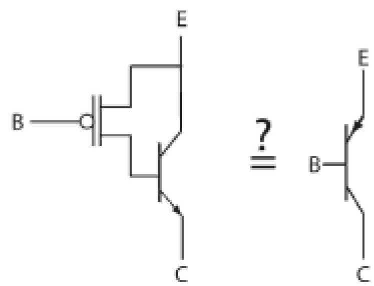
Problem Set 4

Additional Problems

1.



a) The definition of a transistor (Transfer Resistor) is a device that has transconductance, in other words, the device takes an input voltage, and gives an output current. To operate a PNP bipolar device in the forward active region, we need to forward bias the Emitter-base junction, and reverse bias the Base-collector junction. In terms of terminal voltages, Ve-Vb>Vbi, and Vb>Vc. Looking at our PMOS/NPN composite device, if we make Veb greater than the threshold voltage of the PMOS device (Vthp), i.e., Ve-Vb>Vthp, assuming the initial drain voltage Vd is at zero Volt, then the device is in strong inversion, and a current will flow, charging up node Vd, as soon as Vd goes above Vbi of the NPN device, the NPN transistor turns on, giving rise to a current Ic. At this point, Vd is pinned at Vbi of the NPN device, because any additional current will now go to the emitter. At least operation wise, the composite device operates as an PNP bipolar transistor.

b) Assuming PMOS is in saturation

$$I_d = K_p'(V_{eb} - V_{thp})^2 = Ib_{NPN}$$
$$Ic_{NPN} = \beta \cdot Ib_{NPN}$$
$$Ic = \beta \cdot K_p'(V_{eb} - V_{thp})^2$$

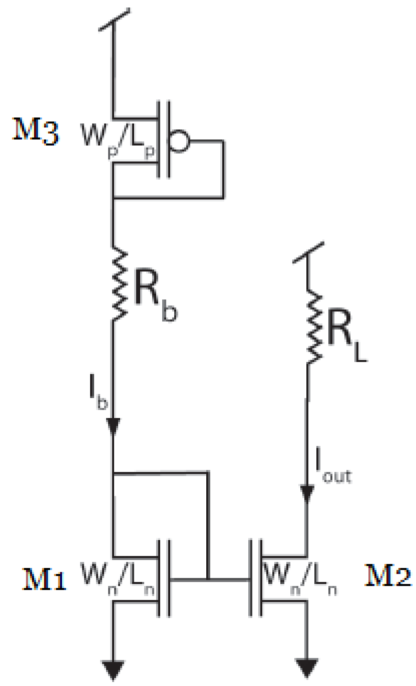
c) The device will show linear[1] and saturation behaviors like that of a PMOS transistor

d) Vbeon on the device is the threshold voltage of the PMOS device, vthp

e) The device does not take any input current, so β is infinite

f) compared to a normal PMOS, the output current of the composite device is increased by factor of (β+1).

2. (a)



The drain and gate of M1 and M3 are connected. So you can think of the two transistors as nonlinear resistors. Then Ib is determined by the three resistors (M1, Rb, and M3). Now we have M1 and M2 current mirror, which means Ib of M1 is copied to Iout of M2. Depending on the size ratio of M1 and M2, Iout is proportional to Ib.

$$I_b = \frac{K_n}{2}\left(\frac{W}{L}\right)_{n1}\left(V_{gsn} - V_{Tn}\right)^2$$

$$I_{out} = \frac{K_n}{2}\left(\frac{W}{L}\right)_{n2}\left(V_{gsn} - V_{Tn}\right)^2$$

$$\Rightarrow I_{out} = \frac{\left(\frac{W}{L}\right)_{n2}}{\left(\frac{W}{L}\right)_{n1}} I_b$$

If $\left(\frac{W}{L}\right)_{n1} = \left(\frac{W}{L}\right)_{n2}$, $I_{out} = I_b$

(b)

