ECE 137A Winter 2009

Homework 4 Solutions

4.33

(a) Since $V_{DS} = V_{GS}$ and $V_{TN} > 0$ for both transistors, both devices are saturated.

Therefore $$I_{D1} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2 \quad and \quad I_{D2} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2.$$

From the circuit, however, I_{D2} must equal I_{D1} since $I_G = 0$ for the MOSFET:

$$I = I_{D1} = I_{D2} \quad or \quad \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2 = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2$$

which requires $V_{GS1} = V_{GS2}$. Using KVL:

$$V_{DD} = V_{DS1} + V_{DS2} = V_{GS1} + V_{GS2} = 2V_{GS2}$$

$$V_{GS1} = V_{GS2} = \frac{V_{DD}}{2} = 5V$$

$$I = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2 = \frac{100}{2}\frac{\mu A}{V^2}\frac{10}{1}\left(5 - 0.75\right)^2 V^2 = 9.03\ mA$$

(b) The current simply scales by a factor of two (see last equation above), and $I_D = 18.1$ mA.

(c) For this case,

$$I_{D1} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda V_{DS1}\right) \quad and \quad I_{D2} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2\left(1 + \lambda V_{DS2}\right).$$

Since $V_{GS} = V_{DS}$ for both transistors

$$I_{D1} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda V_{GS1}\right) \quad and \quad I_{D2} = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2\left(1 + \lambda V_{GS2}\right)$$

and $I_{D1} = I_{D2} = I$

$$\frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda V_{GS1}\right) = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS2} - V_{TN}\right)^2\left(1 + \lambda V_{GS2}\right)$$

which again requires $V_{GS1} = V_{GS2} = V_{DD}/2 = 5V$.

$$I = \frac{K_n^{'}}{2}\frac{W}{L}\left(V_{GS1} - V_{TN}\right)^2\left(1 + \lambda V_{DS}\right) = \frac{100}{2}\frac{\mu A}{V^2}\frac{10}{1}\left(5 - 0.75\right)^2 V^2\left(1 + (.04)5\right) = 10.8\ mA$$

4.54

(a) For $V_{IN} = 0$, the NMOS device is on with $V_{GS} = 5$ and $V_{SB} = 0$, and the PMOS transistor is off with $V_{GS} = 0$, $V_O = 0$, and $V_{SB} = 0$.

$$R_{on} = \frac{1}{\left(100x10^{-6}\right)(10)(5 - 0.75)} = 235\ \Omega$$

(b) For $V_{IN} = 5V$, the NMOS device is off with $V_{GS} = 0$, and the PMOS transistor is on with $V_{GS} = -5V$, $V_O = 5V$, and $V_{SB} = 0$.

$$R_{on} = \frac{1}{\left(40x10^{-6}\right)(25)\left|(-5 + 0.75)\right|} = 235\ \Omega$$

4.103

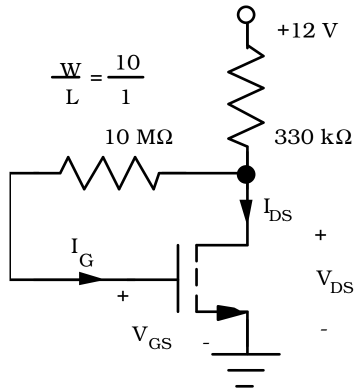
(a) The transistor is saturated by connection.

