ECE 130C HW5 Solution

May 26, 2008

1. (a) The incidence matrix is:

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

   Entry (i, j) in the incidence matrix relates edge i with node j of the graph. $\begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & -1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & -1 & 1 \end{pmatrix}$ are the vectors that span $N(A^T)$.These come from sending the currents around each of the loops.

   (b) $\mathbf{C}(A)$: $r = 3$, the first 3 columns are a basis. $\mathbf{N}(A)$: $(1, 1, 1, 1)$ is a basis. $\mathbf{C}(A^T)$: $r = 3$, rows 1,2,4 are a basis. $\mathbf{N}(A^T)$: the three vectors in part(a) are a basis.

2. $z = (x + y)/2 \Rightarrow Az = (Ax + Ay)/2$

3. $(2 + 3t)1 = (2 + 3t)$, $(2 + 3t)t = 2t + 3t^2$,$(2 + 3t)t^2 = 2t^2 + 3t^3$, $(2 + 3t)t^3 = 2t^3 + 3t^4$

$$T = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

4. Let $p(x), q(x) \epsilon \mathbf{S} \Rightarrow \int_0^1 (cp(x) + dq(x))dx = c \int_0^1 p(x)dx + d \int_0^1 q(x)dx = 0.$

   So, $cp(x) + dq(x)\epsilon\mathbf{S}$ and $\mathbf{S}$ is a subspace.$\frac{-1}{2} + x$, $\frac{-1}{3} + x^2$, $\frac{-1}{4} + x^3$ is a basis for $\mathbf{S}$. We obtain this basis by integrating the polynomial basis and subtracting them from the basis themselves. This works because the measure of that interval is 1.

5. Linearly independent but not orthogonal: $(1,2),(1,0)$

   Orthogonal but not linearly independent: $(1,2),(0,0)$

6. $(y_1, y_2, y_3) = (1, 1, -1)$.

7. Figure not shown. Effect of multiplying by A is pretty clear from the figure in the book.

8. Consider the matrix,

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 1 & 3 \\ 3 & 1 \end{pmatrix}$$

   Then, $\mathbf{S} = \mathbf{C}(A)$. We also know that $\mathbf{C}(A) \perp \mathbf{N}(A^T)$. Hence we are essentially looking for a basis for $\mathbf{N}(A^T)$

$$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 1 & 3 & 3 & 1 \end{pmatrix} y = \mathbf{0}$$

   The general solution to the above homogenous equation takes the form:

$$\begin{pmatrix} -7 \\ 2 \\ 0 \\ 1 \end{pmatrix} \alpha + \begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \end{pmatrix} \beta$$

   , which has the required basis.

9. $\mathbf{S} \subset \mathbf{V} \Rightarrow dim(S) < dim(V)$. Suppose $\mathbf{S}^{\perp}$ is the orthogonal complement of $\mathbf{S}$ in some space of dimension K, then, $dim(\mathbf{S}^{\perp}) = K - dim(\mathbf{S})$. Similarly, $dim(\mathbf{V}^{\perp}) = K - dim(\mathbf{V})$. $dim(\mathbf{S}) < dim(\mathbf{V}) \Rightarrow K - dim(\mathbf{V}) < K - dim(\mathbf{S}) \Rightarrow \mathbf{V}^{\perp} \subset \mathbf{S}^{\perp}$

10. Let $u = 2x - y, v = -x + 2y - z, w = z - y \Rightarrow u + 2v + 2w = 2x - y - 2x + 4y - 2z + 2z - 2y = y \Rightarrow x = \frac{u+y}{2} = u + v + w, z = w + y = u + 2v + 3w$. Hence,

$$\mathbf{T}^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$