ECE 132: Homework 5 Solutions

Problem 1:

A Si *n-p-n* BJT has emitter, base and collector doping levels of 10^{19} cm^{-3}, $5\text{x}10^{18}$ cm^{-3}, and 10^{17} cm^{-3}, respectively. It is biased in the normal active mode, with a base-emitter voltage V_{BE} = 1V and a collector-emitter voltage V_{CE} = 4V. During operation, the current through the device causes is to heat up to 400 K, such that $n_i = 10^{12}$ cm^{-3}, and ε_r = 15. Assume electron and hole mobilities of 500 and 100 cm^2/V-s, respectively, in the emitter, and 800 and 250 cm^2/V-s, respectively, in the base. Assume the minority carrier lifetimes are 1 ns everywhere. If the neutral base width is 500 nm and the neutral emitter is 3 µm wide, calculate the emitter current density J_E, the emitter injection efficiency γ, and the base transport factor α_T. Qualitatively sketch the device structure showing the minority carrier concentrations in the emitter and the base, and sketch the band diagram under bias below it.

Solution:

V_{BE} = 0.7V, V_{CE} = 4V, T = 400K, $n_i = 10^{12}$ cm^{-3}, ε_r = 15, W_B = 500 nm, W_E = 3 µm, $\tau_n = \tau_p = 10^{-9}$ s.

In the emitter: $\mu_{n,e}$ = 500 cm^2/V-s, $\mu_{p,e}$ = 100 cm^2/V-s

In the base: $\mu_{n,b}$ = 800 cm^2/V-s, $\mu_{p,b}$ = 250 cm^2/V-s.

The thermal voltage at 400K is given by $\frac{kT}{q} = 0.0259 * \frac{400}{300} = 0.0345V$.

We first need to determine whether the minority carrier profile is linear in the emitter (which occurs when $W_E \ll L_{p,E}$):

$$D_{p,E} = \frac{kT}{q}\mu_p = 0.0345*100\ \text{cm}^2/\text{V-s} = 3.45\ \text{cm}^2/\text{s}$$

$$L_{p,E} = \sqrt{D_{p,E}\,\tau_p} = \sqrt{(3.45\ cm^2 s^{-1} * 10^{-9} s)} = 5.87 \times 10^{-5} cm = 0.587\ \mu m$$

$W_E > L_{p,E}$, so this is a long-emitter BJT. We can (optionally) verify that the minority carrier profile is linear in the base (which occurs when $W_B \ll L_{n,B}$):

$$D_{n,B} = \frac{kT}{q}\mu_{n,B} = 0.0345\text{V} * 800\ \text{cm}^2/\text{V-s} = 27.6\ \text{cm}^2/\text{s}$$

$$L_{n,B} = \sqrt{D_{n,B}\,\tau_p} = \sqrt{(27.6\ cm^2 s^{-1} * 10^{-9} s)} = 1.66 \times 10^{-4} cm = 1.66\ \mu m$$

$W_B < L_{n,B}$, so minority carrier profile in the base is linear (as it should be). Next calculate $J_{n,E}$ and $J_{p,E}$ to determine J_E and γ.

$$J_{n,E} = \frac{qD_{n,B}n_i^2}{W_B N_{A,B}}\left(e^{qV_{BE}/kT} - 1\right)$$

$$\frac{qD_{n,B}n_i^2}{W_B N_{A,B}} = \frac{(1.6 * 10^{-19})(27.6)(10^{12})^2}{(500 * 10^{-7})(5 * 10^{18})} = 1.76 * 10^{-8}\ A/cm^2$$

$$J_{n,E} = \frac{qD_{n,B}n_i^2}{W_B N_{A,B}}\left(e^{qV_{BE}/kT} - 1\right) = 1.76 * 10^{-8} * \left(e^{1V/0.0345V} - 1\right) = 6.84 \times 10^4\ A/cm^2$$

$$J_{p,E} = \frac{qD_{p,E}n_i^2}{L_{p,E} N_{D,E}}\left(e^{qV/kT} - 1\right)$$

$$\frac{qD_{p,E}n_i^2}{L_{p,E} N_{D,E}} = \frac{(1.6 * 10^{-19})(3.45)(10^{12})^2}{(0.587 * 10^{-4})(10^{19})} = 9.4 * 10^{-10}\ A/cm^2$$