$I_{out} = 25\mu A = I_b$

Assume, $\left(\frac{W}{L}\right)_n = 10, \left(\frac{W}{L}\right)_p = 20$

$$V_{DD} = V_{gsn} + I_b R_b + V_{sgp}$$
$$=> V_{DD} = V_{Tn} + \sqrt{\frac{2I_b}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}} + I_b R_b + \left|V_{Tp}\right| + \sqrt{\frac{2I_b}{\mu_p C_{ox}\left(\frac{W}{L}\right)_p}}$$
$$=> 5 = 1.5 + 25\times10^{-6} R_b + 0.16 + 0.18$$
$$=> 3.16 = 25\times10^{-6} R_b$$
$$=> R_b = 126 \ k\Omega$$

(c)

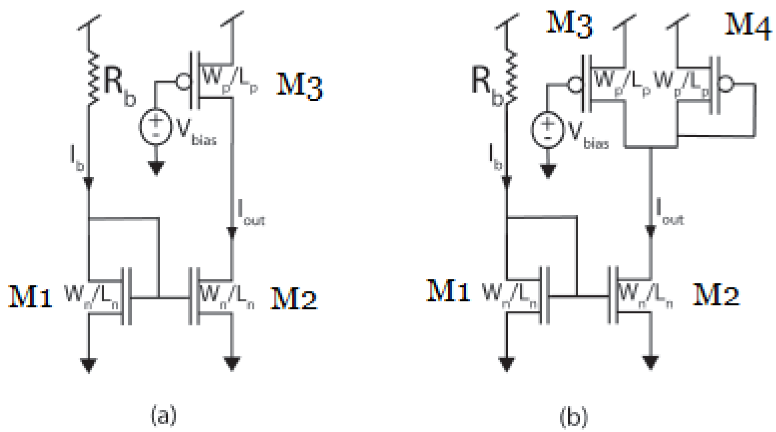
Since the circuit works as a current source, I_{out} should not depend on R_L

The circuit does not work for all values of R_L

$$V_{DD} - I_{out} R_L = V_{gsn} - V_{Tn}$$
$$=> 5 - 25\times10^{-6} R_L = 0.16$$
$$=> R_L = 193.6 \ k\Omega$$

When $R_L > 193.6\ k\Omega$, M2 will operate in linear region and therefore, it becomes a linear resistor.

3.



(a)

$$\left(\frac{W}{L}\right)\left(\frac{100\mu A}{V^2}\right)(V_{gs}-V_T)^2=\frac{V_{DD}-V_{gs}}{R_b}$$

$$=> \frac{W}{L}=\frac{V_{DD}-V_{gs}}{\left(\frac{100\mu A}{V^2}\right)(V_{gs}-V_T)^2 R_b}\geq 1$$

$$=> V_{DD}-V_{gs}\geq 100R_b\left(V_{gs}^2-2V_{gs}V_T+V_T^2\right)$$

$$=> 100R_bV_{gs}^2-(200R_bV_T-1)V_{gs}+100R_bV_T^2-V_{DD}\leq 0$$

$$=> V_{gs}\leq\frac{(200R_bV_T-1)\pm\sqrt{400R_b(V_{DD}-V_T)+1}}{200R_b}$$

So the maximum value of V_{gs} is

$$V_{gs}=\frac{(200R_bV_T-1)+\sqrt{400R_b(V_{DD}-V_T)+1}}{200R_b}$$

(b)

$$Assume,\quad \left(\frac{W}{L}\right)_n=1$$

$$I_b = \left(\frac{W}{L}\right)_n \left(\frac{100\mu A}{V^2}\right)(V_{gsn} - V_T)^2$$

$$=> 100\mu A = 1\left(\frac{100\mu A}{V^2}\right)(V_{gsn} - 0.75)^2$$

$$=> V_{gsn} = 1.75V$$

$$R_b = \frac{V_{DD} - V_{gs\,n}}{I_b} = \frac{5 - 1.75}{100\mu A} = 32.5k\Omega$$

(c)

Assume, V_{bias} = 2V

$$R_3 = R_b$$

$$=> \frac{1}{40 \times 10^{-6} A/V^2 \left(\frac{W}{L}\right)_p (V_{DD} - V_{bias} - |V_{Tp}|)} = 32.5 \times 10^3$$

$$=> \left(\frac{W}{L}\right)_p = \frac{1}{40 \times 10^{-6} A/V^2 (5 - 2 - 0.75) \times 32.5 \times 10^3} = 0.34$$

(d)

R4 is diode connected device, the small-signal resistance of R4 is given by 1/gm4.

