UNIVERSITY OF CALIFORNIA, SANTA BARBARA

DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

CIRCUITS & ELECTRONICS II ECE 137B

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MIDTERM EXAM I, APRIL 26, 2010

Name: MODEL ANSWER

This is open book and open notes exam. For all questions make reasonable approximations. Show all your work, any answers without explanations will not be given credit . GOOD LUCK!

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Question	Points	Score
1	30	
2	70	
Total:	100	

1. (30 points) Question 1: 30points

   Calculate the gain of the differential amplifier shown in figure 1, in terms of I_{bias}, I_1. ***Do not assume*** $g_{mp}R \gg 1$. *Hint:It would be wise to first calculate the output impedance and then calculate the gain as* $g_m^{in} r_{out}$

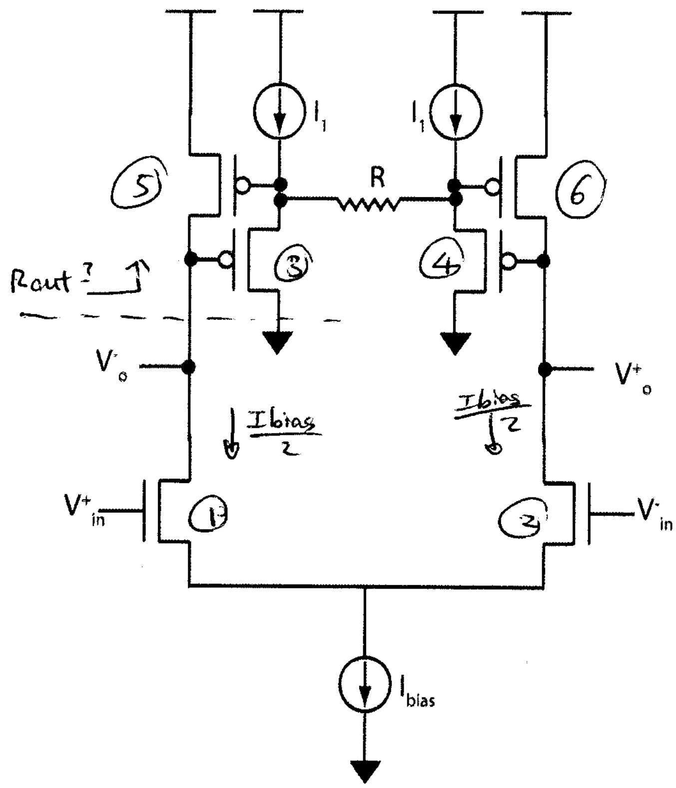
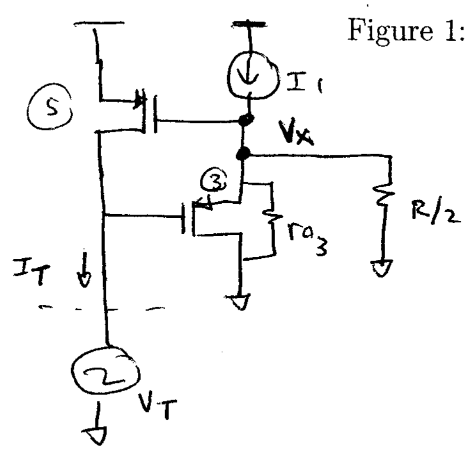


Figure 1: Circuit for Question 2



Transistor M3 → Source follower.

$$V_X = V_T \frac{g_{m3}(r_{o3} // R/2)}{1 + g_{m3}(r_{o3} // R/2)}$$

Transistor M5 → Common-Source.

Output current :

$$I_T = g_{m5} V_X$$

$$I_T = g_{m5} \times V_T \frac{g_{m3}(r_{o3} // R/2)}{1 + g_{m3}(r_{o3} // R/2)}$$

$$\Rightarrow R_{out} \triangleq \frac{V_T}{I_T} = \frac{1 + g_{m3}(r_{o3} // R/2)}{g_{m5}\, g_{m3}(r_{o3} // R/2)}$$

contd..