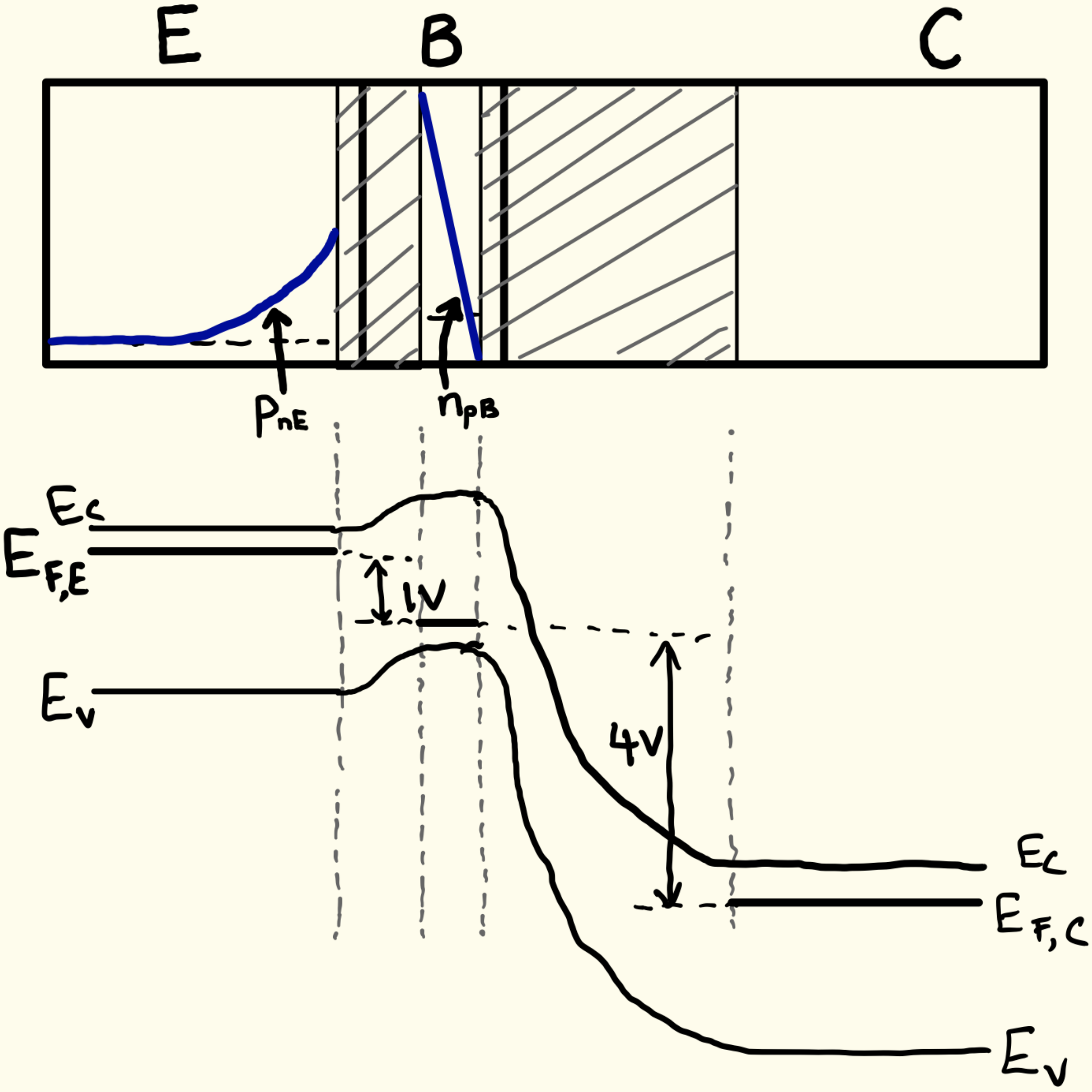
$$J_{p,E} = \frac{qD_{p,E}n_i^2}{L_{p,E} N_{D,E}}\left(e^{qV/kT} - 1\right) = 9.4 * 10^{-10} * \left(e^{1V/0.0345V} - 1\right) = 3.64 \times 10^3\ A/cm^2$$

$$J_E = J_{n,E} + J_{p,E} = \boxed{7.2 \times 10^4\ A/cm^2}$$

$$\gamma = \frac{J_{n,E}}{J_{n,E} + J_{p,E}} = \frac{6.84 \times 10^4}{7.2 \times 10^4} = \boxed{0.95}$$

Next we determine α_T:

$$\alpha_T = 1 - \frac{(w_B)^2}{2L_{n,B}^2} = 1 - \frac{(500 \times 10^{-7}\ cm)^2}{2(1.66 \times 10^{-4} cm)^2} = \boxed{0.954}$$