$V_{GS} = 12 - 10^5 I_D$ and $I_D = \frac{100x10^{-6}}{2}\left(\frac{10}{1}\right)\left(\frac{A}{V^2}\right)\left(V_{GS} - 0.75V\right)^2$

$50V_{GS}^2 - 74V_{GS} + 16.13 = 0 \Rightarrow V_{GS} = 1.214V,\ -0.266V \Rightarrow V_{GS} = 1.214\ V$ since V_{GS} must exceed the threshold voltage. | $I_D = \frac{100x10^{-6}}{2}\left(\frac{10}{1}\right)\left(\frac{A}{V^2}\right)\left(1.214 - 0.75V\right)^2$

$I_D = 104\mu A$ | Checking: $I_D = \frac{12 - 1.21}{10^5} = 108\ \mu A$ | Q-Point: $\left(108\ \mu\text{A}, 1.21\ \text{V}\right)$

(b) Using KVL, $V_{DS} = 10^7\ I_G + V_{GS}$. But, since $I_G = 0$, $V_{GS} = V_{DS}$. Also $V_{TN} = 0.75\ V > 0$, so the transistor is saturated by connection.



$$I_D = \frac{K_n'}{2}\frac{W}{L}(V_{GS} - V_{TN})^2 = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\left(\frac{10}{1}\right)(V_{GS} - 0.75)^2$$

$$V_{GS} = 12 - 330k\Omega(I_D + I_G) - 10M\Omega(I_G) \quad \text{but} \quad I_G = 0$$

$$V_{GS} = 12 - 330k\Omega(I_D)$$

$$V_{GS} = 12 - (3.30x10^5)\left(\frac{1.00x10^{-3}}{2}\frac{A}{V^2}\right)(V_{GS} - 0.75)^2$$

$$165V_{GS}^2 - 246.5V_{GS} + 80.81 = 0 \quad \text{yields} \quad V_{GS} = 1.008V,\ 0.486V$$

V_{GS} must be 1.008 V since 0.486 V is below threshold.

$$I_D = \left(\frac{100}{2}\frac{\mu A}{V^2}\right)\frac{10}{1}(1.008 - 0.75)^2 = 33.3\ \mu A \text{ and } V_{DS} = V_{GS}$$

Q-Point: (33.3 µA, 1.01 V)   Checking: I_D =(12-1.01)V/330kΩ = 33.3 µA. ✔

4.121

For V_{GS} = 5 V and V_{DS} = 0.5 V, the transistor will be in the triode region.

$$I_D = \frac{(5-0.5)V}{82k\Omega} = 54.88\mu A \ \mid\ 54.88x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)\left(5 - 0.75 - \frac{0.5}{2}\right)0.5 \ \mid\ \frac{W}{L} = \frac{0.274}{1} = \frac{1}{3.64}.$$

4.122

For V_{GS} = 3.3 V and V_{DS} = 0.25 V, the transistor will be in the triode region.

$$I_D = \frac{(3.3-0.25)V}{180k\Omega} = 16.94\mu A \ \mid\ 16.94x10^{-6} = 100x10^{-6}\left(\frac{W}{L}\right)\left(3.3 - 0.75 - \frac{0.25}{2}\right)0.25 \ \mid\ \frac{W}{L} = \frac{0.280}{1} = \frac{1}{3.57}$$

4.123

(a) The transistor is saturated by connection. For this circuit,

$$V_{GS} = V_{DD} + I_D R = -15 + 75000 I_D$$

$$I_D = \frac{4x10^{-5}}{2}\left(\frac{1}{1}\right)(-15 + 75000 I_D + 0.75)^2 \Rightarrow 153\ \mu A$$

$$V_{GS} = -15 + 75000 I_D = -3.525V$$

$$V_{DS} = V_{GS} = -3.525V \ \mid\ \text{Q-point: } (153\ \mu A, -3.53\ V)$$

(b) Here the transistor has V_{GS} = -15 V, a large value, so the transistor is most likely operating in the triode region.

$$I_D = \frac{V_{DS} - (-15)}{75000} = 4x10^{-5}\left(-15 - (-0.75) - \frac{V_{DS}}{2}\right)V_{DS} \Rightarrow V_{DS} = -0.347\ V \text{ and } I_D = 195\ \mu A.$$

$$\text{Checking: } I_D = \frac{15 - 0.347}{785k\Omega}V = 195\mu A \qquad \text{Q - point: } (195\ \mu\text{A}, -0.347\ V)$$

Checking the region of operation: $V_{DS} = -0.347V > V_{GS} - V_{TP} = -15 + 0.75 = -14.25V$

Triode region is correct

4.142

Note: The answers are very sensitive to round-off error and are best solved iteratively using MATLAB, a spreadsheet, HP solver, etc. Hand calculations using the quadratic equation will generally yield poor results.

