ECE 130C HW7 Addendum

June 2, 2009

Problem 5.1.18

Suppose A has eigenvalues 0,3,5 with independent eigen vectors u, v, w.

1. Give a basis for the nullspace and a basis for the column space.
2. Find a particular solution to $Ax = v + w$. Find all solutions.
3. Show that $Ax = u$ has no solution. (If it had a solution, then —— would be in the column space).

Problem 5.1.34

This matrix is singular with rank 1. Find the three λs and the three eigenvectors:

$$A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix}$$

Problem 5.1.36

Find the eigen values of A, B, C:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

Problem 5.2.10

Suppose that A eigen values 1.2.4. What is the trace of A? What is the determinant of ${A^{-1}}^T$?

Problem 5.2.32

Diagonalize A and compute $S\Lambda^k S^{-1}$ to prove thjis formula for A^k: $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ has $A^k = \frac{1}{2} \begin{bmatrix} 3^k + 1 & 3^k - 1 \\ 3^k - 1 & 3^k + 1 \end{bmatrix}$.

Problem 5.2.40

Substitute $A = S\Lambda S^{-1}$ into the product $(A - \lambda_1 I)(A - \lambda_2 I)...(A - \lambda_n I)$ and explain why this produces the zero matrix. The *Cayley-Hamilton Theorem* says that every matrix satisfies its characteristic equation which produces the eigen values. We are essentially proving this.

Problem 5.3.4

Suppose each "Gibonacci" number G_{k+2} is the average of the two previous numbers, then $G_{k+2} = (G_{k+1} + G_k)/2$ and $G_{k+1} = G_{k+1}$: $\begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = A \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}$.

1. Find the eigen values and eigen vectors of A
2. Find the limit as $n- > \infty$ of the matrices A^n.
3. If $G_0 = 0, G_1 = 1$, show that Gibonacci numbers approach $\frac{2}{3}$.

Problem 5.4.40

Put $A = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$ into the infinite series to find e^{At}.First compute A^2: $e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} t & 3t \\ 0 & 0 \end{bmatrix} + \frac{1}{2}[] + ... = \begin{bmatrix} e^t & 0 \\ & \end{bmatrix}$.