$$R_4 = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_{DD} - V_{out} - |V_{Tp}|)}$$

R3 is working in linear region, but if Vsd is large, the square term in its current equation takes effect. Let's write its current equation in linear region first. Then find its output resistance by taking derivative over V_{out}.

$$I_3 = \mu_p C_{ox} \left(\frac{W}{L}\right)_p \left[(V_{DD} - V_{bias} - |V_{Tp}|)(V_{DD} - V_{out}) - \frac{1}{2}(V_{DD} - V_{out})^2\right]$$

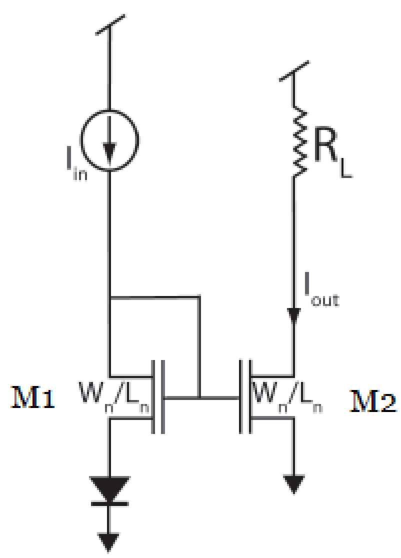
$$R_3 = \left(\frac{\partial I_3}{\partial (V_{DD} - V_{out})} \right)^{-1} = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L} \right)_p \left(V_{out} - V_{bias} - |V_{Tp}| \right)}$$

$$R_3 \| R_4 = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L} \right)_p \left(V_{DD} - V_{out} - |V_{Tp}| + V_{out} - V_{bias} - |V_{Tp}| \right)}$$

$$= \frac{1}{\mu_p C_{ox} \left(\frac{W}{L} \right)_p \left(V_{DD} - V_{bias} - 2|V_{Tp}| \right)}$$

Hence, adding another diode connected PMOS transistor in parallel with M3 eliminate nonlinearity or in other words remove the dependence on Vout.

4.



(a)

$$V_{gs2} = V_{gs1} + V_{on}$$

$$V_{gs1} = V_{Tn} + \sqrt{\frac{2I_{in}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}}$$

$$V_{gs2} = V_{Tn} + \sqrt{\frac{2I_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}}$$

$$V_{gs2} = V_{gs1} + V_{on}$$

$$\Rightarrow \sqrt{\frac{2I_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}} = \sqrt{\frac{2I_{in}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}} + V_{on}$$

(b)

If the voltage across the diode is V_1

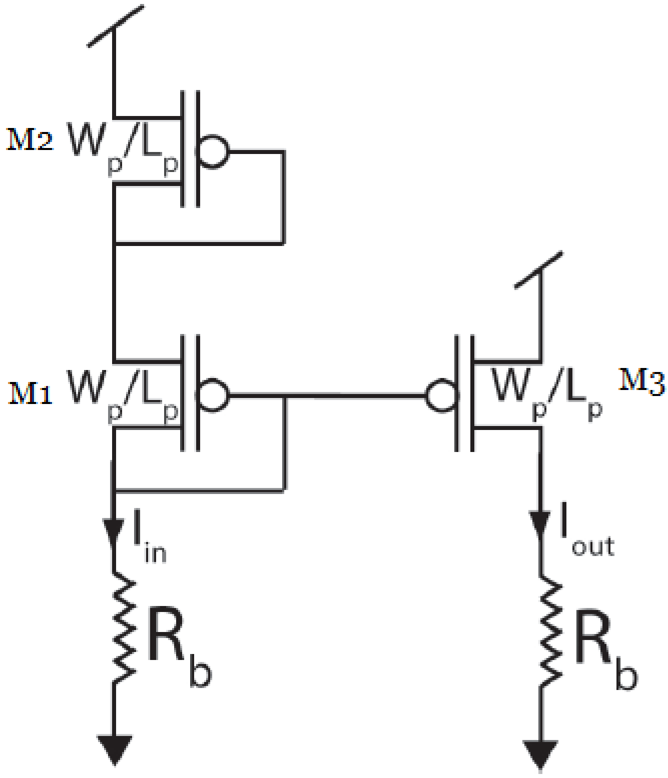
$$I_{in} = I_s\left(e^{\frac{qV_1}{kT}} - 1\right)$$

$$\Rightarrow V_1 = \frac{kT}{q}\ln\left(\frac{I_{in}}{I_s} + 1\right)$$

$$V_{gs2} = V_{gs1} + V_1$$

$$\Rightarrow \sqrt{\frac{2I_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}} = \sqrt{\frac{2I_{in}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}} + \frac{kT}{q}\ln\left(\frac{I_{in}}{I_s} + 1\right)$$

5.