Saturated by connection with $V_{TP} = -1$

$$I_D = \frac{4x10^{-5}}{2}\left(\frac{10}{1}\right)\left[3.3x10^5 I_D - 12 - (-1)\right]^2 \rightarrow 121 - 7.265x10^6 I_D + 1.089x10^{11} I_D^2 = 0$$

$I_D = 34.6\mu A,\ 32.1\mu A$ | $V_{DS} = 3.3x10^5 I_D - 12 = -0.582V, -1.407V$ | Q - point: $(32.1\ \mu A, -1.41\ V)$

since the transistor would not be conducting for $V_{GS} = -0.582V$.

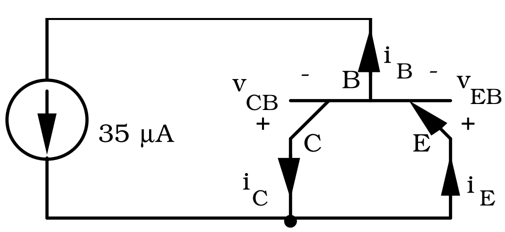
5.9

Using $v_{BC} = 0$ in Eq. 5.13 and recognizing that $i = i_C + i_B = i_E$:

$i = i_E = I_S\left(1 + \frac{1}{\beta_F}\right)\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$, and the reverse saturation current

of the diode connected transistor is $I_S^{'} = I_S\left(1 + \frac{1}{\beta_F}\right) = (2\ fA)\left(1 + \frac{1}{100}\right) = 2.02\ fA$

5.17



(a)-(c)

(b) pnp transistor(d)

$$v_{EB} = v_{CB} \quad i_C = -\frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right] \quad i_E = +\frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right] \quad i_B = +I_S\left(\frac{1}{\beta_F}+\frac{1}{\beta_R}\right)\left[\exp\left(\frac{v_{EB}}{V_T}\right)-1\right]$$

$$\frac{I_E}{I_B} = \frac{\frac{1}{\beta_F}}{\frac{1}{\beta_F}+\frac{1}{\beta_R}} = \frac{\beta_R}{\beta_F+\beta_R} = \frac{4}{79} = 0.0506 \qquad \frac{I_E}{I_B} = -\frac{\beta_R}{\beta_F} = -\frac{4}{75} = -0.0533$$

$$I_B = 35\ \mu A \quad I_E = \frac{4}{79}I_B = 1.77\ \mu A \quad I_C = -\frac{75}{4}I_E = -33.2\ \mu A$$

$$V_{EB} = V_T \ln\left(1-\frac{\beta_R I_C}{I_s}\right) \qquad V_{CB} = V_{EB} = 0.025V \ln\left(1-\frac{4\left(-33.2x10^{-6}A\right)}{2x10^{-15}A}\right) = 0.623\ V$$

5.37

(a) I_B is forced to be negative by the current source, and the largest negative base current according to the Transport model is

$$I_B = -I_S\left(\frac{1}{\beta_F}+\frac{1}{\beta_R}\right) = -10^{-15}A\left(\frac{1}{50}+\frac{1}{0.5}\right) = -2.02\ fA$$

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(b) I_B is forced to be -1 mA by the current source. One or both of the junctions must enter the breakdown region in order to supply this current. For the case of a normal BJT, the base-emitter junction will break down and supply the current since it has the lower reverse breakdown voltage.

5.65

Both transistors are in the forward-active region. For simplicity, assume $V_A = \infty$.