E
B
C
p_{nE}
n_{pB}
E_c
$E_{F,E}$
1V
E_v
4V
E_c
$E_{F,C}$
E_v

Problem 2: A Si p-n-p transistor has the following properties at room temperature:

$\tau_p = \tau_n = 0.1\mu s, D_n = D_p = 10\ cm^2/s\ , \ N_E = 10^{19}cm^{-3} = emitter\ conc.$

$N_B = 10^{16}cm^{-3} = base\ conc,\ N_C = 10^{16}cm^{-3} = collector\ \ conc$

$W_E = emitter\ width = 3\mu m$

$W = metallurgical\ base\ width = 1.5\mu m$
$= distance\ between\ base\ emitter\ junction\ and\ base\ collector\ junction$

$A = cross - sectional\ area = 10^{-5}cm^2$

Calculate the neutral base width W_b for V_{CB} = 0V and V_{EB} = 0.2V. Repeat for V_{EB} = 0.6V.

Solution:

The built-in voltage at the base-emitter junction $(V_{bi})_{BE}$ is given by:

$$(V_{bi})_{BE} = \frac{kT}{q} \ln\left[\frac{N_B N_E}{(n_i)^2}\right]$$

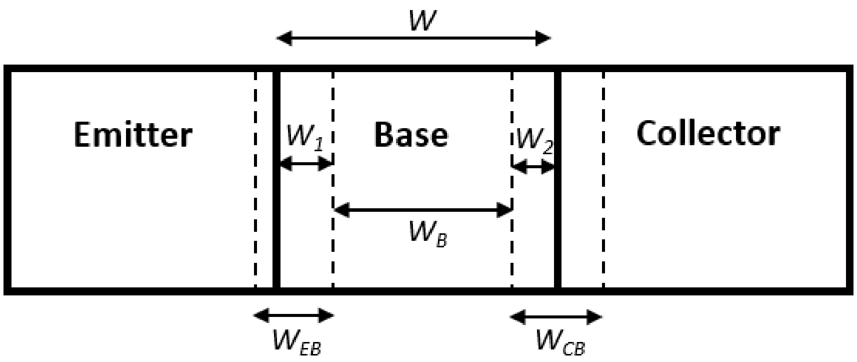
$$(V_{bi})_{BE} = 0.026 \ln\left(\frac{10^{16} * 10^{19}}{2.25 * 10^{20}}\right) = 0.877V$$

The built-in voltage at the base-collector junction $(V_{bi})_{BC}$ is given by:

$$(V_{bi})_{BC} = \frac{kT}{q} \ln\left[\frac{N_B N_C}{(n_i)^2}\right]$$

$$(V_{bi})_{BC} = 0.026 \ln\left(\frac{10^{16} * 10^{16}}{2.25 * 10^{20}}\right) = 0.697V$$

We now need to find W_B:



Referring to the figure above, $W_B = W - W_1 - W_2$, so we need to first find W_1 and W_2:

$$W_{EB} = \sqrt{\frac{2\varepsilon_S}{qN_B}(V_{bi} - V_{EB})} = \sqrt{\frac{2*11.8*8.854*10^{-14}}{1.6*10^{-19}*10^{16}}(0.877V - V_{EB})}$$

$$W_1 = W_{EB}\left(\frac{N_E}{N_E + N_B}\right) \cong W_{EB} = \sqrt{\frac{2 * 11.8 * 8.854 * 10^{-14}}{1.6 * 10^{-19} * 10^{16}}(0.877V - V_{EB})}$$

For V_{EB} = 0.2V, W_1 = 2.97 x 10^{-5} cm = 0.30 μm.

For V_{EB} = 0.6V, W_1 = 1.90 x 10^{-5} cm = 0.19 μm.

$$W_{CB} = \sqrt{\frac{2\varepsilon_S}{q}\frac{(N_C+N_B)}{N_C N_B}(V_{bi} - V_{CB})} = \sqrt{\frac{2*11.8*8.854*10^{-14}*2}{1.6*10^{-19}*10^{16}}(0.697V)} = 4.27 \times 10^{-5} cm$$

$$W_2 = W_{CB}\left(\frac{N_C}{N_C + N_B}\right) = {}^{W_{CB}}/_{2} = 2.14 \times 10^{-5} cm = 0.21\ \mu m$$

Finally: $W_B = W - W_1 - W_2$

When V_{EB} =0.2V: W_B = 1.5μm – 0.30μm – 0.21μm = $\boxed{0.99\ \mu\text{m}}$

When V_{EB} =0.6V: W_B = 1.5μm – 0.19μm – 0.21μm = $\boxed{1.10\ \mu\text{m}}$

Problem 3: For the BJT in the previous problem, calculate the values of γ, α_T, β, I_E, I_B, and I_C for the two values of V_{EB}.

