HW 7 ECE 130B

Page 1

1) Reading assignment

2)

1 a) $x_1[n] = x[2-n] + x[n-2]$

By using d & e $\quad X_1(\omega) = e^{-2j\omega}\left(X(\omega) + X(-\omega)\right)$

1 b) $x_2[n] = \overline{x[-n]}$

$x[-n] \xrightarrow{F} X(-\omega) \Rightarrow \overline{x[-n]} \xrightarrow{F} X^*(-(-\omega)) = X^*(\omega)$

1 c) $x_3[n] = (1+n+n^2)x[n]$

$x_3[n] = x[n] + n\,x[n] + n\cdot(n\,x[n])$

$\Rightarrow X_3(\omega) = X(\omega) + j\frac{dX(\omega)}{d\omega} + j\frac{d}{d\omega}\left(j\frac{dX(\omega)}{d\omega}\right)$

$\Rightarrow X_3(\omega) = \left(X(\omega) + j\frac{dX(\omega)}{d\omega} - \frac{d^2X(\omega)}{d\omega}\right)$

1 d) $x_4[n] = x[n-n_0] \qquad X_4(\omega) = e^{-j\omega n_0}X(\omega)$

1 e) $x_5[n] = x[-n+n_0] \qquad X_5(\omega) = e^{-j\omega n_0}X(-\omega)$

$x[n+n_0] \xrightarrow{F} X(\omega)\cdot e^{j\omega n_0}$

$n \to -n \; : \; x[-n+n_0] \xrightarrow{F} X(-\omega)\,e^{-j\omega n_0}$

3) $X(\omega) = \frac{1}{1-e^{-j\omega}}\frac{\sin(5/2\,\omega)}{\sin(\frac{\omega}{2})} + 2\pi\delta(\omega) \qquad -\pi \leq \omega \leq \pi$

3 $X(\omega) = \frac{1}{1-e^{-j\omega}}\frac{\sin(5/2\,\omega)}{\sin \omega/2} + 5\pi\delta(\omega) - 3\pi\delta(\omega) = \frac{\sin 5/2\,\omega}{\sin\frac{\omega}{2}}\left(\frac{1}{1-e^{-j\omega}} + \frac{\sin\omega/2}{\sin(\frac{5\omega}{2})}5\pi\delta(\omega)\right)$

$-3\pi\delta(\omega)$

$f(t)\,\delta(\omega-k) = f(k)$

$X(\omega) = \underbrace{\frac{\sin(5/2\,\omega)}{\sin(\omega/2)}}_{x_1[n]}\left(1/1-e^{-j\omega} + \pi\delta(\omega)\right) \underbrace{-3\pi\delta(\omega)}_{-3/2}$ $\qquad x_1[n] = \begin{cases} 1 & |n| \leq 2 \\ 0 & \text{else} \end{cases}$

$\sum_{k=-\infty}^{n} x_1[k]$

$\Rightarrow x[n] = \begin{cases} -3/2 & n < -2 \\ n + 3/2 & -2 \leq n \leq 2 \\ n + 7/2 & n > 2 \end{cases}$

4 $$x[n] = \int_0^{\pi} \left(B(\omega)\cos(\omega n) + C(\omega)\sin(\omega n) \right) d\omega \qquad X(\omega) = Re(X(\omega)) + j\,Im(X(\omega))$$

3 $$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{+j\omega n}\, d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(Re(X(\omega)) + j\,Im(X(\omega)) \right)\left(\cos\omega n + j\sin\omega n\right) d\omega$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{\underset{\text{even}}{Re(X(\omega))}\,\underset{\text{even}}{\cos(\omega n)}}_{\text{even}} - \underbrace{\underset{\text{odd}}{Im(X(\omega))}\,\underset{\text{odd}}{\sin(\omega n)}}_{\text{even}} \; d\omega$$

$$+ \frac{1}{2\pi} \int_{-\pi}^{\pi} j\Big(\underbrace{\underset{\text{even}}{Re(X(\omega))}\,\underset{\text{odd}}{\sin\omega n}}_{\text{odd}} + \underbrace{\underset{\text{odd}}{Im(X(\omega))}\cdot\underset{\text{even}}{\cos\omega n}}_{\text{odd}} \Big) d\omega$$

