ECE130C HW3 Addendum

April 16, 2009

Problem 2.1.20

True or False for $\mathbf{M}$ = all 3×3 matrices. Check addition using an example.

1. The skew-symmetric matrices in $\mathbf{M}$ (with $A^T = -A$) form a subspace.
2. The asymmetric matrices in $\mathbf{M}$ with ($A^T \neq A$) form a subspace.
3. The matrices that have $(1, 1, 1)$ in their nullspace form a subspace.

Problem 2.1.26

The columns of AB are combinations of columns of A. This means that anything in the column space of AB is in the column space of A. Give an example where the column spaces of AB and A are not equal.

Problem 2.1.28

True or False with counter example if False.

1. The vectors b that are not in the column space $\mathbf{C}(A)$ form a subspace.
2. If $\mathbf{C}(A)$ contains only the zero vector, then A is the zero matrix.
3. The column space of $2A$ equals the column space of A.
4. The column space of $A - I$ equals the column space of A.

Problem 2.2.2

Reduce A and B to echelon form, to find their ranks. Which variables are free?

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Find special solutions to $Ax = 0$ and $Bx = 0$. Find all solutions.

Problem 2.2.8

Under what conditions on b_1 and b_2 (if any) does $Ax = b$ have a solution?

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 7 \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Find two vectors in the nullspace of A, and the complete solution to $Ax = b$.

Problem 2.2.10

Find a 2 by 3 system $Ax = b$, whose complete solution is of the form:

$$x = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + w \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Find a 3 by 3 system with these solitions exactly when $b_1 + b_2 = b_3$.

Problem 2.2.20

If A has rank r, the it has and r by r submatrix S that is invertible. Find that submatrix S from the pivot rows and pivot columns of each A.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}, A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 2.2.36

Which vectors (b_1, b_2, b_3) are in the column space of A? Which combinations of the rows of A give zero?

1. $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$

Problem 2.2.42

If $Ax = b$ has infinitely many solutions, why is it impossible for $Ax = B$ to have only one solution (new right hand side)?Could $Ax = B$ have no solution?

Problem 2.2.50

The complete solution to $Ax = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is $x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find A.