ECE 130C HW-7 Solutions

June 3, 2008

Problem 4.2.12

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = (b-a)(b-c) \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} = (b-a)(c-a)(c-b)$$

Problem 4.2.16

(a)

$\det(\mathrm{A}) = 0$ since $R_1 + R_3 = 2R_2$

(b)

$\det(\mathrm{A}) = (1-t^2)^3$ after LU factorization.

Problem 4.3.26

$$|B_4| = 2 \begin{vmatrix} 1 & -1 & \\ -1 & 2 & -1 \\ & -1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 & \\ -1 & 2 & \\ & -1 & -1 \end{vmatrix} = 2|B_3| - |B_2|$$

Pivots are all Ones.

Problem 4.3.34

(a)

Gaussian elimination leads to simultaneous triangularization of both the blocks A and D.

NOTE: A, B,C and D are matrices. So the given matrix is as such not triangular.

If A and D are made triangular, then the overall matrix itself is triangular and so the determinant of the matrix is the product of the determinants of A and D.

(b),(c)

Choose A=[1,0;0 0], B=[0 0;1 0], C=[0 1;0 0] and D=[0 0;0 1]

Problem 4.4.38

Partial derivatives and evaluation of the determinant gives the result. Do not forget the dV. This is the volume increment that is used for volume evaluation.

Problem 4.4.42

$$\begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow x - y + z = 0$$

This is the volume of the bounding box enclosed by the the planes through these points. This is just a plane as a plane passes through these points and so the volume, which is the determinant is ZERO.

Problem 4.4.44

By Cramer's rule, the components of $x = A^{-1}b$ are the ratios $|B_k/|A||$. If $b = e_1$, then $|B_k|$ = Cofactor C_{1k}. Therefore x is the first row of the cofactor matrix divided by det(A), which is the first column of inv(A).