Internal and External Op-Amp Compensation: A Control-Centric Tutorial

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Abstract—Frequency compensation of two-stage integrated-circuit operational amplifiers is normally accomplished with a capacitor around the second stage. This compensation capacitance creates the desired dominant-pole behavior in the open-loop transfer function of the op amp. Circuit analysis of this compensation leads to a mathematical observation of “pole splitting”: that as the compensation capacitance is increased, the parasitic poles of the amplifier separate in frequency.

Treatment of op-amp compensation as minor-loop feedback, instead of pole splitting, greatly simplifies and generalizes the analysis and design of op-amp frequency response. Using classical-control techniques instead of direct circuit analysis, insight and intuition into the behavior and flexibility of the system are gained.

I. INTRODUCTION

Operational amplifiers have been used by control engineers for many decades as key components in compensators [1], sensor circuitry [2], and analog computers [3], [4]. They are still one of the most ubiquitous electronic elements in the world. However, despite the required use of feedback in all op-amp applications, and the presence of feedback in the internal circuitry, the design of operational amplifiers is often presented and completed without a useful control framework.

Op amps require a deliberately designed frequency response to ensure stability and satisfactory transient performance in end-user applications. Standard frequency compensation is designed for general-purpose op-amp applications such as amplifiers, buffers, and integrators. Sophisticated compensation techniques can be employed in specific applications in which standard compensation methods perform poorly.

Internally compensated op amps have a fixed transfer function set by the manufacturer. In the design of the circuit, the op-amp designer must choose a compensation network that is appropriate for the intended applications of the op amp. Externally compensated op amps [5] allow the end user to select the compensation network that determines the transfer function of the op amp. The determination and implementation of appropriate op-amp transfer functions in various applications is easily understood with the tools of classical control.

Popular textbooks in analog circuit design [6], [7], [8] treat op-amp compensation in a network-theory context, writing out many node equations and discussing the concept of “pole splitting” [9]. This approach is unnecessarily abstruse. Treatment of op-amp compensation as minor-loop feedback, instead of pole splitting, greatly simplifies and generalizes the analysis and design of op-amp frequency-compensation networks.

This paper demonstrates the use of classical-control techniques instead of direct circuit analysis in the design of compensation for general-purpose and special-purpose operational amplifiers. Intuition and insight into the solution are gained by using these feedback techniques.

II. THE GENERAL-PURPOSE TRANSFER FUNCTION

The frequency response of general-purpose op amps is designed to be stable in the largest number of applications. The schematic for a simple non-inverting amplifier circuit is shown in Figure 1. This amplifier circuit is implemented with a negative-feedback loop around the op amp, and the closed-loop gain is

\[
\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_1}.
\]

For general-purpose use the op amp must be designed such that this circuit is stable for any resistor values \(R_1\) and \(R_2\).

The block diagram of this circuit is shown in Figure 2. The circuit loop transfer function is

\[
L(s) = \frac{A(s)R_1}{R_1 + R_2} = A(s)F.
\]

For stability in this application, this loop transfer function must create a stable feedback system for any value of \(F\) less than
Fig. 2. Block diagram for the non-inverting amplifier circuit in Figure 1. The feedback path is $F = R_1/(R_1 + R_2)$. The op-amp transfer function $A(s)$ must be designed to guarantee stability for any such attenuative feedback.

Fig. 3. Frequency response of the desired op-amp transfer function $A(s)$. The single-pole roll-off (slope of $-1$) behavior over a wide frequency range gives the desired transfer function for a general-purpose op amp. The frequency $\omega_u$ is the unity-gain frequency of the op amp.

Fig. 4. Frequency response of the op-amp-circuit loop transfer function $L(s)$ with a variety of feedback terms. Since the loop transfer function always crosses over with 60° or more of phase margin for any attenuative feedback, stability is guaranteed.

Fig. 5. Simplified schematic of the uncompensated Fairchild $\mu$A741 op amp, showing the signal-path transistors. The full schematic is shown and explained in Appendix II.