$$r_{o3} = \frac{1}{\lambda I_1}$$

$$g_{m3} = \frac{2I_1}{V_{ov3}}$$

$$g_{m5} = \frac{2 I_{bias}}{2\, V_{ov5}} = \frac{I_{bias}}{V_{ov5}}$$

$$r_{o3} /\!/ R/2 = \frac{\frac{r_{o3} R}{\cancel{2}}}{\frac{2R_{o3}}{\cancel{2}} + \frac{R}{\cancel{2}}} = \frac{r_{o3} R}{2r_{o3} +}$$

$$= \frac{\frac{R}{\lambda I_1}}{\frac{2}{\lambda I_1} + \frac{R\lambda I_1}{\lambda I_1}}$$

$$= \frac{R}{2 + \lambda I_1 R}$$

In terms of Ibias, I_1 :

$$R_{out} = \frac{1 + \frac{2I_1}{V_{o3}}\left(\frac{R}{2+\lambda I_1 R}\right)}{\frac{I_{bias}}{V_{ov5}} \cdot \frac{2I_1}{V_{ov3}}\left(\frac{R}{2+\lambda I_1 R}\right)}$$

$$\text{Gain} = -g_{m1} \times (R_{out} /\!/ r_{o1})$$

$$\text{Gain} = -\frac{I_{bias}}{V_{ov1}} \times \left[\underbrace{\frac{1 + \frac{2I_1}{V_{ov3}}\left(\frac{R}{2+\lambda I_1 R}\right)}{\frac{I_{bias}}{V_{ov5}}\, \frac{2I_1}{V_{ov3}}\left(\frac{R}{2+\lambda I_1 R}\right)}}_{r_{out}} /\!/ r_{o1} \right]$$

Assuming $R_{out} << r_{o1}$

$$\boxed{\text{Gain} \simeq \frac{1 + \frac{2I_1}{V_{ov3}}\left(\frac{R}{2+\lambda I_1 R}\right)}{\frac{V_{ov1}}{V_{ov5}}\, \frac{2I_1}{V_{ov3}}\left(\frac{R}{2+\lambda I_1 R}\right)}}$$

we can substitute in device parameters instead of overdrive voltage.

$$I_D = \frac{1}{2}\mu c_{ox}\left(\frac{W}{L}\right) V_{ov}^2$$

$$\sqrt{\frac{2I_D}{\mu c_{ox}\left(\frac{W}{L}\right)}} = V_{ov}$$

Or :

$$\text{Gain} \simeq \frac{1 + \frac{2I_1}{\sqrt{\frac{2I_1}{\mu_p c_{ox}\left(\frac{W}{L}\right)_3}}}\left(\frac{R}{2+\lambda I_1 R}\right)}{\sqrt{\frac{\frac{2I_{bias}}{\mu_n c_{ox}\left(\frac{W}{L}\right)_1}}{\frac{2I_{bias}}{\mu_p c_{ox}\left(\frac{W}{L}\right)_5}}}\; \frac{2I_1}{\sqrt{\frac{2I_1}{\mu_p c_{ox}\left(\frac{W}{L}\right)_3}}}\left(\frac{R}{2+\lambda I_1 R}\right)} = \boxed{\frac{1 + \sqrt{\frac{2I_1}{\mu_p c_{ox}\left(\frac{W}{L}\right)_3}}\left(\frac{R}{2+\lambda I_1 R}\right)}{\sqrt{\frac{\mu_p\left(\frac{W}{L}\right)_5}{\mu_n\left(\frac{W}{L}\right)_1} \cdot \frac{2I_1}{\cancel{\mu_p} c_{ox}\left(\frac{W}{L}\right)_3}}\left(\frac{R}{2+\lambda I_1 R}\right)}}$$

2. Question 2: 70points

Consider the circuit shown in figure 2, this is transistor level implementation of an oscillator.

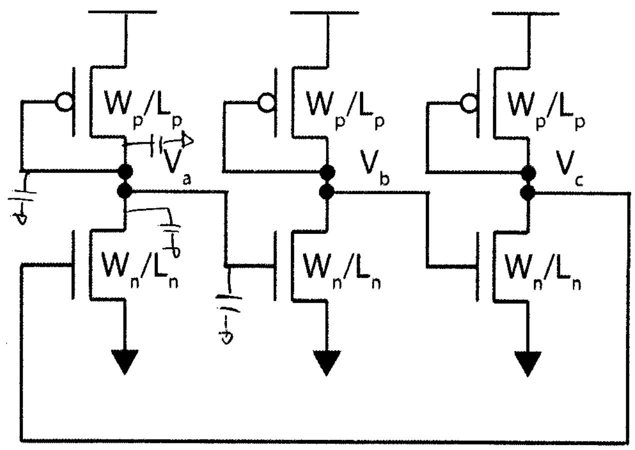


Figure 2: Circuit for Question 1

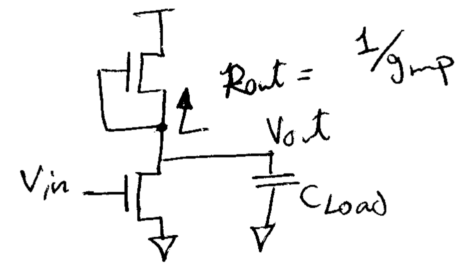
(a) (5 points) Assuming $C_{ov} = 0$ what is the capacitive load seen by each amplifier?

