ECE 130B: HW 8

1. Reading assignment

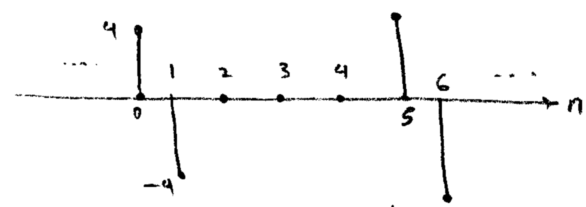
8 2. $N=5$ $\quad a_0 = 1 \quad a_1 = 2e^{j\pi/3} \quad a_2 = e^{j\pi/4} \quad$ $x[n]$ is real

$x[n]$ is real $\Rightarrow a_k = a_{-k}^* \quad a_{-1} = 2e^{-j\pi/3} \quad a_{-2} = e^{-j\pi/4}$

$$x[n] = \sum_{k=-2}^{2} a_k e^{jk\frac{2\pi}{5}n} = e^{-j\pi/4} e^{j(-2)\frac{2\pi}{5}n} + 2e^{-j\pi/3} e^{j(-1)(\frac{2\pi}{5})n} + 1$$
$$+ 2e^{j\pi/3} e^{j(1)\frac{2\pi}{N}n} + e^{j\pi/4} e^{j(2)\frac{2\pi}{N}n}$$

$$= 1 + 2\cos\left((2)\tfrac{2\pi}{5}n + \pi/4\right) + 4\cos\left((1)\tfrac{2\pi}{5}n + \pi/3\right)$$
$$= 1 + 2\sin\left((2)\tfrac{2\pi}{5}n + \tfrac{\pi}{4} + \pi/2\right) + 4\sin\left(\tfrac{2\pi n}{5} + \pi/3 + \pi/2\right)$$

8 3. $N=5$



$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{5}\sum_{n=0}^{4} x[n] e^{-jk\frac{2\pi}{5}n}$$
$$= \frac{4}{5}e^{j(0)} - \frac{4}{5}e^{-j(1)\frac{2\pi}{5}k} = \frac{4}{5}\left(1 - e^{-j\frac{2\pi k}{5}}\right) \quad k=0,1,2,3,4$$

$$a_{k+N} = a_k$$

9 4. $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k] \quad N=4 \qquad a_k = \frac{1}{4}\sum_{n=0}^{3} x[n] e^{-jk\frac{2\pi}{4}n} = \frac{1}{4}x[0]e^{-j(0)} = 1/4$

$$x[n] = \sum_{k=\langle 4\rangle} \frac{1}{4} e^{jk\frac{2\pi}{4}n}$$

$$y[n] = \cos\left(\tfrac{3\pi}{2}n - \pi/4\right) = \tfrac{1}{2}e^{-j\pi/4}\cdot e^{j\frac{3\pi}{2}n} + \tfrac{1}{2}e^{+j\pi/4}\cdot e^{-j3\frac{\pi}{2}n}$$

$$\Rightarrow y[n] = \underbrace{\left(\tfrac{1}{2}e^{-j\pi/4}\right)}_{b_3} e^{j3\frac{2\pi}{4}n} + \underbrace{\tfrac{1}{2}e^{j\pi/4}}_{b_{-3} = b_1}\cdot e^{j(-3)\frac{2\pi}{4}n}$$

$b_0 = a_0 H(0) \Rightarrow H(0) = 0$

$b_1 = a_1 H(1\,\tfrac{2\pi}{4}) \Rightarrow H(\pi/2) = 2e^{j\pi/4}$

$b_2 = a_2 H(2\,\tfrac{2\pi}{4}) \Rightarrow H(\pi) = 0$

$b_3 = a_3 \quad H(3\,\tfrac{2\pi}{4}) \Rightarrow H(3\pi/2) = 2e^{-j\pi/4}$

It's not possible to find the value of $H(\omega)$ for any other values of $\omega \in [0, 2\pi]$ from the given information