(a)

$$V_{sg1} + V_{sg2} = V_{sg3}$$

$$=> |V_{Tp}| + \sqrt{\frac{2I_{in}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_1}} + |V_{Tp}| + \sqrt{\frac{2I_{in}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}} = |V_{Tp}| + \sqrt{\frac{2I_{out}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}}$$

$$=>=> |V_{Tp}| + \sqrt{\frac{2I_{in}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_1}} + \sqrt{\frac{2I_{in}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}} = \sqrt{\frac{2I_{out}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}}$$

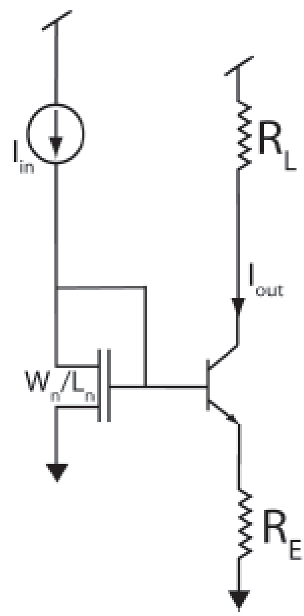
(b) Assume, $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 20$

$$\left|V_{Tp}\right| + \sqrt{\frac{2I_{in}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_1}} + \left|V_{Tp}\right| + \sqrt{\frac{2I_{in}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}} + I_{in}R_b = V_{DD}$$

$$=> 1.5 + 0.176 + 0.176 + 25\times10^{-6} R_b = 5$$

$$=> R_b = 125.6\ k\Omega$$

6.



$$V_{gsn} = V_{on} + I_{out}R_E$$

$$=> V_{Tn} + \sqrt{\frac{2I_{in}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}} = V_{on} + I_{out}R_E$$

$$=> R_E = \frac{V_{Tn} + \sqrt{\frac{2I_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_n}} - V_{on}}{I_{out}} \qquad \left[\sin ce, \quad I_{in} = I_{out}\right]$$

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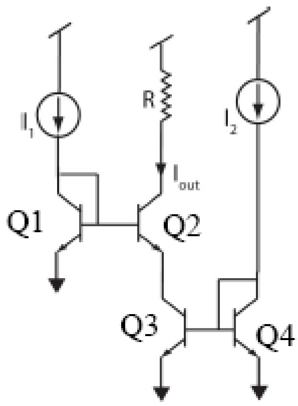

$$V_{be1} = \frac{kT}{q} \ln\left(\frac{I_1}{I_s}\right)$$

$$I_{out} = I_s \exp\left(q \frac{(V_{be1} - V_x)}{kT}\right)$$

$$I_{out} = I_s \exp\left(\frac{qV_{be1}}{kT}\right) \exp(-qV_x/kT) = I_1 \exp(-qV_x/kT)$$

$$V_x = \frac{-kT}{q} \ln\left(\frac{I_{out}}{I_1}\right)$$

$$I_3 = I_{out} = Is \exp\left(\frac{qV_{be4}}{kT}\right)\left(1 - \exp\left(\frac{-qV_x}{kT}\right)\right) = I_2\left(1 - \exp\left(\frac{-qV_x}{kT}\right)\right)$$

$$I_{out} = I2 - I2 \exp\left(\frac{-qV_x}{kT}\right)$$

plugging in Vx to the preceding equation, we have

$$I_{out} = I_2 - I_2 \exp\left(\ln\left(\frac{I_{out}}{I_1}\right)\right)$$

$$I_{out} = I_2 - I_2 \frac{I_{out}}{I_1}$$

$$I_{out} = \frac{I_1 I_2}{I_1 + I_2}$$