HW2 Solution

1.

$$I_S = \frac{27-9}{15k\Omega} = 1.20mA \rightarrow I_L < 1.20\ mA\ \mid\ R_L > \frac{9V}{1.2mA} = 7.50\ k\Omega$$

2.

$$I_Z = \frac{V_S - V_Z}{R_S} - \frac{V_Z}{R_L} = \frac{V_S}{R_S} - V_Z\left(\frac{1}{R_S} + \frac{1}{R_L}\right) \quad \mid \quad P_Z = V_Z I_Z$$

$$I_Z^{nom} = \frac{(60-15)V}{150\Omega} - \frac{15V}{100\Omega} = 150\ mA \quad \mid \quad P_Z^{nom} = 15V(150mA) = 2.25\ W$$

$$I_Z^{\max} = \frac{60V(1.1)}{150\Omega(0.90)} - 15V(0.90)\left(\frac{1}{150\Omega(0.90)} + \frac{1}{100\Omega(1.1)}\right) = 266\ mA$$

$$P_Z^{\max} = 15V(0.90)(266mA) = 3.59\ W$$

$$I_Z^{\min} = \frac{60V(0.90)}{150\Omega(1.1)} - 15V(1.1)\left(\frac{1}{150\Omega(1.1)} + \frac{1}{100\Omega(0.9)}\right) = 43.9\ mA$$

$$P_Z^{\min} = 15V(1.1)(43.9mA) = 0.724\ W$$

3.

$V_1 = V_P - V_{on} = 49.3$ V and $V_2 = -(V_P - V_{on}) = -49.3$V.

4.

$$i_D(0^+) = \frac{5V}{1k\Omega} = 5\ mA\ \mid\ I_F = \frac{5-V_D}{1k\Omega} = \frac{5-0.6}{1k\Omega} = 4.4\ mA$$

$$I_r = \frac{-3-0.6}{1k\Omega} = -3.6\ mA\ \mid\ \tau_S = (7ns)\ \ln\left(1 - \frac{4.4mA}{-3.6mA}\right) = 5.59\ ns$$

5.

$$i_D(0^+) = \frac{5V}{5\Omega} = 1\ A\ \mid\ I_F = \frac{5-V_{on}}{5\Omega} = \frac{5-0.6}{1\Omega} = 0.880\ A$$

$$I_R = \frac{-3-0.6}{5\Omega} = -0.720\ A\ \mid\ \tau_S = (250ns)\ \ln\left(1 - \frac{0.880A}{-0.720A}\right) = 200\ ns$$

Additional Problems

1. (a)

$$\omega = \frac{1}{\sqrt{LC}} \qquad C_{diode} = \frac{C_{j0}A}{\sqrt{1-\frac{V}{\phi_j}}}$$

Change in voltage results in variation of depletion width. This results in change in capacitance which changes frequency as a result. Thus, output frequency can be modulated by changing the input voltage.

(b)

$$\omega = 2\pi f = 2\times\pi\times 13.56\times 10^{6} = 85.2\times 10^{6}\, rad.\sec^{-1}$$

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow LC = \frac{1}{\omega^2} = \frac{1}{\left(85.2\times 10^{6}\right)^2} = 1.38\times 10^{-16}$$

Assume, $V_{bias} = 5V$ [To operate the diode in reverse bias]

Area = 100um x 125 um = 1.25x10^{-8} m^{2}

$N_A = 10^{17} cm^{-3}, N_D = 10^{20} cm^{-3}$

$$\varphi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.025\times\ln\left(\frac{10^{17}\times 10^{20}}{10^{20}}\right) = 0.979V$$

$$W_{d0} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\varphi_j} = \sqrt{\frac{2\times 11.7\times 8.854\times 10^{-14}\times 10^{-4}}{1.6\times 10^{-19}}\times\left(\frac{1}{10^{17}}+\frac{1}{10^{20}}\right)\times 0.979} = 0.1126\times 10^{-6}\, m$$

$$C_{j0} = \frac{\varepsilon_s}{W_{d0}} = \frac{11.7\times 8.854\times 10^{-14}\times 10^{2}}{0.1126\times 10^{-6}} = 9.2\times 10^{-4}\, F/m^2$$

$$C_{diode} = \frac{C_{j0}A}{\sqrt{1-\frac{V}{\phi_j}}} = \frac{9.2\times 10^{-4}\times 1.25\times 10^{-8}}{\sqrt{1+\frac{5}{0.979}}} = 4.65\times 10^{-12} F$$

$$L = \frac{1.38\times 10^{-16}}{C_{diode}} = \frac{1.38\times 10^{-16}}{4.65\times 10^{-12}} = 2.97\times 10^{-5} H$$