$I = I_{C1} + I_{B1} + I_{B2}$ | Since the transistors are identical and have the same V_{BE},

$I_{C2} = I_{C1}$ *and* $I_{B1} = I_{B2}$ | $I = I_{C1} + 2I_{B1} = (\beta_F + 2)I_{B1}$ | $I_{C2} = \beta_F I_{B2} = \beta_F I_{B1}$

$I_{C2} = \frac{\beta_F}{\beta_F+2} I = \frac{25}{25+2} 25\mu A$ | $I_{C2} = 23.2\ \mu A$ | See the Current Mirror in Chapter 15.

5.96

$$V_{CE} = 1.5 - (I_C + I_B)R_C \rightarrow R_C = \frac{1.5 - 0.9}{20\mu A + \frac{20\mu A}{50}} = 29.4k\Omega \rightarrow 30\ k\Omega$$

$$R_B = \frac{V_{CE} - V_{BE}}{I_B} = \frac{0.9 - 0.65}{\frac{20\mu A}{50}} = 625k\Omega \rightarrow 620\ k\Omega$$

For $R_C = 30k\Omega$: $V_{CE} = 1.5 - 30k\Omega(I_C + I_B)R_C = 1.5 - 30k\Omega(126)I_B \ \ | \ \ I_B = \frac{V_{CE} - 0.65}{620k\Omega}$

$$V_{CE} = 1.5 - 30k\Omega(126)\frac{V_{CE} - 0.65}{620k\Omega} \rightarrow V_{CE} = 0.770V$$

$$I_C = 125 I_B = 125\frac{0.770 - 0.65}{620k\Omega} = 24.2\mu A \ \ | \ \ Q\text{-}point: (24.2\ \mu A,\ 0.770\ V)$$

5.97

$$12 = R_C(I_C + I_B) + V_Z + V_{BE} = 500(I_E) + 7.7 \ \ | \ \ I_E = \frac{12 - 7.7}{500} = 8.60mA$$

$$I_B = \frac{I_E}{\beta_F + 1} = \frac{8.60mA}{101} = 85.2\mu A \ \ | \ \ I_C = \beta_F I_B = 8.52mA \ \ | \ \ V_{CE} = 7.70V$$

Q-point = (8.52 mA, 7.70 V)

5.98

$$V_{EQ} = 6 + 100\frac{15 - 6}{7800 + 100} = 6.114V \ \ | \ \ R_{EQ} = 100\Omega \| 7800\Omega = 98.73\Omega$$

$$I_B = \frac{20mA}{51} + \frac{V_O}{51(4700\Omega)} = \frac{20mA}{51} + \frac{6.14 - 98.7I_B - V_{BE}}{51(4700\Omega)} \rightarrow I_C = 50I_B = 50\frac{101.1 - V_{BE}}{2.398x10^5}$$

$$V_{BE} = 0.025\ln\frac{I_C}{10^{-16}}$$

Using MATLAB: fzero('IC107',.02) ---> ans =0.0207

```
function f=IC107(ic)

vbe=0.025*log(ic/1e-16);

f=ic-50*(101.1-vbe)/2.398e5;
```

$$V_O = 6.14 - 98.7\frac{20.7mA}{51} - .025\ln\frac{20.7mA}{10^{-16}} = 5.276\ V$$

5.100

$$v_O = 7 - 100i_B - v_{BE} = 7 - 100i_B - V_T \ln\frac{i_C}{I_S} = 7 - 100i_B - V_T \ln\frac{\alpha_F i_L}{I_S}$$

$$v_O = 7 - 100i_B - V_T \ln i_L - V_T \ln\frac{\alpha_F}{I_S}$$

$$R_o = -\frac{dv_O}{di_L} = -\left(-100\Omega\frac{di_B}{di_L} - \frac{V_T}{i_L}\right) = \frac{100\Omega}{51} + \frac{0.025V}{0.02A} = 3.21\Omega$$

5.101

Since the voltage across the op-amp input must be zero, $v_O = V_Z = 10\ V$.