We know that $x[n]$ is real $\longrightarrow$ $Re(X(\omega)) = Re(X(-\omega))$ Even function in ω

$Im(X(\omega)) = -Im(X(-\omega))$ Odd function in ω

if $f(\omega)$ & $g(\omega)$ are even $\longrightarrow$ $f(\omega)\,g(\omega)$ is even

if $f(\omega)$ & $g(\omega)$ are odd $\longrightarrow$ $f(\omega)\,g(\omega)$ is even

if $f(\omega)$ is even & $g(\omega)$ is odd $\longrightarrow$ $f(\omega)\,g(\omega)$ is odd

$f(\omega)$ even: $\int_{-\alpha}^{\alpha} f(\omega)\,d\omega = 2\int_0^{\alpha} f(\omega)\,d\omega$ $\qquad$ $f(\omega)$ odd: $\int_{-\alpha}^{\alpha} f(\omega)\,d\omega = 0$

$$\Rightarrow x[n] = 2 \cdot \frac{1}{2\pi} \int_0^{\pi} \left[Re(X(\omega))\cos(\omega n) - Im(X(\omega))\sin(\omega n) \right] d\omega$$

$$\Rightarrow B(\omega) = \frac{1}{\pi}\, Re(X(\omega))$$

$$C(\omega) = -\frac{1}{\pi}\, Im(X(\omega))$$

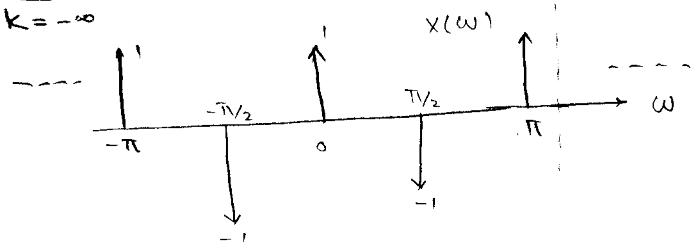
5. Problems from the textbook:

2 5.22 (d) $X(\omega) = \cos^2\omega + \sin^2(3\omega)$ $\quad x[n] = ?$

$$X(\omega) = \frac{1+\cos 2\omega}{2} + \frac{1-\cos(6\omega)}{2} = 1 + \tfrac{1}{4}e^{j2\omega} + \tfrac{1}{4}e^{-j2\omega} - \tfrac{1}{4}e^{j6\omega} - \tfrac{1}{4}e^{-j6\omega}$$

$$\Rightarrow x[n] = \delta[n] + \tfrac{1}{4}\delta[n+2] + \tfrac{1}{4}\delta[n-2] - \tfrac{1}{4}\delta[n+6] - \tfrac{1}{4}\delta[n-6]$$

2 5.22 (e) $X(\omega) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \frac{\pi}{2}k)$



$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}\,d\omega$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi+\pi/4}^{\pi+\pi/4} \left[\delta(\omega+\pi) - \delta(\omega+\pi/2) + \delta(\omega) - \delta(\omega-\pi/2) + \delta(\omega-\pi)\right] e^{+j\omega n}\,d\omega$$

$$= \tfrac{1}{2\pi}\left(-e^{-j\pi/2\, n} + 1 - e^{j\pi/2\, n} + e^{j\pi n}\right)$$

$$= \tfrac{1}{2\pi} + \tfrac{1}{2\pi}e^{j\pi n} - \tfrac{1}{\pi}\cos\left(\frac{\pi n}{2}\right) = \tfrac{1}{2\pi} + \frac{1}{2\pi}(-1)^n - \frac{1}{\pi}\cos\left(\frac{\pi n}{2}\right)$$