Fig. 6. Equivalent-circuit block diagram for the non-inverting amplifier circuit in Figure 1. The feedback path is $F = R_1/(R_1 + R_2)$. The op-amp transfer function $A(s)$ must be designed to guarantee stability for any such attenuative feedback.

one. The ideal transfer function that meets this requirement is

$$A(s) = \frac{A_0}{\tau s + 1}.$$  (1)

With this op-amp transfer function, the closed-loop circuit will be stable for any choice of resistive feedback. The frequency response of this desired op-amp transfer function $A(s)$ rolls off with a slope of $-1$ over a wide frequency range, as shown in Figure 3. In the ideal case, this transfer function gives 90° of phase margin, regardless of the feedback $F$.

A real op amp will have additional high-frequency poles beyond its unity-gain frequency $\omega_u$. Including the effect of an additional pole at $2\omega_u$, the frequency response of the loop transfer function of the op-amp circuit with a variety of feedback terms is shown in Figure 4. Even with this additional high-frequency pole, the loop transfer function always crosses over with 60° (or more) of phase margin for any attenuative feedback. Thus, stability is guaranteed for any set of feedback resistors.

The implementation of this desired op-amp transfer function is easier said than done. Even a simple op-amp circuit model gives an unacceptable op-amp transfer function.

For example, a simplified schematic of the Fairchild $\mu$A741 [10] op amp is shown in Figure 5. This circuit can be modeled by the equivalent-circuit block diagram shown in Figure 6. The frequency response of this circuit, when uncompensated, is shown in Figure 7. The two low-frequency poles severely degrade the phase margin at crossover. Additional high-frequency poles in the circuit make matters worse.

For stability in amplifier applications, the op amp must be compensated to achieve a frequency response similar to the ideal transfer function in equation (1) and shown in Figure 3. This general-purpose compensation is usually accomplished with a capacitor [5]. (This technique is often called “Miller compensation.” See Appendix I.) The simplified schematic of the $\mu$A741 op amp with a compensation capacitor is shown in Figure 8. The compensation capacitor goes around the high-gain stage as shown in the equivalent-circuit block diagram in Figure 9.

Using two-port circuit models for each stage, the equivalent-circuit schematic in Figure 10 can be drawn. Each gain stage is represented by a Norton-equivalent two-port model with input resistance, output resistance, output capacitance, and a transconductance generator. The output buffer is ignored in this equivalent circuit since the output voltage of the
The compensation capacitor goes around the high-gain second stage. The locations of the transfer-function poles can be found by assuming that the pole locations are widely separated.

\[
\text{The pole-splitting approach } [9] \text{ uses brute-force circuit analysis to determine this transfer function. The approach starts with the constitutive current equations at the two circuit nodes } V_1 \text{ and } V_o.
\]

\[
G_{M1}V_{in} - \frac{V_1}{R_1} - sC_1V_1 - sC(V_1 - V_o) = 0 \quad (2)
\]

\[
sC(V_1 - V_o) - G_{M2}V_1 - \frac{V_o}{R_2} - sC_2V_o = 0. \quad (3)
\]

After a page of algebra (as shown in detail in Appendix III) the transfer function is found

\[
A(s) = \frac{V_o}{V_{in}}(s) = \frac{G_{M1}R_1G_{M2}R_2(Cs/G_{M2} - 1)}{a_2s^2 + a_1s + 1}
\]

where the coefficients of the denominator are

\[
a_2 = R_1R_2(C_1C_2 + C_1C + C_2C)
\]

\[
a_1 = R_1C_1 + R_1C + R_2C_2 + R_2C + G_{M2}R_2R_1C.
\]

Assuming that the gain of the second stage is large \(G_{M2}R_2 \gg 1\), the final term in the first-order coefficient \(a_1\) dominates the sum, and the transfer function can be simplified as

\[
A(s) \approx \frac{G_{M1}R_1G_{M2}R_2(Cs/G_{M2} - 1)}{R_1R_2(C_1C_2 + C_1C + C_2C)s^2 + G_{M2}R_2R_1Cs + 1}
\]

The locations of the transfer-function poles can be found by assuming that the pole locations are widely separated

\[
A(s) \approx \frac{A_0}{(\tau_1s + 1)(\tau_2s + 1)} = \frac{A_0}{\tau_1\tau_2s^2 + (\tau_1 + \tau_2)s + 1}.
\]