$$C_{Load} = C_{gsn} + C_{dbn} + C_{dbp} + C_{gsp}$$

$$C_{db} = C_j WL + C_{jsw}(W + 2E)$$

$$C_{gs}\ (\text{sat.}) = \frac{2}{3} C_{ox} WL$$

(b) (5 points) What is the transfer function of each amplifier in the frequency domain i.e $H(s) = \frac{V_a}{V_c} = \frac{V_b}{V_a} = \frac{V_c}{V_b} = ?$



DC gain $A_v = g_{mn} r_{out} = g_{mn}\left(r_{on} \| \frac{1}{g_{mp}}\right)$

$$A_v \simeq \frac{g_{mn}}{g_{mp}}$$

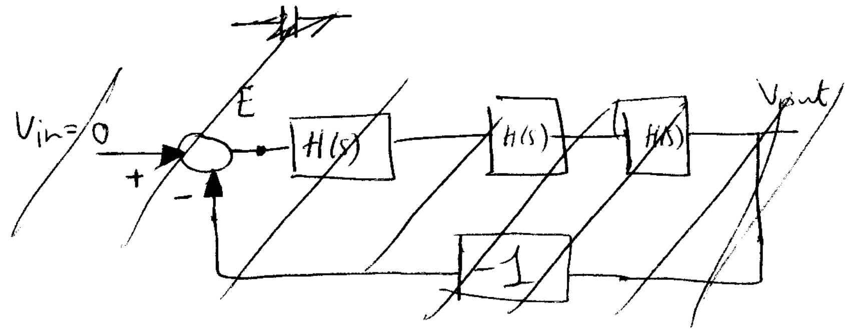
$$\tau = R_{out} C_{load} = \cancel{\frac{g_{mp}}{C_{load}}} = \frac{C_{load}}{g_{mp}}$$

$$\boxed{H(s) = \frac{A_v}{s\tau + 1} = \frac{\frac{g_{mn}}{g_{mp}}}{s\left(\frac{C_{load}}{g_{mp}}\right) + 1}}$$

(c) (10 points) What is the transfer function of the oscillator?

$$\cancel{E = V_{out} = H^3(s)\, E(s)}$$

$$\cancel{1 - H^3(s) = 0}$$



$$\boxed{T(s) = \frac{H^3(s)}{1 + H^3(s)}} = \frac{\left(g_{mn}/g_{mp}\right)^3}{\left[1 + s\frac{C_{load}}{g_{mp}}\right]^3 + \left(\frac{g_{mn}}{g_{mp}}\right)^3}$$

(d) (15 points) Given the expression for your transfer function, find the gain of each amplifer such that the roots of the transfer function of the oscillator are either purely real or imaginary so that the circuit will function as an oscillator, given $\sqrt[3]{-a^3} = -a, \frac{a}{2} \pm j\frac{a}{2}\sqrt{3}$

roots at $\left[1 + s\frac{C_{load}}{g_{mp}}\right]^3 + \left(\frac{g_{mn}}{g_{mp}}\right)^3 = 0$

$\therefore \quad 1 + s\frac{C_{load}}{g_{mp}} = \sqrt[3]{-\left(\frac{g_{mn}}{g_{mp}}\right)^3} = \sqrt[3]{-a^3}$

$\therefore a = \frac{g_{mn}}{g_{mp}}$ #

$\therefore$ Either

$1 + s\frac{C_{load}}{g_{mp}} = -a = -\frac{g_{mn}}{g_{mp}} \Rightarrow \boxed{s_1 = -\frac{g_{mp}}{C_{load}}\left(1 + \frac{g_{mn}}{g_{mp}}\right)}$

-ve real pole

or

$1 + s\frac{C_{load}}{g_{mp}} = \frac{g_{mn}}{2g_{mp}} \pm j\frac{\sqrt{3}}{2}\frac{g_{mn}}{g_{mp}}$

$\therefore \boxed{s_{2,3} = \frac{g_{mp}}{C_{load}}\left[\left(\frac{g_{mn}}{2g_{mp}} - 1\right) \pm j\frac{\sqrt{3}}{2}\frac{g_{mn}}{g_{mp}}\right]}$ complex poles

