ECE 137A
Quiz #2 Solution
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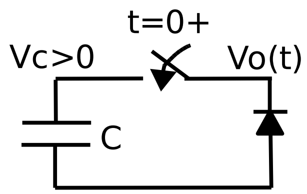

$Na=10^{17}$
$Nd=10^{15}$
Lossless switch
Vo(t)=0 for t<0
1. Calculate total energy before switch closes
2. Calculate total energy after switch closes
3. Is Ei=Ef, if not, why not

Here, we assume ideal diode, i.e. No reverse current. Initially,

$$E_i=\frac{1}{2}C\cdot V_C^{\,2}$$

At t>0, switch closes, charges flow from the capacitor to the diode. The diode is reverse biased, and has depletion capacitance C_{depl}.

To calculate, Ef, we need to find Vo(t) in steady state (Vf).

$$Q_i=C\,V_C$$
$$Q_f=Q_i=(C+C_{Depl})V_f$$
$$C\,V_C=(C+C_{Depl})V_f$$
$$so\ V_f=Vc\frac{C}{C+C_{Depl}}$$
$$where\ C_{Depl}=\frac{C_{j0}}{\sqrt{1-V_f/\phi_i}}\,,\ C_{j0}=\sqrt{\frac{q\,\epsilon_s}{2\,\phi_i}\frac{Na\cdot Nd}{Na+Nd}}$$
$$V_f=Vc\frac{C}{C+\frac{C_{j0}}{\sqrt{1-V_f/\phi_i}}}$$

Vf is difficult to solve symbolically, Had we been given the values of C, Vc and Area, then we can solve it easily.

Let's take some numbers and see what we get,

Assume C=20fF, Vc=1V, and A=1e-6 cm^2,

$$\phi_i = \frac{kT}{q}\ln\left(\frac{NaNd}{n_i^2}\right) = 26\text{mV}\cdot\ln\left(\frac{10^{15}\cdot 10^{17}}{(1.5\times 10^{10})^2}\right) = 0.7\text{V}$$

$$C_{j0} = \sqrt{\frac{1.6\times 10^{-19}\cdot 11.7\cdot 8.85\times 10^{-14}}{2\cdot 0.7\text{V}}\times\left(\frac{10^{15}\cdot 10^{17}}{10^{15}+10^{17}}\right)} = \frac{10.8\text{nF}}{cm^2}\times 1\times 10^{-6}\,cm^2 = 10.8\,fF$$

$$V_f = Vc\frac{C}{C+C_{Depl}} = \frac{1\text{V}\times 20\text{fF}}{20\text{fF}+\frac{10.8\text{fF}}{\sqrt{1-\frac{V_f}{0.7\text{V}}}}}$$

$$\textit{Solving for } V_f,$$
$$V_f = 0.498\,V$$

This is consistent, from the equation for Cdepl, we know that Vf < 0.7V, and we assume that there is some final voltage, hence, 0<Vf<0.7V

Now that we found Vf, we can calculate Ef

$$E_f = \frac{1}{2}(C+C_{Depl})V_f^{\,2}$$

Plugging in Vf

$$E_f = \frac{1}{2}(C+C_{Depl})\cdot V_C^{\,2}\frac{C^2}{(C+C_{Depl})^2} = E_i\frac{C}{C+C_{Depl}}$$

Clearly, Ef is less than Ei, *where did the rest of the energy go*?
The energy we calculated is the electric energy stored in the capacitors. Movement of charges (current) converts some of the initial energy into magnetic energy, and it is radiated into space in the form of magnetic field (Ampere's law).