If the two poles are widely separated \(\tau_1 \gg \tau_2\), then

\[
A(s) \approx \frac{A_0}{\tau_1\tau_2s^2 + \tau_1s + 1}.
\]
Therefore, the approximate pole locations of the op-amp transfer function are

\[
\begin{align*}
\omega_1 &= \frac{1}{\tau_1} = \frac{1}{a_1} = \frac{1}{G_{M2}R_2R_1C} \\
\omega_2 &= \frac{\tau_1}{\tau_1\tau_2} = \frac{a_1}{a_2} = \frac{G_{M2}C}{C_1C_2 + CC_1 + CC_2}. 
\end{align*}
\]

(4) (5)

Figure 11 shows the resulting “pole-splitting” behavior in the frequency response of this transfer function. It is observed that as the size of the compensation capacitor is increased, the low-frequency pole location \(\omega_1\) decreases in frequency, and the high-frequency pole \(\omega_2\) increases in frequency. The poles appear to “split” in frequency. For a large enough compensation capacitor, a single-pole roll off over a wide range of frequency results, as shown in Figure 11, which matches the desired transfer function in Figure 3.

IV. MINOR-LOOP FEEDBACK

While the above results are correct and useful, they are an impediment to intuition [11]. Treating the compensation capacitor \(C\) as a minor-loop feedback path, instead of as just another circuit element, simplifies the analysis of the compensated op amp. The concept of op-amp compensation by minor-loop feedback provides useful design insight into the flexibility of this topology and opens up a wide range of applications for special-purpose compensation schemes.

In the minor-loop approach, the capacitor \(C\) is treated as a feedback path as shown in Figure 12. If the gain of the second stage is large, then the first-stage voltage \(V_1\) will be much smaller than the second-stage voltage \(V_o\). Comparatively, the node \(V_1\) appears to be a virtual ground. Therefore the effect of the capacitor can be modeled as an admittance \(Y_c(s)\) that injects a current \(I_c\) into the first stage that depends only on the voltage of the second stage \(V_o\):

\[
I_c = Y_c(s)V_o = sCV_o.
\]

When the compensation capacitor \(C\) is removed from the circuit and replaced with this block, the capacitive loading on each stage must be maintained. A high-frequency model of the effective capacitive loading of the compensation capacitor is shown in Figure 13. Therefore the capacitors \(C_3\) and \(C_4\) in Figure 12 are replaced with the capacitors \(C_3\) and \(C_4\), where

\[
C_3 = C_1 + \frac{C_2C}{C_2 + C}
\]

and

\[
C_4 = C_2 + C.
\]

In the equivalent circuit in Figure 12, the voltage \(V_1\) is the total current flowing into the first node times the impedance of \(R_1\) and \(C_3\)

\[
V_1 = (G_{M1}V_{in} + I_c) \left( \frac{R_1}{R_1C_3s + 1} \right).
\]

(6)

The voltage \(V_o\) is the current flowing into the second stage times the impedance of \(R_2\) and \(C_4\)

\[
V_o = -G_{M2}V_1 \left( \frac{R_2}{R_2C_4s + 1} \right).
\]

(7)

From equations (6) and (7) the block diagram of the equivalent circuit with minor-loop feedback can be drawn, as shown in Figure 14. The block diagram can be rearranged into Figure 15 by pushing the \(G_{M1}\) block inside the loop.

As shown in Figure 15, the forward path of the op amp is

\[
G(s) = \left( \frac{G_{M1}R_1}{R_1C_3s + 1} \right) \left( \frac{G_{M2}R_2}{R_2C_4s + 1} \right)
\]

(8)

and the feedback path is

\[
H(s) = \frac{Y_c(s)}{G_{M1}} = \frac{sC}{G_{M1}}.
\]

(9)
Therefore, the intersection occurs when
\[ \text{at a frequency of } \frac{1}{\omega_2} = \frac{G_{M1} R_1}{R_1 C_3 s + 1}. \]

This result agrees exactly with the result (4) found in Section III using brute-force circuit analysis.

