## ECE 132: Introduction to Electronic Devices Fall 2018 Homework 2 Solutions

1. $\mathrm{V}_{\mathrm{AB}}$ has an error associated with it due to the misaligned contacts. Let $\mathrm{V}_{\mathrm{H}}$ represent the value of the Hall voltage without error.

First, take the measurement when the B-field is in the $+z$ direction. Then we can write,

$$
\begin{equation*}
V_{A B 1}=V_{H}-\Delta x E_{x} \tag{1}
\end{equation*}
$$

Where $\Delta x$ represents the misalignment of the probe.

Next, switch the direction of the B-field to the -z direction and again repeat the above measurement. We can then write,

$$
\begin{equation*}
V_{A B 2}=-V_{H}-\Delta x E_{X} \tag{2}
\end{equation*}
$$

Subtracting the two equations, we get

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{AB} 1}-\mathrm{V}_{\mathrm{AB} 2}=2 \mathrm{~V}_{\mathrm{H}} \\
& \mathrm{~V}_{\mathrm{H}}=\left(\mathrm{V}_{\mathrm{AB} 1}-\mathrm{V}_{\mathrm{AB} 2}\right) / 2
\end{aligned}
$$

Note that other solutions are possible as well.
2. n-type Silicon sample with $\mathrm{n}_{\mathrm{o}}=4 \times 10^{16} \mathrm{~cm}^{-3}$

At $t=0,10^{14} \mathrm{EHP} / \mathrm{cm}^{-3}$ are generated as a result of a pulse of light.
$\delta \mathrm{n}=\delta \mathrm{p}=10^{14} \mathrm{~cm}^{-3}$
a) $\tau_{\mathrm{p}}=1 \mu \mathrm{~s}$

Since we know that for a semiconductor at equilibrium, $n_{o} p_{o}=n_{i}{ }^{2}$

$$
p_{o}=\frac{n_{i}^{2}}{n_{o}}=\frac{\left(1.5 \times 10^{10}\right)^{2}}{\left(4 \times 10^{16}\right)}=5.6 \times 10^{3} \mathrm{~cm}^{-3}
$$

The hole concentration would decay exponentially due to the carrier lifetime.

$$
p(t)=10^{14} e^{\frac{-t}{1 \mu s}}+5.6 \times 10^{3} \mathrm{~cm}^{-3}
$$

b) At $\mathrm{t}=1 \mu \mathrm{~s}$, the hole carrier concentration can be calculated as:

$$
p(1 \mu s)=10^{14} e^{\frac{-1 \mu s}{1 \mu s}}+5.6 \times 10^{3}=3.7 \times 10^{13} \mathrm{~cm}^{-3}
$$

3. Given: $\mathrm{N}_{\mathrm{D}}=10^{17} \mathrm{~cm}^{-3}, \mathrm{~g}_{\mathrm{op}}=10^{20} \mathrm{~cm}^{-3} \mathrm{~s}^{-1}, \tau_{\mathrm{p}}=\tau_{\mathrm{n}}=10 \mu \mathrm{~s}, \mathrm{D}_{\mathrm{p}}=12 \mathrm{~cm}^{2} \mathrm{~s}^{-1}$,

$$
D_{n}=36 \mathrm{~cm}^{2} \mathrm{~s}^{-1}, \mathrm{n}_{\mathrm{i}}(450 K)=10^{14} \mathrm{~cm}^{-3}
$$

Assuming complete ionization of donors, $\mathrm{N}_{\mathrm{D}}=\mathrm{n}_{0}=10^{17} \mathrm{~cm}^{-3}$.

$$
\mathrm{p}_{o}(450 \mathrm{~K})=\frac{\left[n_{i}(450 \mathrm{~K})\right]^{2}}{n_{o}}=10^{11} \mathrm{~cm}^{-3}
$$

The excess carriers generated in the system within the hole/electron lifetime $\delta n=\delta p=g_{o p} \tau_{n}=10^{20} \times 10 \times 10^{-6} \mathrm{~s}=10^{15} \mathrm{~cm}^{-3}$
$\mathrm{n}(450 \mathrm{~K})=\mathrm{n}_{0}(450 K)+\delta \mathrm{n}=10^{17} \mathrm{~cm}^{-3}+10^{15} \mathrm{~cm}^{-3} \approx 10^{17} \mathrm{~cm}^{-3}$
$\mathrm{p}(450 \mathrm{~K})=\mathrm{p}_{\mathrm{o}}(450 \mathrm{~K})+\delta \mathrm{p}=10^{11} \mathrm{~cm}^{-3}+10^{15} \mathrm{~cm}^{-3} \approx 10^{15} \mathrm{~cm}^{-3}$
Let us now find the position of the Quasi Fermi levels.

At 450 K , the thermal voltage $\mathrm{V}_{\mathrm{T}}=\mathrm{kT}=1.38 \times 10^{-23} \times 450 \mathrm{~K}=0.039 \mathrm{eV}$.

