Homework 4 Solutions:

PROBLEM 1: An abrupt Si p^{+}-n diode has $N_D = 10^{16}$ cm^{-3} on the *n* side and $N_A = 10^{17}$ cm^{-3} on the *p* side. For Si at room temperature, E_G = 1.1 eV, N_C = 2.8 x 10^{19} cm^{-3}, and N_V = 1.8 x 10^{19} cm^{-3}. Assume the minority carrier lifetime is 8 μs (for both electrons and holes), the electron mobility is 1400 cm^2/V·s, and the hole mobility is 500 cm^2/V·s.

PART A: Find the depletion region width under zero bias on the p-side (w_{p0}) and on the n-side (w_{n0}), and the total depletion width $w_{tot,0}$.

$$V_0 = \frac{kT}{q} ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.0259 * \ln\left(\frac{10^{33}}{2.25 * 10^{20}}\right) = 0.754\ V$$

$$W_{tot}(V_A) = \sqrt{\left(\frac{2\epsilon}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(V_o - V_A)\right)}$$

$$\epsilon_{Si} = 11.7\epsilon_o = 11.7 * 8.85 * 10^{-14} = 1.04 * 10^{-12} F/cm$$

$$W_{tot} = \sqrt{\frac{2 * 1.04 * 10^{-12} F/cm}{1.6 * 10^{-19}\ C} * \left(\frac{1}{10^{16}\ cm^{-3}} + \frac{1}{10^{17}\ cm^{-3}}\right) * (V_o - V_A)}$$

$$= (3.77 * 10^{-5}\ cm \cdot V^{-1/2}) * \sqrt{V_o - V_A} = (0.377\ \mu m \cdot V^{-1/2}) * \sqrt{V_o - V_A}$$

At V_A = 0:

$$W_{tot}(0\,V) = (0.377\ \mu m \cdot V^{-\frac{1}{2}})\sqrt{0.754\,V} = 0.33\ \mu m$$

$$x_p = \frac{W * N_d}{N_a + N_d} = 0.33 * \frac{10^{16}}{1.1 * 10^{17}} = 0.03\ \mu m$$

$$x_n = \frac{W * N_a}{N_a + N_d} = 0.33 * \frac{10^{17}}{1.1 * 10^{17}} = 0.30\ \mu m$$

PART B: If a forward bias of 0.2 V is applied, find the resulting depletion widths (w_p, w_n, and w_{tot}), the electron current density J_n through the depletion region, the hole current density J_p through the depletion region, and the total current density J_{tot} through the diode.

At V_A = 0.2 V:

$$W_{tot}(0.2\,V) = (0.377\ \mu m \cdot V^{-\frac{1}{2}})\sqrt{0.754\,V - 0.2\,V} = 0.28\ \mu m$$

$$x_p = \frac{W * N_d}{N_a + N_d} = 0.28 * \frac{10^{16}}{1.1 * 10^{17}} = 0.026\ \mu m$$

$$x_n = \frac{W * N_a}{N_a + N_d} = 0.28 * \frac{10^{17}}{1.1 * 10^{17}} = 0.255\ \mu m$$

$$J_n = \frac{qD_n n_p}{L_n}\left(e^{qV_A/kT} - 1\right) \qquad J_p = \frac{qD_p p_n}{L_p}\left(e^{qV_A/kT} - 1\right)$$

$$D_n = \frac{kT}{q}\mu_n = 36.3\ \frac{cm^2}{s} \qquad D_p = \frac{kT}{q}\mu_p = 12.95\ \frac{cm^2}{s}$$

$$L_n = \sqrt{D_n \tau_n} = 1.70 * 10^{-2}\ cm \qquad L_p = \sqrt{D_p \tau_p} = 1.02 * 10^{-2}\ cm$$

$$n_p = \frac{n_i^2}{N_A} = 2.25 * 10^3 cm^{-3} \qquad p_n = \frac{n_i^2}{N_D} = 2.25 * 10^4 cm^{-3}$$

$$J_n = (7.7 \times 10^{-13}\ A/cm^2)\left(e^{.2/.0259} - 1\right) = 1.7 \times 10^{-9}\ A/cm^2$$

$$J_p = (4.6 \times 10^{-12}\ A/cm^2)\left(e^{.2/.0259} - 1\right) = 1.02 \times 10^{-8}\ A/cm^2$$

$$J_{tot} = J_n + J_p = 1.2 \times 10^{-8}\ A/cm^2$$

Problem 2:

$$\mathbf{W} = A * \sqrt{\frac{N_A + N_D}{N_A N_D}} \qquad A = \sqrt{\frac{2\epsilon}{q}(V_0 - V_A)}$$

PART A:

In Problem 1, $N_D = 0.1N_A$.

$$If\ \boldsymbol{N_{D,new} = 2N_D \rightarrow N_{D,new} = 0.2N_A}$$

$$W = \mathrm{A} * \sqrt{\frac{1.1\mathrm{N_A}}{0.1N_A^2}} \qquad W_{new} = A * \sqrt{\frac{1.2N_A}{0.2N_A^2}}$$

$$\frac{W_{new}}{W} = \sqrt{\frac{6}{11}} = 73.9\%$$

$$\boxed{\frac{\Delta W}{W} = -26.1\%}$$

$$If\ \boldsymbol{N_{A,new} = 2N_A \rightarrow N_{A,new} = 20N_D}$$

$$W = \mathrm{A} * \sqrt{\frac{11\mathrm{N_D}}{10N_D^2}} \qquad W_{new} = A * \sqrt{\frac{21N_D}{20N_D^2}}$$

$$\frac{W_{new}}{W} = \sqrt{\frac{21}{22}} = 97.7\%$$

$$\boxed{\frac{\Delta W}{W} = -2.3\%}$$

PART B:

N doping is increased by a factor of 2

$$\mathrm{P_{new}} = \frac{1}{2}P_n = 1.125 * 10^4 cm^{-3}$$

$$\mathrm{J} = \mathrm{q}(4.8 * 10^6 + 14.325 * 10^6) = 19.125 * 10^6 * q = 3.06 * 10^{-16}\ A/cm^2$$

$$\frac{\mathrm{J_{new}}}{\mathrm{J}} = \frac{3.06}{5.352} = 57.2\%$$

$$\frac{\Delta \mathrm{J}}{\mathrm{J}} = -42.8\%$$

P doping is increased by a factor of 2

$$\mathrm{N_{new}} = \frac{1}{2} n_p = 1.125 * 10^3 cm^{-3}$$

$$\mathrm{J} = \mathrm{q}(2.4 * 10^6 + 28.65 * 10^6) = 31.05 * 10^6 * q = 4.968 * 10^{-12}\ A/cm^2$$

$$\frac{\mathrm{J_{new}}}{\mathrm{J}} = \frac{4.968}{5.352} = 92.9\%$$

$$\frac{\Delta \mathrm{J}}{\mathrm{J}} = -7.1\%$$

PART C:

As seen above, varying the doping on the heavily doped side only causes very small changes in the depletion width and current density in the diode, whereas varying the doping on the lightly doped side causes much larger variations in the resulting depletion width and current density.

Problem 3: In a P^+N junction, the hole diffusion current in the neutral *n* material is given by Eq. 5.32 in Streetman. What are the electron diffusion and electron drift components of current at point x_n in the neutral *n* region?

$$I_p(x_n) = \frac{qAD_p}{L_p} * p_n e^{qV/kT} e^{-x_n/L_p} \qquad for\ V \gg \frac{KT}{q}$$

$$I_{tot} \cong I_p(x_n = 0) = \frac{qAD_p}{L_p} p_n e^{qV/kT}$$

Assuming space charge neutrality, the excess hole distribution is equal to the excess electron distribution:

$$\delta n(x_n) = \delta p(x_n)$$

$$I_{n,diff}(x_n) = qAD_n \frac{d\delta n}{dx_n} = \frac{qAD_n}{L_p} p_n e^{qV/kT}\ e^{-x_n/L_p}$$

$Since\ I_{tot}\ is\ constant\ we\ have:$

$$I_{n,drift}(x_n) = I_{tot} - I_{n,diff}(x_n) - I_p(x_n)$$

$$I_{n,drift}(x_n) = \frac{qAp_n}{L_p} e^{\frac{qV}{KT}} \left[D_p \left(1 - e^{-\frac{x_n}{L_p}}\right) + D_n e^{-\frac{x_n}{L_p}} \right]$$

Problem 4: Assume that a P^+N diode is built with a quasi-neutral *n* region having a width *l* which is smaller than the hole diffusion length ($l < L_p$). This is a so-called *narrow base diode*. Since for this case holes are injected into a shorter *n* region under forward bias, we cannot use the boundary condition $\delta p(x' = \infty) = 0$ as in Eq. 4-35 in Streetman. Instead, our boundary condition becomes $\delta p(x' = l) = 0$.

Part A: Solve the diffusion equation for this case to obtain $\delta p(x') = \frac{\Delta p_n\left[e^{\frac{l-x'}{L_P}} - e^{-\frac{l-x'}{L_p}}\right]}{e^{\frac{l}{L_p}} - e^{-\frac{l}{L_p}}}$

$$\frac{d^2\delta p}{d(x')^2} = \frac{\delta p(x')}{L_p^2}$$

$$\delta p(x') = Ce^{-\frac{x'}{L_p}} + De^{\frac{x'}{L_p}}$$

Most of the holes will diffuse across the narrow *n* region without recombining. At the contact:

$$At\ x' = l: \quad \delta p = 0$$

$$At\ x' = 0: \delta p = \Delta p_n$$

$$\Delta p_n = C + D$$

$$0 = Ce^{-\frac{l}{L_p}} + De^{\frac{l}{L_p}}$$

$\boldsymbol{Solving}$:

$$C = \frac{\Delta p_n e^{\frac{l}{L_p}}}{e^{\frac{l}{L_p}} - e^{-\frac{l}{L_p}}}$$

$$D = \Delta p_n - C = -\frac{\Delta p_n e^{-\frac{l}{L_p}}}{e^{\frac{l}{L_p}} - e^{-\frac{l}{L_p}}}$$

plugging C and D:

$$\delta p(x') = \frac{\Delta p_n\left[e^{\frac{l-x'}{L_p}} - e^{\frac{-l-x'}{L_p}}\right]}{e^{\frac{l}{L_p}} - e^{-\frac{l}{L_p}}}$$

Part B: If l << L_p show $\delta p(x') = \Delta p_n \left(1 - \frac{x\prime}{l}\right)$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$$

$$e^x - e^{-x} = 2\left[2 + \frac{x^3}{3!} + \cdots\right]$$

$$e^{\frac{l}{L_p}} - e^{\left(-\frac{l}{L_p}\right)} = 2\left[\frac{l}{L_p}\right] = 2 \qquad since\,\frac{l}{L_p} \ll 1 \qquad \frac{x^3}{3!} \ll x$$

$$e^{\frac{l-x\prime}{L_p}} - e^{\left(\frac{-(l-x\prime)}{L_p}\right)} = 2\left[\frac{l-x'}{L_p}\right]$$

$$\delta p(x') = \Delta p_n\left[\frac{l-x'}{l}\right] = \Delta p_n\left[1 - \frac{x'}{l}\right]$$