The transfer function of the op amp can now be calculated from this block diagram with Black’s formula
\[ A(s) = \frac{V_o}{V_{in}}(s) = \frac{G(s)}{1 + G(s) H(s)} \]
or it can be determined from \( G(s) \) and \( 1/H(s) \) on the asymptotic Bode plot shown in Figure 16 (as explained in Appendix IV).

An accurate rendering of the Bode magnitude plot of the forward-path and the inverse-feedback-path transfer functions is shown in Figure 17. The Bode magnitude plot of the resulting op-amp transfer function \( A(s) \) is shown in Figure 18.

Finally, the locations of the resulting poles of the op-amp transfer function \( A(s) \) can be found from the intersections of the two curves in Figure 16. The low-frequency intersection occurs when the low-frequency behavior of the forward path (below the frequencies its poles) intersects with the inverse of the feedback path. At low frequency
\[ \lim_{\omega \to 0} |G(j\omega)| = G_{M1} R_1 G_{M2} R_2 \]
therefore, the intersection occurs when
\[ G_{M1} R_1 G_{M2} R_2 = \frac{G_{M1}}{\omega C} \]
at a frequency of
\[ \omega_1 = \frac{1}{R_1 G_{M2} R_2 C}. \]
This result agrees exactly with the result (4) found in Section III using brute-force circuit analysis.

The high-frequency intersection occurs when the high-frequency behavior of the forward path intersects with the inverse of the feedback path. At high frequency
\[ \lim_{\omega \to \infty} |G(j\omega)| = \frac{G_{M1} G_{M2}}{\omega^2 C_3 C_4} \]
therefore, the intersection occurs when
\[ \frac{G_{M1} G_{M2}}{\omega^2 C_3 C_4} = \frac{G_{M1}}{\omega C} \]
at a frequency of
\[ \omega_2 = \frac{G_{M2}C}{C_3C_4} \]
where
\[ C_3C_4 = C_1C_2 + C_1C + C_2C. \]
Again, this result agrees exactly with the result (5) found above using brute-force circuit analysis.

Using this minor-loop feedback approach to calculating the compensated transfer function of the op amp produced the same results with less work. In addition, a better understanding of the internals of the op amp is achieved. The minor-loop feedback path created by the compensation capacitor (or the compensation network) allows the frequency response of the op-amp transfer function to be easily shaped.

V. COMPENSATION FOR STEADY-STATE ERROR

This feedback approach to op-amp compensation can be exploited in the design of special-purpose op-amp transfer functions. Such transfer functions can be used to improve the performance characteristics of many op-amp circuits. Using these minor-loop techniques, these special-purpose op-amp transfer functions are easier to design.

For example, the dynamic-tracking behavior of an op-amp amplifier circuit can be modified with appropriate changes to the op-amp transfer function \( A(s) \). An inverting op-amp amplifier is shown in Figure 19. With standard capacitive compensation, the steady-state error to a step input is nearly zero, since the op-amp transfer function looks like an integrator

\[ A(s) \approx \frac{G_{M1}}{sC} \]

with a single pole near the origin. If zero steady-state error to a ramp input is desired, the transfer function of the op amp can be changed to achieve this specification. As seen in Section IV, the op-amp transfer function can be designed by appropriate choice of the compensation network. An op-amp transfer function such as

\[ A(s) \approx \frac{G_{M1}(\tau s + 1)}{C\tau s^2} \]

can be achieved with a compensation admittance of the form

\[ Y_c(s) = \frac{C\tau s^2}{\tau s + 1}. \]

This double-integrator transfer function will exhibit zero steady-state error to an input ramp.

The transfer admittance of the two-port compensation network shown in Figure 20 has two zeros at the origin

\[ Y_c(s) = \frac{I_c}{V_o} = \frac{RC^2s^2}{2RCs + 1}. \]

Therefore, with this compensation, the open-loop transfer function of the op amp is approximately

\[ A(s) \approx \frac{G_{M1}(2RCs + 1)}{RC^2s^2} \]

with two poles at the origin. This “two-pole” compensation network [12] creates a slope of \(-2\) in the frequency response of the op-amp transfer function.

