ECE 130B HW#4

②

2.5 A) $5^{n-6}u[-n+6]$ $\quad * a^n u[n] = \frac{1}{1-az^{-1}}, \ |z|>|a|$ $\quad * \begin{cases} x[n] \xrightarrow{z} X(z) & R_1 \\ x[-n] \to X(z^{-1}) & R_2 = 1/R_1 \end{cases}$

$(\frac{1}{5})^n u[n] \to \frac{1}{1-\frac{z^{-1}}{5}}, \ |z|>5$ $\quad * x[n-n_0] \longrightarrow z^{-n_0}X(z)$ ROC=R except o or ∞

$\Rightarrow 5^n u[-n] \to \frac{1}{1-\frac{z}{5}}, \ |z|<5 \xrightarrow{n \to n-6} 5^{n-6}u[-n+6] = \frac{z^{-6}}{1-z/5}$, $0<|z|<5$

↳ ! $z \neq 0$

2.5 B) $e^{j\omega_0(n-2)}u[n-1] = e^{-j\omega_0}e^{j\omega_0(n-1)}u[n-1] \Rightarrow X(z) = e^{-j\omega_0}\frac{z^{-1}}{1-z^{-1}e^{j\omega_0}}$

$|z| > |e^{j\omega_0}| = 1 \Rightarrow |z|>1$

2.5 C) $\sin(\alpha n - \pi)u[n] = -\sin\alpha n\, u[n] = -\frac{e^{j\alpha n}+e^{-j\alpha n}}{2j}u[n]$ $\quad * \sin(\omega) = \frac{e^{j\omega}-e^{-j\omega}}{2j}$

$X(z) = \frac{1}{2j}\left[\frac{-1}{1-z^{-1}e^{j\alpha}} + \frac{1}{1-z^{-1}e^{-j\alpha}}\right]$ $\quad$ ROC $= R_1 \cap R_2 \Rightarrow |z|>1$

$R_1: |z|>|e^{j\alpha}| \quad R_2: |z|>|e^{-j\alpha}|$

$R_1: |z|>1 \quad R_2: |z|>1$

$X(z) = \frac{1}{2j}\frac{-1+z^{-1}e^{-j\alpha}+1-z^{-1}e^{j\alpha}}{(1-z^{-1}e^{j\alpha})(1-z^{-1}e^{-j\alpha})} = \frac{-z^{-1}\left(\frac{e^{j\alpha}-e^{-j\alpha}}{2j}\right)}{1-2\frac{e^{j\alpha}+e^{-j\alpha}}{2}z^{-1}+z^{-2}} = \frac{-z^{-1}\sin(\alpha)}{1-2\cos\alpha z^{-1}-z^{-2}}$

$|z|>1$

2.5 D) $\frac{1}{5}\left(\frac{1}{4}\right)^{2n-1}u[n] + 4^{3n}u[-n-5]$ $\quad * -a^n u[-n-1] \xrightarrow{z} \frac{1}{1-az^{-1}}, \ |z|<|a|$

I) $4^{3n}u[-n-5]$: $(4^3)^n u[-n-1] \xrightarrow{z} \frac{1}{1-4^3z^{-1}}, \ |z|<4^3$

$n \to n+4 \quad (4^3)^4\cdot(4^3)^n u[-n-5] \xrightarrow{z} \frac{z^4}{1-4^3z^{-1}}, \ |z|<4^3$

$\Rightarrow 4^{3n}u[-n-5] \longrightarrow \frac{4^{-12}z^4}{1-4^3z^{-1}}, \ |z|<4^3 : R_1$

II) $\frac{4}{5}\left(\frac{1}{16}\right)^n \cdot u(n) \xrightarrow{z} \frac{4}{5}\cdot\frac{1}{1-\frac{z^{-1}}{16}}, \ |z|>\frac{1}{16} : R_2$

$\Rightarrow$ I + II $\longrightarrow \frac{4}{5}\frac{1}{1-z^{-1}/16} + \frac{4^{12}z^4}{1-64z^{-1}}$ $\quad R = R_1 \cap R_2 = \{z : \frac{1}{16}<|z|<64\}$

2.5 3) A) $\dfrac{1-4z^{-2}}{1+\frac{5}{2}z^{-1}+z^{-2}} = \dfrac{1-4z^{-2}}{(1+\frac{1}{2}z^{-1})(1+2z^{-1})} = \left(\dfrac{-1/3}{1+\frac{1}{2}z^{-1}} + \dfrac{4/3}{1+2z^{-1}}\right)(1-4z^{-2})$

$|z|>2>\frac{1}{2}$: $(-\frac{1}{3})(-\frac{1}{2})^n u[n] + 4/3\,(-2)^n u[n] + (\frac{4}{3})(-\frac{1}{2})^{n-2} u[n-2] - (\frac{8}{3})(-2)^{n-2} u[n-2]$

$\frac{1}{2}<|z|<2$: $(+\frac{1}{3})(-\frac{1}{2})^n u[n] - 4/3\,(-2)^n u[-n-1] + (\frac{4}{3})(-\frac{1}{2})^{n-2} u[n-2] + (8/3)(-2)^{n-2} u[-n+1]$

