ECE 130 B

HW-3 Solutions

2) A) $y[n] - 5y[n-1] + 4y[n-2] = 0$.

$$y[n] = c_1[1]^n + c_2[4]^n = c_1 + c_2 4^n.$$

B) $y[n] - 9y[n-2] = 0$.

$$y[n] = c_1 3^n + c_2(-3)^n.$$

c) $2y[n] - 11y[n-1] + 5y[n-2] = 0$.

$z = +\frac{1}{2}$ and 5.

$$\therefore y[n] = c_1\left(\frac{1}{2}\right)^n + c_2 5^n$$

d) $5y[n-1] - 11y[n] + 2y[n+1] = 0$.

$z = \frac{1}{2}$ and 5.

$$\therefore y[n] = c_1\left(\frac{1}{2}\right)^n + c_2 5^n.$$

e) $2y[n] + 5y[n-2] = 0$.

$$z = \pm\sqrt{\frac{5}{2}}$$

$$y[n] = c_1\left(\sqrt{\frac{5}{2}}\right)^n + c_2\left(-\sqrt{\frac{5}{2}}\right)^n.$$

F) $y[n+1] - 2e^{\frac{j\pi}{6}} y[n] + y[n-1] = 0$.

$$y[n] = C_1 \left(e^{-\frac{j\pi}{3}} + e^{-\frac{j\pi}{2}} \sqrt{e^{\frac{j\pi}{3}} - e^{\frac{j\pi}{2}}} \right)^n + C_2 \left(e^{-\frac{j\pi}{3}} - e^{-\frac{j\pi}{2}} \sqrt{e^{\frac{j\pi}{3}} - e^{\frac{j\pi}{2}}} \right)^n .$$

3) A) $y[n] - 3y[n-2] = u[n+1]$ $\quad y[0] = y[-1] = 2$.

As $u[n+1]$ is right sided, pick right sided impulse response

(i) $y_n(n) = C_1 (\sqrt{3})^n + C_2 (-\sqrt{3})^n$.

(ii) $y[n] - 3y[n-2] = \delta[n]$.

$n < 0$ $\quad h[n] = C_{L_1} (\sqrt{3})^n + C_{L_2} (-\sqrt{3})^n$. $= 0$ (As left sided).

$n > 0$ $\quad h[n] = \cancel{C}$ ~~(As right)~~ $C_{R_1} (\sqrt{3})^n + C_{R_2} (-\sqrt{3})^n$

$y[0] - 3y[-2] = 1 \Rightarrow y[0] = 1$.

$y[1] - 3y[-1] = 0 \Rightarrow y[1] = 0 \Rightarrow C_{R_1} = C_{R_2}$.

$y[2] - 3y[0] = 0 \Rightarrow y[2] = 3 \Rightarrow C_{R_1} + C_{R_2} = 1$

$\therefore C_{R_1} = C_{R_2} = \frac{1}{2}$.

$\therefore h[n] = \dfrac{(\sqrt{3})^n + (-\sqrt{3})^n}{2}$.

$$y_p[n] = h[n] * u[n+1] = \frac{1}{2} \left[\frac{1 - (\sqrt{3})^{n+2}}{1 - \sqrt{3}} u[n+1] + \frac{1 - (-\sqrt{3})^{n+2}}{1 + \sqrt{3}} u[n+1] \right]$$

(iii) $y[n] = y_n[n] + y_p[n]$.

$y[n] = C_1(\sqrt{3})^n + C_2(-\sqrt{3})^n + y_p[n]$.

$y[0] = 2 \Rightarrow C_1 + C_2 = 1$ (as $y_p[0] = 1$).

$y[1] = 2 \Rightarrow C_1 - C_2 = \sqrt{3}$.

$$\therefore y[n] = \frac{1+\sqrt{3}}{2}(\sqrt{3})^n + \frac{1-\sqrt{3}}{2}(-\sqrt{3})^n + \frac{1}{2}\left[\frac{1-(\sqrt{3})^{n+2}}{1-\sqrt{3}} + \frac{1-(-\sqrt{3})^{n+2}}{1-\sqrt{3}}\right] \times u[n+1]$$

B) $2y[n] - 5y[n-2] = u[-n]$ (Pick Left sided impulse response)

(i) $y_n[n] = C_1\left(\sqrt{\frac{5}{2}}\right)^n + C_2\left(-\sqrt{\frac{5}{2}}\right)^n$.

(ii) $n < 0 \quad h[n] = C_{L_1}\left(\sqrt{\frac{5}{2}}\right)^n + C_{L_2}\left(-\sqrt{\frac{5}{2}}\right)^n$

$n > 0 \quad h[n] = 0$.

$2h[0] - 5h[-2] = 1$.

$h[-1] = 0 \Rightarrow C_{L_1} = C_{L_2}$.

$h[0] = 0$

$\Rightarrow C_{L_1} = C_{L_2} = \frac{-1}{4}$.

$$\therefore h[n] = \left[\frac{-1}{4}\left(\sqrt{\frac{5}{2}}\right)^n - \frac{1}{4}\left(-\sqrt{\frac{5}{2}}\right)^n\right] u[-n-1].$$

$$\therefore y_p[n] = \frac{-1}{4}\left[\frac{1-\left(\sqrt{\frac{5}{2}}\right)^{-n}}{1-\sqrt{\frac{5}{2}}}\left(\sqrt{\frac{5}{2}}\right)^{n} + \frac{1-\left(-\sqrt{\frac{5}{2}}\right)^{-n}}{1-\left(-\sqrt{\frac{5}{2}}\right)}\left(-\sqrt{\frac{5}{2}}\right)^{n}\right] u[-n-1].$$

$$y[n] = c_1\left(\sqrt{\frac{5}{2}}\right)^{n} + c_2\left(-\sqrt{\frac{5}{2}}\right)^{n} + y_p[n].$$

$$y[0] = 2 \Rightarrow c_1 + c_2 = 2.$$

$$y[-1] = 2 \Rightarrow c_1 - c_2 = \sqrt{10}.$$

$$c_1 = \frac{1+\sqrt{10}}{2} = 1+\sqrt{\frac{5}{2}}.$$

$$c_2 = 1-\sqrt{\frac{5}{2}}.$$

$$\therefore y[n] = \left(1+\sqrt{\frac{5}{2}}\right)\left(\sqrt{\frac{5}{2}}\right)^{n} + \left(1-\sqrt{\frac{5}{2}}\right)\left(-\sqrt{\frac{5}{2}}\right)^{n} - \frac{1}{4}\left[\frac{\left(1-\left(\sqrt{\frac{5}{2}}\right)^{-n}\right)}{1-\sqrt{\frac{5}{2}}}\left(\sqrt{\frac{5}{2}}\right)^{n} + \frac{\left(1-\left(-\sqrt{\frac{5}{2}}\right)^{-n}\right)}{1-\left(-\sqrt{\frac{5}{2}}\right)} \times \left(-\sqrt{\frac{5}{2}}\right)^{n}\right] \times u[-n-1].$$