A Bode magnitude plot of the forward-path and inverse-feedback-path transfer functions for single-pole and two-pole compensation is shown in Figure 21. A comparison of the resultant op-amp transfer functions is shown in Figure 22. The open-loop gain of the two-pole op-amp transfer function exceeds the gain for the single-pole transfer function for all frequencies between \(10^2\) and \(10^7\) radians per second (rps). The response of the error signal for a fast input ramp of 1 V/\(\mu\)s is shown in Figure 23. The increased gain of the two-pole op-amp transfer function significantly
presented at the 2004 American Control Conference

−100
−50
0
50
100
Magnitude (dB)

−180
−135
−90
−45
0
Phase (deg)

Bode Diagram

Fig. 22. Comparison of the single-pole and two-pole op-amp transfer functions \( A(s) \). The open-loop gain of the two-pole op-amp transfer function exceeds the gain for the single-pole transfer function for all frequencies between \( 10^2 \) and \( 10^7 \) rps.

reduces the magnitude of the steady-state error, and this op-amp transfer function is easier to design using minor-loop feedback techniques.

VI. COMPENSATION FOR CAPACITIVE LOADS

This feedback approach to op-amp design can also illuminate and diagnose subtle problems in compensation network design. Some special-purpose op-amp transfer-function designs exhibit insidious stability problems that can be difficult to diagnose using only circuit analysis techniques.

A. Compensation that Introduces a Zero

Consider an op-amp unity-gain buffer circuit with capacitive load, as shown in Figure 24. The output resistance of the op-amp and the capacitance of the load create a low-pass filter in the feedback loop

\[
L(s) = \frac{A(s)}{R_OC_L s + 1}.
\]

Because the output resistance is inside the op amp, feedback from the output terminal always includes the effects of this additional pole. The block diagram of the buffer circuit showing the minor loop of the op amp is shown in Figure 25. The low-frequency pole from the capacitive load appears between the second stage of the op amp and the output node of the circuit.

As an example, consider the op-amp model in Figure 12 with an input-stage transconductance of

\[
G_m = G_{M1} = 10^{-4} \, \Omega,
\]

a second-stage transresistance of

\[
G_a(s) = \frac{R_1 G_M R_2}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)} = \frac{10^{10} \, \Omega}{(10^{−5} s + 1)^2},
\]

and standard capacitive compensation of

\[
Y_c(s) = sC = 10^{-11} s \, \Omega.
\]

The Bode plot of the forward-path and inverse-feedback-path transfer functions for this op amp is shown in Figure 26. With this choice of \( Y_c(s) \), the high-frequency pole of the op amp occurs two orders of magnitude above crossover.

The op-amp transfer function is

\[
A(s) = \frac{G_m G_a(s)}{1 + G_a(s) Y_c(s)} \approx \frac{G_m}{Y_c(s)} = \frac{10^7}{s}.
\]

The Bode plot of the op-amp transfer function is shown in Figure 27. This transfer function \( A(s) \) has nearly ninety
Fig. 26. Bode plot of the forward-path and inverse-feedback-path transfer functions for the example op amp with standard compensation. This op amp appears to be over-compensated since the second pole of the op amp occurs two orders of magnitude above crossover.

Fig. 27. Bode plot of the closed-minor-loop transfer function of the op amp $A(s)$. This transfer function has nearly ninety degrees of phase margin at unity-gain crossover. This op amp is over-compensated and is much more stable than it would need to be for most applications.

Fig. 28. Step response of the op-amp buffer without the capacitive load. With ninety degrees of phase margin, the response is first order.

Fig. 29. Bode plot of the loop transfer function $L(s)$ for the op-amp buffer with capacitive load. The pole due to the capacitive load occurs below crossover, and stability is significantly compromised. The load capacitor reduces the phase margin from ninety degrees (as shown in Figure 27) to less than eighteen degrees shown here.

Fig. 30. Step response of the op-amp buffer with capacitive load. With less than eighteen degrees of phase margin, the response exhibits considerable

The whole circuit is approximately

$$L(s) = A(s)G_I(s) \approx \frac{G_m G_I(s)}{Y_c(s)} = \frac{10^7}{s(10^{-6}s + 1)}.$$

The Bode plot of the loop transfer function for the op-amp circuit with capacitive load is shown in Figure 29. The pole due to the capacitive load occurs below crossover, so the loop transfer function crosses over with a slope of $-2$ and a small phase margin. Stability of the circuit is significantly compromised and the phase margin has decreased from ninety degrees to less than eighteen degrees.

