ECE 130C HW-3 Solution

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1. Problem 2.1.20
   (a) True
   (b) False(consider $A + A^T$)
   (c) True: $Ax = 0 and Bx = 0 \implies (cA + dB)x = 0$
2. Problem 2.1.26
   $C(AB) \subseteq C(A)$
   If $B = 0$, then $C(AB) \subset C(A)$
3. Problem 2.1.28
   (a) False
   (b) True
   (c) True
   (d) False take $A = I$
4. Problem 2.2.2
   Echelon form:
   $$U = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
   u and v are the pivot variables; w and y are free variables.

The complete solution is

$$x = \begin{bmatrix} 2w - y \\ -w \\ w \\ y \end{bmatrix} = w \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$Rank(A) = 2$

$$EchelonForm, U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

w is free, all solutions are $(w, -2w, w)$.

5. Problem 2.2.8

   No condition on b_1, b_2

   $(-2, 1, 0, 0), (0, 0, 1, 0)$ are basis for $N(A)$.

   The complete solution is

$$x = \begin{bmatrix} 7b_1 - 3b_2 \\ 0 \\ 0 \\ b_2 - 2b_1 \end{bmatrix} + v \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

   $N(A^T) = (0, 0)$.

6. Problem 2.2.10

   Rank is 2. A *can be* $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -3 \end{bmatrix}$ with $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

7. Problem 2.2.20

   $S = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$ and $S = \begin{bmatrix} 1 \end{bmatrix}$ and $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

   itemProblem 2.2.36

   (a) Every b is in the column space: *independent rows.*

   (b) Need $b_3 = 2b_2$ for (b_1, b_2, b_3) to be in the column space. Row 3 - 2*Row 2 = 0.

8. Problem 2.2.42

   If $Ax_1 = b$ and $Ax_2 = b$ then we can add $c(x_1 - x_2)$ to any solution of $Ax = B$.

   But there will be *no* solution to $Ax = B$ if B is not in the column space.

9. Problem 2.2.50

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$

10. Problem 3 (not in text)

    $A_{m*n} = RX$ where R is an $m * r$ matrix with the columns as the basic columns of A and R is an $r * n$ matrix with the r rows corresponding to the non-zeros rows from the echelon form of A.

    Note that a direct LU factorizarion may not always be possible for all A and a permutation may be needed.

$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 9 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$