$0<|z|<\frac{1}{2}<2$: $(\frac{1}{3})(-\frac{1}{2})^n u[-n-1] - 4/3\,(-2)^n u[-n-1] - (4/3)(-\frac{1}{2})^{n-2} u[-n+1] + (8/3)(-2)^{n-2} u[-n+1]$

2.5 B) $\dfrac{2}{z-\frac{1}{4}-\frac{1}{8}z^{-1}} = \dfrac{2z}{z^2-\frac{1}{4}z-\frac{1}{8}} = \dfrac{2z}{(z-\frac{1}{2})(z+\frac{1}{4})} = \dfrac{\frac{2}{3}}{z-\frac{1}{2}} + \dfrac{4/3}{z+\frac{1}{4}}$

$= \frac{2}{3}z^{-1}\dfrac{1}{1-\frac{1}{2}z^{-1}} + \frac{4}{3}z^{-1}\dfrac{1}{1-z^{-1}(\frac{-1}{4})}$

$(\frac{2}{3})\frac{z^{-1}}{1-\frac{z^{-1}}{2}}$: $\begin{cases} |z|>\frac{1}{2} & (\frac{2}{3})(\frac{1}{2})^{n-1}u(n-1) = A \\ |z|<\frac{1}{2} & (\frac{-2}{3})(\frac{1}{2})^{n-1}u(-n) = B \end{cases}$ $\quad \frac{4}{3}\,\frac{z^{-1}}{1+\frac{1}{4}z^{-1}}$ $\begin{cases} |z|>\frac{1}{4} & (\frac{4}{3})(\frac{-1}{4})^{n-1}u(n-1) = C \\ |z|<\frac{1}{4} & (\frac{-4}{3})(\frac{-1}{4})^{n-1}u(-n) = D \end{cases}$

$|z|>\frac{1}{2}$ $\quad A+C$

$\frac{1}{4}<|z|<\frac{1}{2}$ $\quad B+C$

$0<|z|<\frac{1}{4}$ $\quad B+D$

2.5 C) $\dfrac{1-4z^{-1}}{1-\frac{1}{4}z^{-1}} = \dfrac{1}{1-\frac{1}{4}z^{-1}} - 4z^{-1}\dfrac{1}{1-\frac{1}{4}z^{-1}}$

$|z|>\frac{1}{4} \Rightarrow (\frac{1}{4})^n u(n) + 4(\frac{1}{4})^{n-1}u(n-1) = (\frac{1}{4})^n\left[u(n)+u(n-1)\right]$

$0<|z|<\frac{1}{4} \Rightarrow -(\frac{1}{4})^{+n}u[-n-1] + 4(\frac{1}{4})^{n-1}u[-n] = (\frac{1}{4})^{-n}\left[u(-n)+u(-n-1)\right]$

2.5 D) $\dfrac{1-\frac{1}{3}z^{-1}}{1+\frac{1}{3}z^{-1}} = \dfrac{1}{1+\frac{1}{3}z^{-1}} - \frac{1}{3}\dfrac{z^{-1}}{1+\frac{1}{3}z^{-1}}$

$|z|>\frac{1}{3}$: $x[n] = (-\frac{1}{3})^n u[n] - \frac{1}{3}(-\frac{1}{3})^{n-1}u[n-1]$

$0<|z|<\frac{1}{3}$ $\quad x[n] = -(-\frac{1}{3})^n u[-n-1] + \frac{1}{3}(-\frac{1}{3})^{n-1}u[-n]$

2.5 E)

$$\frac{z^{-1}-1/3}{1-1/3z^{-1}} = \frac{z^{-1}}{1-1/3z^{-1}} - 1/3 \cdot \frac{1}{1-1/3z^{-1}}$$

$|z|>1/3$: $(1/3)^{n-1}u[n-1] - 1/3(+1/3)^n u[n]$

$0<|z|<1/3$: $-(1/3)^{n-1}u[-n] + 1/3(1/3)^n u[-n+1]$

2.5 F)

$$\frac{z^{-1}-\frac{1}{3}}{(1-1/3z^{-1})^2}$$

$$nx[n] \longrightarrow -z\frac{dX(z)}{dz} \quad \text{ROC}=R$$

$$\frac{z^{-1}}{(1-1/3z^{-1})^2} - 1/3 \cdot \frac{1}{1-1/3z^{-1}} = 3\frac{1/3z^{-1}}{(1-1/3z^{-1})^2} - \frac{(1/3)z^{-1}}{(1-1/3z^{-1})^2}\cdot z$$

$$x[n] \Rightarrow \begin{cases} \infty > |z| > 1/3 : & 3n(1/3)^n u[n] - (n+1)(1/3)^{n+1}u[n+1] \\ |z| < 1/3 : & -3n(1/3)^n u[-n-1] + (n+1)(1/3)^{n+1}u[-n-2] \end{cases}$$

* Please, pay attention to zero and infinity when you are finding the Roc of a signal.