This reduction in phase margin degrades the transient performance of the circuit. The step response of the op-amp buffer with capacitive load is shown in Figure 30. Clearly, the response is no longer first order. With less than eighteen degrees of phase margin, the response exhibits considerable...
loop transfer function \( L \) has a zero near the frequency of the additional pole in the circuit with capacitive load. The op-amp transfer function \([13]\) is shown in Figure 31. The single-pole roll-off behavior of the op-amp transfer function

\[
Y_{c}(s) = \frac{I_{o}}{V_{o}} = \frac{C_{cs}}{R_{c}C_{cs} + 1}.
\]

The pole in the admittance \( Y_{c}(s) \) introduces a zero into the op-amp transfer function \( A(s) \). This zero will be used to cancel the capacitive-load pole.

A compensation network that introduces a zero in the op-amp transfer function \([13]\) is shown in Figure 31. The admittance of this network is

\[
Y_{c}(s) = \frac{I_{o}}{V_{o}} = \frac{C_{cs}}{R_{c}C_{cs} + 1}.
\]

The pole in the admittance \( Y_{c}(s) \) becomes a zero in the op-amp transfer function \( A(s) \). This zero will be used to cancel the effects of the capacitive load at the output. With this compensation network the op-amp transfer function is

\[
A(s) \approx \frac{G_{m}}{Y_{c}(s)} = \frac{G_{m}(R_{c}C_{cs} + 1)}{C_{cs}}
\]

and the loop transfer function of the circuit is

\[
L(s) = A(s)G_{t}(s) \approx \frac{G_{m}(R_{c}C_{cs} + 1)}{C_{cs}} \left( \frac{1}{R_{o}C_{L}s + 1} \right).
\]

If the time constant of the admittance is chosen to be equal to the time constant of the output pole \( (R_{c}C_{c} \approx R_{o}C_{L}) \) then the term from the minor loop and the term from the capacitive load will cancel, and the loop transfer function returns to single-pole roll-off behavior.

For this example, the minor-loop transfer admittance is

\[
Y_{c}(s) = \frac{10^{-11}s}{3 \cdot 10^{-7}s + 1}
\]

and the circuit loop transfer function is approximately

\[
L(s) \approx \frac{10^{7}(3 \cdot 10^{-7}s + 1)}{s} \left( \frac{1}{10^{-6}s + 1} \right)
\]

(the \( RC \) products are chosen to be slightly different to remain distinct on the Bode plots and to be a bit more realistic). The Bode plot of the forward-path and inverse-feedback-path transfer functions for the op amp with this compensation is shown in Figure 32. The Bode plot of the closed-minor-loop transfer function of the op amp \( A(s) \) is shown in Figure 33. The transfer-function zero is clearly visible near \( 3 \cdot 10^{6} \) rps. The resonance is blithely ignored.

This zero will be used to cancel the capacitive-load pole.

Fig. 30. Step response of the op-amp buffer with capacitive load. With less than eighteen degrees of phase margin, the response exhibits considerable peak overshoot and ringing.

Fig. 31. Compensation network with an admittance pole. This pole in the feedback-path \( Y_{c}(s) \) introduces a zero into the op-amp transfer function \( A(s) \). This zero will be used to cancel the capacitive-load pole.

Fig. 32. Bode plot of the forward-path and inverse-feedback-path transfer functions for the op amp with compensation that introduces a zero. The pole in \( Y_{c}(s) \) becomes a zero in \( A(s) \).

Fig. 33. Bode plot of the closed-minor-loop transfer function of the op amp \( A(s) \). The transfer-function zero introduced by the compensation network is clearly visible near \( 3 \cdot 10^{6} \) rps. The resonance is blithely ignored.

Fig. 34. Bode plot of the loop transfer function for the op-amp circuit with capacitive load is shown in Figure 34. The zero
Bode plot of the loop transfer function $L(s)$ for the op-amp buffer with capacitive load. The zero from the compensation network nearly cancels the pole from the capacitive load. The phase margin has increased from eighteen degrees in Figure 29 to more than sixty degrees.