From Streetman Eqn (4-15) as shown in the text, we can write:
$n=n_{i} e^{\left(F_{n}-E_{i}\right) / k T} \rightarrow \quad F_{n}-E_{i}=k T \ln \left(\frac{n}{n_{i}}\right)$
Therefore, $F_{n}-E_{i}=0.039 \mathrm{eV} \ln \left(\frac{10^{17}}{10^{14}}\right)=0.27 \mathrm{eV}$.
Similarly, we can write: $p=n_{i} e^{\left(E_{i}-F_{p}\right) / k T} \rightarrow E_{i}-F_{p}=k T \ln \left(\frac{p}{n_{i}}\right)$
Thus, $E_{i}-F_{p}=0.039 \mathrm{eV} \ln \left(\frac{10^{15}}{10^{14}}\right)=0.09 \mathrm{eV}$


Let us now calculate the change in conductivity of the sample.

$$
\Delta \sigma=q\left\{\left[\mu_{\mathrm{n}}(450 \mathrm{~K}) \cdot n(450 \mathrm{~K})+\mu_{\mathrm{p}}(450 \mathrm{~K}) \cdot p(450 \mathrm{~K})\right]-\left[\mu_{\mathrm{n}}(300 \mathrm{~K}) \cdot n(300 \mathrm{~K})+\mu_{\mathrm{p}}(300 \mathrm{~K}) \cdot p(300 \mathrm{~K})\right]\right\}
$$

$\rightarrow$ Calculate the mobility using Einstein's relation.

$$
\begin{aligned}
\frac{D_{n}}{\mu_{n}}=\frac{k T}{q} \rightarrow \mu_{n}(450 K) & =\frac{36 \mathrm{~cm}^{2} s^{-1}}{0.039 V}=923 \mathrm{~cm}^{2} V^{-1} \mathrm{~s}^{-1} \\
\mu_{n}(300 K) & =\frac{36 \mathrm{~cm}^{2} s^{-1}}{0.026 \mathrm{~V}}=1385 \mathrm{~cm}^{2} V^{-1} \mathrm{~s}^{-1}
\end{aligned}
$$

Similarly, for holes, we can write,

$$
\begin{aligned}
\frac{D_{p}}{\mu_{p}}=\frac{k T}{q} \rightarrow & \mu_{p}(450 K)=\frac{12 \mathrm{~cm}^{2} s^{-1}}{0.039 V}=307 \mathrm{~cm}^{2} V^{-1} \mathrm{~s}^{-1} \\
& \mu_{p}(300 K)=\frac{12 \mathrm{~cm}^{2} \mathrm{~s}^{-1}}{0.026 \mathrm{~V}}=460 \mathrm{~cm}^{2} V^{-1} \mathrm{~s}^{-1} \\
n(450 K)= & N_{D}+\delta \mathrm{n} \approx N_{D}=10^{17} \mathrm{~cm}^{-3}=n(300 \mathrm{~K}) \\
p(450 K)= & \frac{\left[n_{i}(450 K)\right]^{2}}{n(450 K)}+\delta \mathrm{p} \approx \delta \mathrm{p}=10^{15} \mathrm{~cm}^{-3} \\
p(300 K)= & \frac{\left[n_{i}(300 K)\right]^{2}}{n(300 K)}=\frac{\left[1.5 \times 10^{10} \mathrm{~cm}^{-3}\right]^{2}}{10^{17} \mathrm{~cm}^{-3}}=2.3 \times 10^{3} \mathrm{~cm}^{-3}
\end{aligned}
$$

We can calculate the change in conductivity as,

$$
\begin{array}{r}
\Delta \sigma=\left(1.6 \times 10^{-19} \mathrm{C}\right) \times\left[\left(923 \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1} \times 10^{17} \mathrm{~cm}^{-3}+307 \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1} \times 10^{15} \mathrm{~cm}^{-3}\right)\right. \\
\left.-\left(1385 \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1} \times 10^{17} \mathrm{~cm}^{-3}+460 \mathrm{~cm}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1} \times 2.3 \times 10^{3} \mathrm{~cm}^{-3}\right)\right]
\end{array}
$$

$\Delta \sigma=-7.42 \mathrm{~S} / \mathrm{cm}$ (conductivity has dropped due to reduced electron mobility)
4. Every $\tau_{p}$ seconds, each hole of the charge distribution $Q_{p}$ recombines and must therefore be replaced by injection of another hole.

Thus, we can write $I_{p}=\frac{Q_{p}}{\tau_{p}}$

From Figure (4-17), we know that $\delta \mathrm{p}(\mathrm{x})=\Delta \mathrm{p} e^{-x / L_{p}}$

$$
\begin{gathered}
Q_{p}=q A \int_{0}^{\infty} \delta \mathrm{p}(x) d x \\
Q_{p}=q A \int_{0}^{\infty} \Delta \mathrm{p} e^{-x / L_{p}} d x \\
Q_{p}=q A \Delta \mathrm{p} \int_{0}^{\infty} e^{-x / L_{p}} d x \\
Q_{p}=-q A \Delta \mathrm{p} L_{p}
\end{gathered}
$$

Now, we know that $L_{p}=\sqrt{D_{p} \tau_{p}} \rightarrow \tau_{p}=\frac{L_{p}{ }^{2}}{D_{p}}$

Thus, $I_{p}=\frac{Q_{p}}{\tau_{p}}=\frac{-q A \Delta \mathrm{p} L_{p}}{\left(L_{p}{ }^{2} / D_{p}\right)}=\frac{-q A \Delta \mathrm{p} D_{p}}{L_{p}}$

