ECE 130C Homework 2 Solution

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1.5.4 L and U are $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$; $\begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 1/3 & 1/4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ 0 & 8/3 & 2/3 \\ 0 & 0 & 5/2 \end{bmatrix}$; $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{bmatrix}$.

1.5.18 Singular, no solution; Singular, infinitely many solutions; Nonsingular, one solution.

1.5.40 $(3, 1, 2, 4)$ and $(2, 3, 1, 4)$ keep only 4 in position; six more even P's keep 1 or 2 or 3 in position; $(2, 1, 4, 3)$ and $(3, 4, 1, 2)$ exchange two pairs. Then $(1, 2, 3, 4)$ and $(4, 3, 2, 1$ make a total of twelve even P's.

1.5.42 $P_1P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \neq P_2P_1$. $P_3P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = P_4P_3$.

1.6.4 (a) $A^{-1}AB = A^{-1}AC \Rightarrow B = C$. (b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$.

1.6.8 $Ax = 0$ for $x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$; $AB = I$ is not solvable.

1.6.18 All of e, f, c must be nonzero; e and $ad - bc$ must be nonzero.

1.6.20 $A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/4 & 1 & 0 & 0 \\ -1/4 & -1/3 & 1 & 0 \\ -1/4 & -1/3 & -1/2 & 1 \end{bmatrix}.$

1.6.44 The solution is $x = (1, 1, \ldots, 1)$: all ones.

1.6.48 $$\begin{aligned} MM^{-1} &= (I_n - UV)(I_n + U(I_m - VU)^{-1}V) \\ &= I_n - UV + U(I_m - VU)^{-1}V - UVU(I_m - VU)^{-1}V \\ &= I_n - UV + U(I_m - VU)(I_m - VU)^{-1}V \\ &= I_n \end{aligned}$$

3. $a + d = 0;\ \ a + e = -1;\ \ a + f = -2\ \Rightarrow$

$$3a + (d + e + f) = -3 \tag{1}$$

similarly, we can get

$$3b + (d + e + f) = 0 \tag{2}$$

and,

$$3c + (d + e + f) = 3 \tag{3}$$

then (3) – (1), we get

$$c - a = 2 \Rightarrow a = c - 2$$

(3) – (2), we get

$$c - b = 1 \Rightarrow b = c - 1$$

Also, we know that $c + d = 2; c + e = 1; c + f = 0$. So

$$d = 2 - c$$

$$e = 1 - c$$

$$f = -c$$