# 130C HW2 Addendum

April 7, 2009

## 1 1.5.4

Aplly elimination to produce L and U factors for:

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}, A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{bmatrix}$$

## 2 1.5.18

Decide whether the following systems are singular or non-singular, and whether they have no solutions, one solution or infinitely many:

$$\begin{aligned} v - w &= 2 \\ u - v &= 2 \\ u - w &= 2 \end{aligned}$$

and

$$\begin{aligned} v - w &= 0 \\ u - v &= 0 \\ u - w &= 0 \end{aligned}$$

and

$$\begin{aligned} v + w &= 1 \\ u + v &= 1 \\ u + w &= 1 \end{aligned}$$

## 3 1.5.40

There are 12 “even” permutations of (1,2,3,4), with an even number of exhanges. They are (1,2,3,4) with no exchanges and (4,3,2,1) with two exchanges. List the other ten. Instead of writing each 4x4 matrix, use the numbers 4,3,2,1 to give the position of 1 in each of them.

## 4 1.5.42

If $P_1$ and $P_2$ are permutation matrices, so is $P_1P_2$. This still has the rows of $I$ in some order. Give examples with $P_1P_2 \neq P_2P_1$ and $P_3P_4 = P_4P_3$.

## 5 1.6.4

1. If $A$ is invertible, and $AB = AC$, prove quickly(!) that $B = C$. Note down the time you took to prove this. Faster you prove, more credits! :)
2. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, find an example with $AB = AC$ but $B \neq C$.

## 6 1.6.8

Show that $A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$ has no inverse, by solving $Ax = 0$ and by failing to solve:

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

## 7 1.6.18

Under what conditions on their entries are $A$ and $B$ invertible?

$$A = \begin{bmatrix} a & b & c \\ d & e & 0 \\ f & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{bmatrix}$$

## 8 1.6.20

Find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

## 9 1.6.44

If $B$ has the columns of $A$ in reverse order, solve $(A - B)x = 0$ to show that $A - B$ is not invertible. An example will lead you to $x$.

## 10 1.6.48

$M^{-1}$ shows the change in $A^{-1}$ when a matrix is subtracted from $A$. Check the following by verifying if $MM^1 = I$: $M = I - UV \Rightarrow M^{-1} = I_n + U(I_m - VU)^{-1}V$.