# ECE 130C Homework 4 Solution

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**2.3.14** The dimension of **S** is (a) 0 when $x = 0$ (b) 1 when $x = (1, 1, 1, 1)$ (c) 3 when $x = (1, 1, -1, -1)$ because all arrangements of this $x$ are perpendicular to $(1, 1, 1, 1)$ (d) 4 when the $x$'s are not equal and don't add to zero. No $x$ gives dim**S**=2.

**2.3.26** (a) True. (b) False because the basis vectors may not be in **S**.

**2.3.34** If $v_1, v_2, v_3$ is a basis for **V**, and $w_1, w_2, w_3$ is a basis for **W**, then these six vectors cannot be independent and some combination is zero: $\sum c_i v_i + \sum d_i w_i = 0$. This puts $\sum c_i v_i = -\sum d_i w_i$ in both subspaces.

**2.3.36** $n - r = 17 - 11 = 6$ is nullspace dimension; $64 - 11 = 53$ is dim$N(A^T)$

**2.3.40** (a) $y(x) = e^{2x}$ (b) $y = x$ (one basis vector in each case).

**2.4.28** $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$; $r + (n - r) = n = 3$ but $2 + 2$ is 4.

**2.4.32** Row space = $yz$ plane; column space = $xy$ plane; nullspace = $x$ axis; left nullspace = $z$ axis. For $I + A$: Row space = column space = $\mathbf{R}^3$, nullspaces contain only zero vector.

**2.4.34** (a) Elimination leads to $0 = b_3 - b_2 - b_1$ so $(-1, -1, 1)$ is in the left nullspace. (b) Elimination leads to $b_3 - 2_b1 = 0$ and $b_4 + b_2 - 4b_1 = 0$, so $(-2,0,1,0)$ and $(-4,1,0,1)$ are in the left nullspace.

**2.4.36** Row space basis $(3,0,3)$, $(1,1,2)$; column space basis $(1,4,2)$, $(2,5,7)$; rank is only 2.

**2.4.38** $AB = 0$ leads to $\dim C(B) \leq \dim N(A)$. This is $r_B \leq n - r_A$, so $r_A + r_B \leq n$.