ECE132 Winter 2018: Homework 1 Solution

P1. Calculate the approximate donor binding energy for GaAs (ϵ_r = 13.2, m_n^* = 0.067m_0).

We can assume hydrogen like orbit to calculate the binding energy:

For n=1:

$$\mathrm{E} = \frac{\mathrm{mq}^4}{8 * (\epsilon_0 \epsilon_\mathrm{r})^2 h^2}$$

$$m = 0.067 * 9.11 * 10^{-31}\ kg$$

$$\epsilon_0 = 8.854 * 10^{-12}\ F/m$$

$$\epsilon_r = 13.2$$

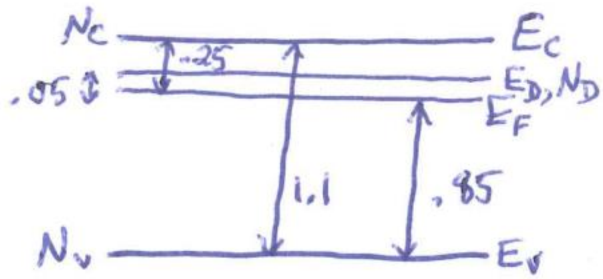
$$h = 6.63 * 10^{-34} J \cdot s$$

So, we can calculate:

$$\mathrm{E} = \frac{\mathrm{mq}^4}{8 * (\epsilon_0 \epsilon_\mathrm{r})^2 h^2} = \frac{0.067 * 9.11 * 10^{-31} kg\ \ * (1.6 * 10^{-19}\ C)^2}{8 * \left(13.2 * 8.854 * \frac{10^{-12} F}{m}\right)^2 * 6.63 * 10^{-34} J \cdot s}$$

$$E = 8.327 * 10^{-22}\ J = 5.204\ meV$$

P2. An unknown semiconductor has Eg = 1.1 eV and Nc = Nv. It is doped with 10^{15} cm^{-3} donors, where the donor level is 0.2 eV below E_c. Given that E_F is 0.25 eV below E_c, calculate n_i and the concentration of electrons and holes in the semiconductor at 300 K.



$$N_d = 10^{15}\ cm^{-3} \qquad donor\ concentration$$

$$1 - f(E_D) \qquad percent\ of\ donors\ which\ are\ ionized$$

$$n = N_d * \left(1 - f(E_D)\right)$$

$$f(E_D) = \frac{1}{1 + e^{\frac{E_D - E_F}{kT}}} = \frac{1}{1 + e^{\left(\frac{0.05}{0.0259}\right)}} = 0.1267$$

$$n = 10^{15} * (1 - 0.1267) = 8.73 * 10^{14} cm^{-3}$$

$$n = N_C \mathrm{e}^{\left(-\frac{E_C - E_F}{kT}\right)}$$

$$N_C = n * e^{\frac{E_C - E_F}{kT}} = 8.73 * 10^{14} cm^{-3} * e^{\frac{0.25}{0.259}}$$

$$N_C = 1.35 * 10^{19} cm^{-3}$$

$$N_C = N_V = 1.36 * 10^{19}\ cm^{-3}$$

$$P = N_V e^{-\frac{E_F - E_V}{kT}} = (1.35 * 10^{-19} cm^{-3}) * e^{-\frac{0.85}{0.259}}$$

$$P = 7.60 * 10^{4} cm^{-3}$$

$$n_i = \sqrt{n * p} = 8.15 * 10^{9} cm^{-3}$$

P3. Calculate the bandgap of Si from $n_i = \sqrt{N_C N_V} e^{-\frac{E_g}{2kT}}$ and from the plot of n_i vs. 1000/*T* (see Fig. 3-17 in Streetman). [*Hint*: The slope cannot be measured directly from a semilogarithmic plot; read the values from two points on the plot and take the natural logarithm as needed for the solution.]

$$n_i = \sqrt{N_C N_V} e^{-\frac{E_g}{2kT}}$$

$$ln(n_i) = \ln\left(\sqrt{N_C N_V}\right) - \frac{E_g}{2kT}$$

Pick two values of n_i from the plot at two different temperatures, we get:

$$\ln(n_{i1}) = \ln\left(\sqrt{N_C N_V}\right) - \frac{E_g}{2kT_1} \qquad eq.1$$

$$\ln(n_{i2}) = \ln\left(\sqrt{N_C N_V}\right) - \frac{E_g}{2kT_2} \qquad eq.2$$

eq.1 – eq.2 we get:

$$\ln(n_{i1}) - \ln(n_{i2}) = \frac{E_g}{2k}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$E_g = \frac{2k}{\left(\frac{1}{T_2} - \frac{1}{T_1}\right)} \ln\left(\frac{n_{i1}}{n_{i2}}\right)$$