Since the input current to the op amp is zero, $I_E = \frac{V_O}{100} = 100\ mA$

$$I_{+15} = I_Z + I_C = I_Z + \alpha_F I_E = \frac{15V - 10V}{47k\Omega} + \frac{60}{61}100mA = 98.5\ mA$$

11.42

$(a)\ A_v(s) = -\frac{Z_2(s)}{Z_1(s)} \quad Z_2 = \frac{1}{sC} \quad Z_1(s) = R \quad A_v(s) = -\frac{1}{sRC}$, an inverting integrator.

$(b)\ \frac{\mathbf{V_s} - \mathbf{V_+}}{R} + \frac{\mathbf{V_o} - \mathbf{V_+}}{KR} = sC\mathbf{V_+}$ and $\mathbf{V_+} = \mathbf{V_-} = \mathbf{V_o}\frac{R_1}{R_1 + KR_1} = \frac{\mathbf{V_o}}{1+K}$

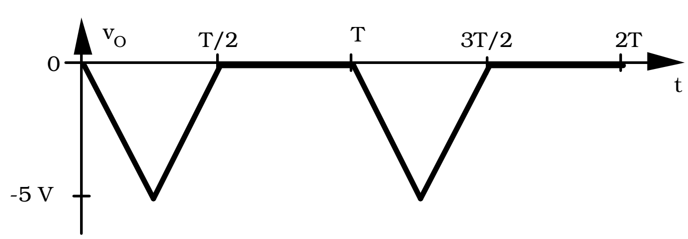
Combining these expressions yields: $A_V(s) = \frac{\mathbf{V_o}}{\mathbf{V_s}} = +\frac{1+K}{sRC}$, a noninverting integrator.

11.79

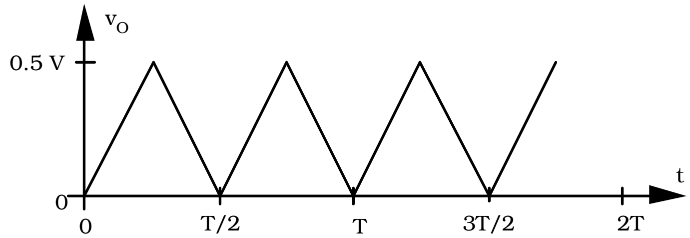
For $C_1 = C = C_2$: $\omega_o = \frac{1}{C\sqrt{R_1 R_2}}$ | $Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$ | $\omega_o = \frac{1}{2R_1 CQ}$

$$S_Q^{\omega_o} = \frac{Q}{\omega_o}\frac{\partial\omega_o}{\partial Q} = \frac{Q}{\omega_o}\left(-\frac{1}{2R_1 CQ^2}\right) = \frac{Q}{\omega_o}\left(-\frac{\omega_o}{Q}\right) = -1 \quad | \quad S_Q^{\omega_o} = -1$$

11.91



11.94



11.98

$$\frac{V_1}{10k\Omega} = I_S \exp\frac{-V_{o1}}{V_T} \quad | \quad V_{O1} = -V_T \ln\frac{V_1}{10^4 I_S} \quad | \quad V_{O2} = -V_T \ln\frac{V_2}{10^4 I_S}$$

$$V_{O3} = -\left(V_{O1} + V_{O2}\right) = V_T\left(\ln\frac{V_1}{10^4 I_S} + \ln\frac{V_2}{10^4 I_S}\right) = V_T \ln\frac{V_1 V_2}{10^8 I_S^2}$$

$$V_O = -10^4 I_D = -10^4 I_S \exp\frac{V_D}{V_T} = -10^4 I_S \exp\left(\ln\frac{V_1 V_2}{10^8 I_S^2}\right) = -\frac{V_1 V_2}{10^4 I_S}$$

The End