Solution: First let's calculate α_T for both values of V_{EB}.

$$\alpha_T = \frac{1}{(\cosh\,(\frac{W_b}{L_p})} \approx 1 - \frac{1}{2}\left(\frac{W_b}{L_p}\right)^2 \quad (Approximation\ can\ be\ made\ if\ \frac{W_b}{L_p} \ll 1)$$

It is given that $\tau_n = \tau_p = 0.1\mu s,\ D_n = D_p = 10\frac{cm^2}{s}$.

Thus, $L_n = L_p = \sqrt{D_n \tau_n} = \sqrt{(10\ cm^2 s^{-1} \ast 0.1 \ast 10^{-6} s)} = 10\mu m$

For both values of V_{EB}, $W_b \ll L_p$, so above approximation holds.

When V_{EB} = 0.2V → $\alpha_T = 1 - \frac{1}{2}\left(\frac{W_b}{L_p}\right)^2 = 1 - 0.5 \ast \left(\frac{0.985\mu m}{10\mu m}\right)^2 = \boxed{0.995}$

When V_{EB} = 0.6V → $\alpha_T = 1 - \frac{1}{2}\left(\frac{W_b}{L_p}\right)^2 = 1 - 0.5 \ast \left(\frac{1.09\mu m}{10\mu m}\right)^2 = \boxed{0.994}$

The next step is to find the injector efficiency (γ): $\gamma = \frac{I_{E,P}}{I_{E,P} + I_{E,N}}$, where

$I_{E,P}$ = diffusion injected across the B-E junction by the emitter (holes for p-n-p transistors)

$I_{E,N}$ = diffusion injected across the B-E junction by the base (electrons for p-n-p transistors)

$$\frac{I_{e,p}}{A} = \frac{qD_p n_i^2}{W_B N_B}\left(e^{qV_{EB}/kT} - 1\right)$$

When V_{EB} = 0.2V: $I_{e,p} = \frac{AqD_p n_i^2}{W_B N_B}\left(e^{qV_{EB}/kT} - 1\right) = \frac{10^{-5}(1.601 \ast 10^{-19})(10)(1.5 \ast 10^{10})^2}{(0.985 \ast 10^{-4})(10^{16})} e^{0.2/0.0259}$

$$I_{e,p} = 8.25 \ast 10^{-12} A$$

$$I_{e,n} = \frac{AqD_p n_i^2}{W_E N_E}\left(e^{qV_{EB}/kT} - 1\right) = \frac{10^{-5}(1.601 \ast 10^{-19})(10)(1.5 \ast 10^{10})^2}{(3 \ast 10^{-4})(10^{19})} e^{0.2/0.0259}$$

$$I_{e,n} = 2.70 \ast 10^{-15} A$$

$$I_E = I_{e,p} + I_{e,n} \cong \boxed{8.25 \ast 10^{-12} A}$$

$$\gamma = \frac{I_{E,P}}{I_{E,P} + I_{E,N}} = \boxed{0.9997}$$

$$I_C = \gamma\alpha_T I_E = 0.9947 \times 8.25 * 10^{-12}A = \boxed{8.21 * 10^{-12}\text{A}}$$

$$\beta = \frac{\gamma\alpha_T}{1 - \gamma\alpha_T} = \frac{0.9947}{1 - 0.9947} = \boxed{187.7}$$

$$I_B = \frac{I_C}{\beta} = \boxed{4.37 * 10^{-14}\text{A}}$$

Similarly for V_{EB} = 0.6V:

$$I_{e,p} = 3.8 * 10^{-5}A$$

$$I_{e,n} = 1.38 * 10^{-8}A$$

$$I_E = I_{e,p} + I_{e,n} \cong \boxed{3.8 * 10^{-5}A}$$

$$\gamma = \frac{I_{E,P}}{I_{E,P} + I_{E,N}} = \boxed{0.9996}$$

$$I_C = \gamma\alpha_T I_E = 0.9936 \times 3.8 * 10^{-5}A = \boxed{3.78 * 10^{-5}\text{A}}$$

$$\beta = \frac{\gamma\alpha_T}{1 - \gamma\alpha_T} = \frac{0.9936}{1 - 0.9936} = \boxed{153.3}$$

$$I_B = \frac{I_C}{\beta} = \boxed{2.47 * 10^{-7}\text{A}}$$