ECE 130C HW4 Addendum

April 23, 2009

Problem 2.3.14

Choose $x = (x_1, x_2, x_3, x_4)$ in $\mathbf{R}^4$. It has 24 rearrangements like (x_2, x_1, x_3, x_4) and (x_4, x_3, x_1, x_2). Those 24 vectors, including the x itself, span a subspace (S). Find the specific vectors x so that the dimensions of $\mathbf{S}$ is : (a)0 (b)1 (c)3 (d)4.

Problem 2.3.26

Suppose $\mathbf{S}$ is a five-dimensional subspace of $\mathbf{R}^6$. True or False?

1. Every basis for $\mathbf{S}$ can be extended to a basis for $\mathbf{S}^6$ by adding one more vector.
2. Every basis for $\mathbf{R}^6$ can be reduced to a basis for $\mathbf{S}$ by removing one vector.

Problem 2.3.34

prove that if $\mathbf{V}$ and $\mathbf{W}$ are three-dimensional subspaces of $\mathbf{R}^5$, then $\mathbf{V}$ and $\mathbf{W}$ must have a non-zero vector in common *Hint:* Start with bases for the two subspaces, making six vectors in all.

Problem 2.3.36

If A is a 64 by 17 matrix of rank 11, how many independent vectors satisfy $Ax = 0$? how many independent vectors satisfy $A^T y = 0$?

Problem 2.3.40

Find a basis for the space of functions that satisfy

1. $\frac{dy}{dx} - 2y = 0$
2. $\frac{dy}{dx} - \frac{y}{x} = 0$

Problem 2.4.28

Construct a matrix with $(1, 0, 1)$ and $(1, 2, 0)$ as a basis for its row space and its column space. Why can't this be a basis for the row space and nullspace?

Problem 2.4.32

Describe the four subspace of $\mathbf{R}63$ associated with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$I + A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 2.4.34

reduce A to echelon form and look at the zero rows. The b column tells which combinations you have taken of the rows:

1. $$\begin{bmatrix} 1 & 2 & b_1 \\ 3 & 4 & b_2 \\ 4 & 6 & b_3 \end{bmatrix}$$

2. $$\begin{bmatrix} 1 & 2 & b_1 \\ 2 & 3 & b_2 \\ 2 & 4 & b_3 \\ 2 & 5 & b_4 \end{bmatrix}$$

Problem 2.4.36

Without multiplying matrices, find bases for row and column spaces of A:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

How do you know from these that A is not invertible?

Problem 2.4.38

if $AB = 0$, the columns of B are in the nullspace of A. If those vectors are in $\mathbf{R}^n$, prove that $\text{rank}(A) + \text{rank}(B) \leq n$