From Figure 3.17:

$$n_{i1} = 3 * 10^4 \qquad \frac{1}{T_1} = 2 * 10^{-3}\left(\frac{1}{K}\right)$$

$$n_{i2} = 10^8 \qquad \frac{1}{T_2} = 4 * 10^{-3}\left(\frac{1}{K}\right)$$

Solving for Eg:

$$E_g = 1.286\ eV$$

Result is off from the known value of Eg since we have neglected the temperature dependence of Nc, Nv.

P4. (a) Show that the minimum conductivity of a semiconductor sample occurs when $n_0 = \frac{n_i\sqrt{\mu_p}}{\mu_n}$ [Hint: Begin with $J_x = (n\mu_n + p\mu_p)\mathcal{E}_x = \sigma\mathcal{E}_x$, and apply $n_0p_0 = n_i^2$.

(b) What is the expression for the minimum conductivity σ_{min}?

(c) Calculate σ_{min} for Si at 300 K and compare with the intrinsic conductivity.

(a)

$$\sigma = q\left(n * \mu_n + p * \mu_p\right) = q\left(n * \mu_n + \frac{n_i^2}{n} * \mu_p\right)$$

$$\frac{\delta\sigma}{\delta n} = q(\mu_n - \frac{n_i^2\mu_p}{n^2})$$

$$Set\ \frac{\delta\sigma}{\delta n} = 0$$

$$n^2 = \frac{n_1^2\mu_p}{\mu_n} \quad => \quad n = n_i\sqrt{\frac{\mu_p}{\mu_n}}$$

(b) For minimum conductivity σ_{min}: $n_o = n_i\sqrt{\frac{\mu_p}{\mu_n}}$, *and we can get:*

$$\sigma_{min} = q(n_i\sqrt{\mu_n\mu_p} + n_i\sqrt{\mu_n\mu_p}) = 2qn_i\sqrt{\mu_n\mu_p}$$

(c) For Si at 300 K:

$$n_i = 1.5 * 10^{10} cm^{-3}$$

$$\mu_n = 1350 \frac{cm^2}{V * s}$$

$$\mu_p = 480 \frac{cm^2}{V * s}$$

$$\sigma_{min} = 3.86 * 10^{-6}\ \frac{1}{\Omega * cm}$$

$$\sigma = q\left(n_i\mu_n + p_i\mu_p\right)$$

$$n_i = p_i$$

$$\sigma = 4.4 * 10^{-6} \frac{1}{\Omega * cm}$$

P5: (a) A silicon sample is doped with 3 × 10^{16} cm^{-3} boron atoms and a certain number of shallow donors. The Fermi level is 0.38 eV above E_i at 300 K. What is the donor concentration N_d?

(b) A silicon sample contains 10^{16} cm^{-3} Indium (In) acceptor atoms and a certain number of shallow donors. The In acceptor level is 0.16 eV above E_v, and E_F is 0.26 eV above E_v at 300K. How many In atoms (cm^{-3}) are un-ionized (i.e., neutral)?

(a)

$$n_0 = n_i e^{\frac{E_F - E_i}{kT}} = 1.5 * 10^{10} e^{\frac{0.38}{0.259}}$$

$$n_o = 3.54 * 10^{16}$$

Since the semiconductor is electrostatically neutral:

$$p_0 + N_d^+ = n_0 + N_a^-$$

Since the sample is n-type:

$$n_0 \gg p_o$$

$$N_d = n_0 + N_a \text{ (Assume all the impurities are ionized)}$$

$$N_d = n_0 + N_a = 3.54 * 10^{16} + 3 * 10^{16}$$

$$N_d = 6.54 \times 10^{16}\ cm^{-3}$$

(b) The occupation probability of an energy level is given by:

$$f(E_a) = \frac{1}{1 + e\left(\frac{E_a - E_F}{kT}\right)}$$

$$f(E_a) = \frac{1}{1 + e^{\frac{0.16 - 0.26}{kT}}}$$

$$f(E_a) = 0.9793$$

The Probability of not being occupied number of In atoms are not Ionized:

$$p_{non-ionized} = 1 - f(E_a)$$

$$p_{non-ionized} = 0.020613$$

$$\text{\# of non-ionized In atoms} = (p_{non-ionized}) * 10^{16} = 2.06 * 10^{14}\ cm^{-3}$$