Home Work 5
Due May 20, 2009
Two pages

1. **Reading Assignment**. Read Chapter 4 of the text book. Also *browse* through section 3.5 of the text book (it will not be included in the finals).

2. (a) Write the 6x4 incidence matrix A for the graph in Figure 1. The vector (1,1,1,1) is in the nullspace of A, but now there will be $m - n + 1 = 3$ independent vectors that satisfy $A^T y = 0$. Find the three vectors y and connect them to the loops in the graph.

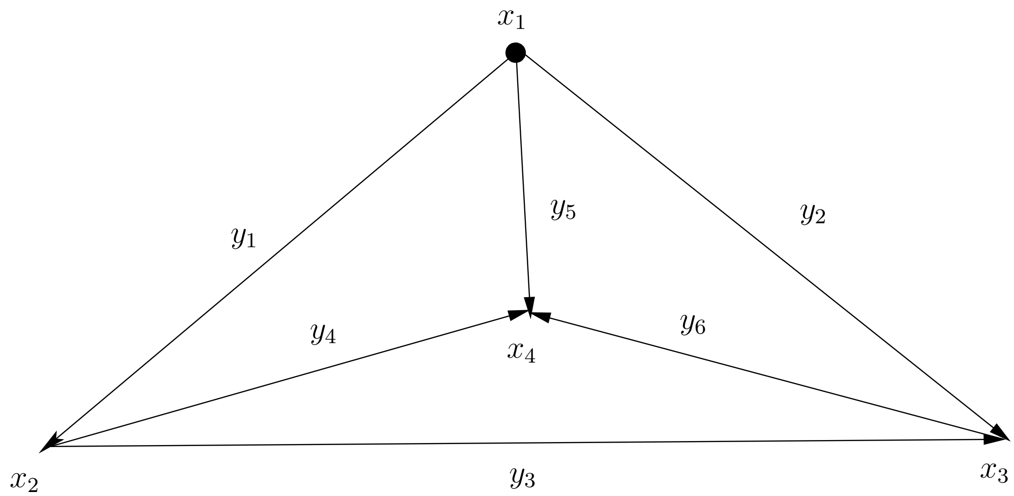

Figure 1: For problem 2.

   (b) Write down the dimensions of the four fundamental subspaces for the incidence matrix A from the previous problem, and a basis for each subspace.

3. Every straight line remains straight after a linear transformation. If z is halfway between x and y, show that Az is halfway between Ax and Ay.

4. From the cubics P_3 of cubic polynomials to the fourth degree polynomials P_4, what matrix represents multiplication by $2 + 3t$? The columns of the 5x4 matrix A come from applying the transformation to $1, t, t^2, t^3$.

5. In the vector space P_3 of all $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$, let **S** be the subset of polynomials with $\int_0^1 p(x)dx = 0$. Verify that **S** is a subspace and find a basis.

6. Give an example in $\mathcal{R}^2$ of linearly independent vectors that are not orthogonal. Also give an example of orthogonal vectors that are not independent (*Hint*: think of a trivial vector).

7. Suppose **S** is spanned by the vectors $\begin{pmatrix}1 & 2 & 1 & 3\end{pmatrix}$ and $\begin{pmatrix}1 & 3 & 3 & 1\end{pmatrix}$. Find two vectors that span $\mathbf{S}^{\perp}$. This is the same thing as solving $Ax = 0$ for which A?

8. Suppose **S** is contained in a subspace **V**, prove that $\mathbf{S}^{\perp}$ contains $\mathbf{V}^{\perp}$.

9. (Problem 3.2.16) Suppose P is the projection matrix onto a line through a.

   (a) Why is the inner product of x with Py equal to the inner product of Px with y?

   (b) Are the two angles the same? Find their cosines if $a = (1, 1, 1), x = (-2, 0, 1), y = (2, -1, 2)$.

   (c) Why is the inner product of Px with Py again the same? What is the angle between those two?

10. (Problem 3.2.20) Construct the projection matrices P_1 and P_2 onto the lines through a's as defined below. Is it true that $(P_1 + P_2)^2 = P_1 + P_2$? This *would* be true if $P_1 P_2 = 0$.

$$a = (0, 1)^T \text{ for } P_1$$

$$a = (1, -1)^T \text{ for } P_2$$

11. (Problem 3.2.26) Project $a_1 = (0, 1)$ onto $a_2 = (1, -2)$. Then project the result back onto a_1. Draw these projections and multiply the projection matrices $P_1 P_2$. Is this a projection? How about $P_1 + P_2$?