To make the poles purely imaginary

$\therefore \frac{g_{mn}}{2g_{mp}} = 1 \Rightarrow \boxed{\text{Each stage gain} = \frac{g_{mn}}{g_{mp}} = 2}$

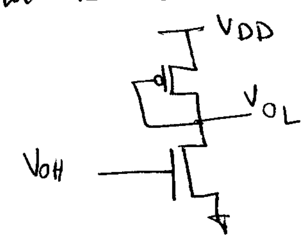
In that case $s_{2,3} = \pm j\frac{\sqrt{3}}{2}\frac{\cancel{C_{load}} * g_{mn}}{\cancel{g_{mp}}\ C_{load}}$

(e) (15 points) Size your amplifier to obtain the gain you calculated in the earlier section given $L_{min} = 0.2\mu m$ in the process, assuming $W_n = 2 * L_{min}, L_n = L_{min}$ and that $V_{DD} = 2V, \frac{1}{2}\mu_n C_{ox} = 100\mu A/V^2, \frac{1}{2}\mu_p C_{ox} = 25\mu A/V^2, V_{TN} = 0.5V$ and $V_{TP} = -0.5V$. Assume that $\lambda = 0$

$V_{OH} = V_{DD} - |V_{TP}| = 1.5V$

$V_{OL} = V_{Tn} = 0.5V$

when the i/p is "high", the output is "low"



$\therefore V_{gs_n} = V_{sg_p} = 1.5V$

$\because g_{mn} = 2 g_{mp}$

$\therefore \cancel{2} I_{dsn} \frac{W_n}{L_n} \mu_n C_{ox} = 2 * \cancel{2} I_{sdp} \frac{W_p}{L_p} \mu_p C_{ox}$

but $I_{dsn} = I_{sdp}$ & using $\boxed{L_n = L_p = L_{min} = 0.2\mu m}$

& $\because \mu_n C_{ox} = 4 \mu_p C_{ox}$

$\therefore 4 * W_n = 2 * W_p$

$\therefore W_p = 2 * W_n = 2 * 2 L_{min} \Rightarrow \boxed{W_p = 0.8\mu m}$

$\boxed{W_n = 0.4\mu m}$

(f) (15 points) Given that $C_{ox} = 10fF/\mu m^2, C_j = 1fF/\mu m^2, C_{jsw} = 0.1fF/\mu m, C_{ov} = 0, E = 0.2\mu m$ and the size of your amplifer, calculate the frequency of oscillation. $C_{gs} = \frac{2}{3}C_{ox}WL, C_{db} = WE * C_j + (2E+W)C_{jsw} + WC_{ov}$

The freq. of oscillation is given by $\boxed{\frac{\sqrt{3}}{2}\frac{g_{mn}}{C_{load}}}$

$$g_{mn} = \mu_n C_{ox}\frac{W_n}{L_n}(V_{gsn} - V_t) = 200\mu * 2 * (1.5 - 0.5)$$

$$\therefore \boxed{g_{mn} = 400\ \mu A/V}$$

$$\therefore \boxed{g_{mp} = 200\ \mu A/V}$$

From part(a) $[W_p = 2W_n]$

$$C_{Load} = \frac{2}{3}C_{ox}L_{min}(3W_n) + C_j L_{min}(3W_n) + C_{jsw}(3W_n + 4E)$$

$$\therefore \cancel{C_{Load} = \frac{2}{3}C_{ox}\ 3W_n\left[\left(\frac{2}{3}C_{ox} + C_j\right.\right.} + C$$

$$\therefore C_{Load} = C_{jsw}4E + 3W_n\left[\left(\frac{2}{3}C_{ox} + C_j\right)L_{min} + C_{jsw}\right]$$

$$= 0.08\ fF + 3 * 0.4\left[\left(\frac{20}{3} + 1\right)0.2 + 0.1\right]$$

$$\therefore \boxed{C_{Load} = 2.04\ fF}$$

$$\therefore \text{oscillation freq} = \cancel{\frac{\sqrt{3} * 400 * 2.04 * 10^{-15}}{2 * 2\pi * 2.04 * 10^{-6}}} = \frac{\sqrt{3} * 400 * 10^{-6}}{2 * 2\pi * 2.04 * 10}$$

$$\therefore \boxed{f_{osc} = 27.026\ GHz}$$

(g) (5 points) What happens if the gain of your ampifier exceeds the gain that you calculated for the oscillation condition?

The complex poles will be in the RHP

∴ you have a +ve real values whichs ~~[illegible]~~ makes the oscillation get multiplied by a growing exponential

∴ The oscillations will grow until it bangs to the rails & ∴ you get a square wave rather than a sine wave.