\( \text{Fig. 34. Bode plot of the loop transfer function } L(s) \text{ for the op-amp buffer with capacitive load.} \)

Bode plot of the minor-loop transfer function $L_m(s)$ for the op amp with the compensation network from Figure 31. The pole in the admittance $Y_C(s)$ becomes a zero in $A(s)$, but the pole appears in the minor-loop transfer function and degrades the stability of the minor loop.

\( \text{Fig. 36. Bode plot of the minor-loop transfer function } L_m(s) \text{ for the op amp with the compensation network from Figure 31.} \)

Step response of the compensated op-amp buffer with capacitive load. The peak overshoot is greatly improved from Figure 30, but the high-frequency ringing indicates that the system is on the edge of instability.

\( \text{Fig. 35. Step response of the compensated op-amp buffer with capacitive load. The peak overshoot is greatly improved from Figure 30, but the high-frequency ringing indicates that the system is on the edge of instability.} \)

Compensation network with a shunt capacitance. The capacitor $C_D$ introduces a zero into the minor-loop transfer function that can be used to increase the stability of the minor loop.

\( \text{Fig. 37. Compensation network with a shunt capacitance. The capacitor } C_D \text{ introduces a zero into the minor-loop transfer function that can be used to increase the stability of the minor loop.} \)

B. Compensation with Minor-Loop Stability

The problem with the compensation network in Figure 31 is that the minor loop is nearly unstable, which can be demonstrated by examining the minor-loop transfer function. As shown in the block diagram in Figure 25, the minor-loop transfer function is

\[
L_m(s) = G_a(s)Y_C(s).
\]

The Bode plot of this minor-loop transfer function is shown in Figure 36. The admittance pole from the compensation network appears directly in the minor-loop transfer function and degrades the stability of the minor loop.

To improve the stability the minor loop, the compensation network is augmented with a shunt capacitance [12] as shown in Figure 37. The capacitor $C_D$ introduces a zero into the minor-loop transfer function and is used to improve the minor-loop phase margin. The compensation network admittance is

\[
Y_C(s) = \frac{s(C_C + C_D)(R_CC_Ss + 1)}{R_CC_Ss + 1}
\]

where

\[
C_S = \frac{C_CC_D}{C_C + C_D}.
\]

Using $R_CC_S = 10^{-8}$ second, the Bode plot of the minor-loop transfer function for the op amp with this improved compensation is shown in Figure 38. The additional zero from the capacitor $C_D$ increases the minor-loop phase margin to thirty-five degrees.
The Bode plot of the forward-path and inverse-feedback-path transfer functions for the op amp are now shown in Figure 39. The zero to cancel the capacitive-load pole is still visible near $3 \cdot 10^6$ rps. The pole from the capacitor $C_D$ appears beyond both minor-loop crossover and major-loop crossover at $10^8$ rps.

The Bode plot of the forward-path and inverse-feedback-path transfer functions for the op amp are now shown in Figure 39. The zero to cancel the capacitive-load pole is still visible near $3 \cdot 10^6$ rps. The pole from the capacitor $C_D$ appears beyond both major-loop crossover and minor-loop crossover frequencies.

The Bode plot of the op-amp transfer function is shown in Figure 40. The resonance from the minor loop has been greatly reduced compared to Figure 33.

The Bode plot of loop transfer function for op-amp circuit with capacitive load is shown in Figure 41. In contrast to Figure 34, the loop transfer function now has adequate gain margin as well as sixty degrees of phase margin.

The step response of the compensated op-amp buffer with capacitive load is shown in Figure 42. The peak overshoot
from Figure 30 is greatly reduced, and the high-frequency ringing from Figure 35 is gone.

As a final note, the step response of the compensated op-amp buffer with the capacitive load removed is shown in Figure 43. This special-purpose compensation developed here requires the capacitive load to be present for proper loop behavior. Without the capacitive load, the loop transfer function of the circuit is the transfer function of the op amp alone from Figure 40, which does not have adequate phase margin. However, with the capacitive load, the compensated op amp performs quite well.