(c)

$$C_{j0} = \frac{\varepsilon_s}{W_{d0}}$$

$$C = \frac{C_{j0} A}{\sqrt{1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}}} = \frac{\varepsilon_s A}{W_{d0}\sqrt{1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{L}} \times \frac{1}{\sqrt{\varepsilon_s A}} \times \sqrt{W_{d0}}\left[1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}\right]^{1/4}$$

$$=> \frac{df}{dV_{in}(t)} = \sqrt{\frac{W_{d0}}{\varepsilon_s AL}} \times \frac{1}{8\pi} \times \frac{1}{\varphi_j} \times \left[1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}\right]^{-3/4}$$

$$=> \frac{df}{dV_{in}(t)} = \frac{1}{8\pi\varphi_j}\sqrt{\frac{W_{d0}}{\varepsilon_s AL}} \times \left[1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}\right]^{-3/4}$$

(d)

$$f(x) = \left[1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}\right]^{-3/4}$$

$$=> f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots\ldots..$$

$where,\ x = V_{in},\ x_0 = -V_{bias}$

$$f(x_0) = \left[1 + \frac{-V_{bias} + V_{bias}}{\varphi_j}\right]^{-\frac{3}{4}} = 1$$

$$f'(x) = -\frac{3}{4} \times \frac{1}{\varphi_j}\left[1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}\right]^{-\frac{7}{4}} => f'(x_0) = -\frac{3}{4\varphi_j}$$

$$f''(x) = \frac{21}{16} \times \frac{1}{\varphi_j^{\ 2}} \times \left[1 + \frac{V_{in}(t) + V_{bias}}{\varphi_j}\right]^{-\frac{11}{4}} => f''(x_0) = \frac{21}{16} \times \frac{1}{\varphi_j^2}$$

So, $f(x) = 1 + \frac{3}{4\varphi_j}(V_{in}(t) + V_{bias}) + \frac{21}{32\varphi_j^2}(V_{in}(t) + V_{bias})^2$

$$f(x)\Big|_{V_b=0} = 1 + \frac{3}{4\varphi_j} V_{in}(t) + \frac{21}{32\varphi_j^2} V_{in}{}^2(t)$$

$$=> 0.68V_{in}(t)^2 - 0.08V_{in}(t) - 0.1 < 0$$

For linear relationship,

$$V_{in}(t) = \frac{0.04 \pm \sqrt{0.0016 + 0.068}}{0.68} = \frac{0.04 \pm 0.26}{0.68} = 0.44V, -0.32V \, (\textit{not acceptable})$$

$$=> V_{in}(t) < 0.44V$$

Substituting $V_{in}(t) = 0.44V$,

$$\frac{df}{dV_{in}(t)} = 5.4 \times 10^6 \, Hz$$

2.

(a) The circuit consists of two pumping clocks, φ and $\bar{\varphi}$, which are anti-phase and have voltage amplitude of V_φ. The diodes operate as self-timed switches characterized by a forward bias voltage, V_d.

The multiplier operates by pumping charge along the diode chain as the capacitors are successively charged and discharged during each clock cycle. When clock phase goes low, diode D1 conducts until the voltage at node 1 becomes $V_{in} - V_d$. When φ is switched to V_φ, the voltage at node 1 becomes $V_{in} + (V_\varphi - V_d)$.

This causes diode D2 to conduct until the voltage at node 2 becomes equal to $V_{in} + (V_\varphi - V_d) - V_d$. When φ goes low again, the voltage at node 2 becomes $V_{in} + 2(V_\varphi - V_d)$. After N stages, it is easy to see that the output voltage is

$$V_{out} = V_{in} + N(V_\varphi - V_d)$$

So, in this case, $V_{out} = V_{in} + 4(V_\varphi - V_d)$

(b) When V_{in} is ac, the diodes will work as half-wave rectifiers. The expression for output voltage will be the same except that V_{in} will be replaced by V_0

$$V_{out} = V_0 + 4(V_\varphi - V_d)$$

3.

The circuit is behaving like a half-wave rectifier. The capacitor should charge during the first 1/2 cycle, but it is not. Therefore, diode D_1 is not functioning properly. It behaves as an open circuit.