaced by 0.05, and a desired frequency resolution of 0.02.
Make sure N is a power of 2, so that you can use the more efficient FFT function.