Homework 2

Due April 15.

1. **Reading assignment:** Sections 1.5 (Triangular Factors and Row Exchanges), 1.6 (Inverses and Transposes), 2.1 (Vector Spaces and Subspaces) and 2.2 (Solving $Ax = 0$ and $Ax = b$) of the textbook.

2. Do problems 1.5.4, 1.5.18, 1.5.40, 1.5.42, 1.6.4, 1.6.8, 1.6.18, 1.6.20, 1.6.44, 1.6.48.

3. Find all possible values for a, b, c, d, e and f such that

$$\begin{pmatrix} a & 1 \\ b & 1 \\ c & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ d & e & f \end{pmatrix} = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}.$$

What follows is the text of the relevant problems from the $200 text book:

1.5.4. Apply elimination to produce L and U factors for:

$$A = \begin{pmatrix} 2 & 1 \\ 8 & 7 \end{pmatrix}, \qquad A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}, \qquad A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{pmatrix}.$$

1.5.18. Decide whether the following systems are singular or non-singular, and whether they have no solutions, one solution or infinitely many:

$$\begin{aligned} v - w &= 2 \\ u - v &= 2 \\ u - w &= 2 \end{aligned}$$

and

$$\begin{aligned} v - w &= 0 \\ u - v &= 0 \\ u - w &= 0 \end{aligned}$$

and

$$\begin{aligned} v + w &= 1 \\ u + v &= 1 \\ u + w &= 1 \end{aligned}$$

1.5.40. There are 12 "even" permutations of (1,2,3,4), with an even number of exhanges. Two of them are (1,2,3,4) with no exchanges, and (4,3,2,1) with two exchanges. List the other ten. Instead of writing each 4×4 matrix, use the numbers 1, 2, 3, 4 to give the position of 1 in each of them.

1.5.42. If P_1 and P_2 are permutation matrices, so is $P_1 P_2$. This still has the rows of I in some order. Give examples with $P_1 P_2 \neq P_2 P_1$ and $P_3 P_4 = P_4 P_3$.

1.6.4. (a) If A is invertible, and $AB = AC$, prove that $B = C$.

(b) If $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, find an example with $AB = AC$ but $B \neq C$.

1.6.8. Show that $A = \begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}$ has no inverse, by solving $Ax = 0$ and by failing to solve:

$$\begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

1.6.18. Under what conditions on their entries are A and B invertible?

$$A = \begin{pmatrix} a & b & c \\ d & e & 0 \\ f & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{pmatrix}.$$

1.6.20. Find the inverse of

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}.$$

1.6.44. If B has the columns of A in reverse order, solve $(A - B)x = 0$ to show that $A - B$ is not invertible. An example will lead you to x.

1.6.48. Check if $MN = I$ when $M = I - UV$ and $N = I_n + U(I_m - VU)^{-1}V$.