2 5.23 ~~f~~) (ii) $\int_{-\pi}^{\pi} \left|\frac{dX(\omega)}{d\omega}\right|^2 d\omega$ $\quad * \; \left|j\frac{dX(\omega)}{d\omega}\right| = \left|\frac{dX(\omega)}{d\omega}\right|$

$$\int_{-\pi}^{\pi}\left|j\frac{dX(\omega)}{d\omega}\right|^2 d\omega \xrightarrow[\text{relation}]{\text{parseval's}} 2\pi\sum_{n=-\infty}^{\infty}|n x[n]|^2 = 316\pi$$

3 5.25 $x_e[n] = Ev\{x[n]\} = \frac{x[n]+x[-n]}{2} \xleftrightarrow{FT} A(\omega)$

$x_o[n] = Od\{x[n]\} = \frac{x[n]-x[-n]}{2} \xleftrightarrow{FT} jB(\omega)$

Therefore $\quad B\omega \xrightarrow{FT^{-1}} -j\,x_o[n]$

$A(\omega)e^{j\omega} \xrightarrow{FT^{-1}} x_e[n+1]$

$$\Rightarrow B(\omega) + e^{j\omega}A(\omega) \longrightarrow x_e[n+1] - jx_o[n]$$

5.26 1 a) $X_2(\omega) = Re(X_1(\omega)) + Re(X_1(\omega - 2\pi/3)) + Re(X_1(\omega + 2\frac{\pi}{3}))$

$$\Rightarrow x_2[n] = (1 + e^{j2\pi/3} + e^{-j2\pi/3})\, Ev\{x_1[n]\}$$

$$x_2[n] = (1 + 2\cos\frac{2\pi}{3})\, Ev\{x_1[n]\}$$

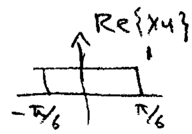
1 b) $X_3(\omega) = Im\{X_1(\omega - \pi)\} + Im\{X_1(\omega + \pi)\}$

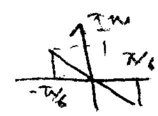
$$\Rightarrow x_3[n] = Od\{x_1[n]\}\,(\underbrace{e^{j\pi n} + e^{-j\pi n}}_{2\cos(\pi n)}) = 2(-1)^n\, Od\{x_1[n]\}$$

1 c)
$$\alpha = \frac{j\left.\frac{dX_1(\omega)}{d\omega}\right|_{\omega=0}}{\left.X_1(\omega)\right|_{\omega=0}} = \frac{j(-6j/\pi)}{1} = \frac{6}{\pi}$$

$$X_1(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \Rightarrow \left.X_1(\omega)\right|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n]$$

$$X_2(\omega) = j\frac{dX_1(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} n x[n] e^{-j\omega n} \Rightarrow \left.X_2(\omega)\right|_{\omega=0} = \sum_{n=-\infty}^{\infty} n x[n]$$





d)

2 5.3 1 a) From the given information, it is clear that when the input to the system is a complex exponential of frequency ω_0, the output is a complex exponential of the same frequency but scaled by $|\omega_0|$

So $\quad H(\omega) = |\omega| \quad 0 \leq |\omega| \leq \pi$

b)
$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} |\omega| e^{j\omega n} d\omega = -\frac{1}{2\pi}\int_{-\pi}^{0} \omega e^{j\omega n} + \frac{1}{2\pi}\int_{0}^{\pi} \omega e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\int_0^{\pi} \omega e^{-j\omega n} d\omega + \frac{1}{2\pi}\int_0^{\pi} \omega e^{j\omega n} d\omega$$

$$= \frac{1}{\pi}\int_0^{\pi} \omega \cos(\omega n)\, d\omega$$

$n \neq 0$
$$= \frac{1}{\pi}\left[\frac{\cos(n\pi) - 1}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

$n = 0 \quad h(n) = \pi/2$