Using the feedback approach to op-amp compensation design helped diagnose and solve the minor-loop stability problem. Without this approach, the solution to the high-frequency ringing from Figure 35 is gone.

VII. CONCLUSIONS

In all applications, op amps require a deliberately designed frequency response to ensure stability and satisfactory transient performance. Standard frequency compensation, using a capacitor around the high-gain stage, is designed for general-purpose op-amp applications such as amplifiers, buffers, and integrators. Sophisticated compensation techniques can be employed in specific applications in which standard compensation methods perform poorly.

These compensation techniques are necessary to understand in the design of internally compensated op amps or in the use of externally compensated op amps. A pole-splitting approach to the compensation design is harmful to understanding. All of these techniques can be easily understood in a simple classical-control framework. Using a feedback approach to the compensation network design, insight and intuition into the behavior and flexibility of the system are gained.

APPENDIX I

MILLER COMPENSATION

The Miller effect is the apparent scaling of an impedance connected from input to output of a gain stage, which was first noticed in vacuum tubes [14]. The input current into the second stage, as shown in Figure 44, depends on the total voltage across the capacitor

\[ I_1 = (V_1 - V_o)sC = V_1(1 + A)sC. \]

Thus for an amplifier with a large negative gain, the effective input capacitance appears \((1 + A)\) times larger than the capacitor \(C\).

From this effective capacitance, the low-frequency pole of the equivalent-circuit in Figure 10 can be estimated as

\[ \omega_1 = \frac{1}{RC_{eff}} = \frac{1}{R_1(1 + A_2)C} \approx \frac{1}{R_1(G_{M2}R_2)C} \]

which agrees with equation (4). For this reason, op-amp compensation with a capacitor around the second gain stage, as shown in Figure 8, is often called “Miller compensation.”

APPENDIX II

FAIRCHILD µA741 COMPLETE SCHEMATIC

The complete schematic for the Fairchild Semiconductor µA741 operational amplifier is shown in Figure 45. This topology is classic and simple.

The primary signal path is comprised of three blocks. The first stage is the differential quad of transistors \(Q_1-Q_4\) with active current-mirror load \(Q_5-Q_7\). The second stage is the Darlington common-emitter amplifier \(Q_{16}\) and \(Q_{17}\) with current source load \(Q_{13}\). A push-pull emitter-follower output buffer is implemented by transistors \(Q_{14}\), \(Q_{20}\), and \(Q_{22}\).

The remaining transistors provide biasing and protection. The network of current mirrors \(Q_{8}-Q_{15}\) produce bias currents for the transistors in the signal path. Compensation of the output-buffer dead-zone region is provided by \(Q_{18}\) and \(Q_{19}\). Output-current limiting and short-circuit protection is implemented by \(Q_{15}\) and \(Q_{21}-Q_{25}\).

And of course, the frequency compensation is accomplished by the 30 pF capacitor around \(Q_{16}\) and \(Q_{17}\), as discussed in Section II.
Solving for the transfer function using the feedback techniques in Section IV and Appendix V. This second-order transfer function is the expected result from the topology in Figure 10. All of this math can be avoided.

APPENDIX III

EXACT TRANSFER FUNCTION MATH

The exact transfer function for the equivalent circuit in Figure 10 can be found directly from the node equations (2) and (3). Solving the output-node equation (3) for $V_1$

\[
V_o \left( sC + \frac{1}{R_2} + sC_2 \right) = V_1(sC - G_{M2})
\]

\[
V_o (R_2(C_2 + C)s + 1) = V_1(R_2Cs - G_{M2}R_2)
\]

\[
V_1 = \left( \frac{R_2(C_2 + C)s + 1}{R_2Cs - G_{M2}R_2} \right) V_o.
\]

Massaging the input-node equation (2)

\[
G_{M1}V_{in} + sCV_o = V_1 \left( \frac{1}{R_1} + sC_1 + sC \right)
\]

\[
G_{M1}R_1V_{in} + R_1CsV_o = (R_1(C_1 + C)s + 1) \left( \frac{R_2(C_2 + C)s + 1}{R_2Cs - G_{M2}R_2} \right) V_o
\]

\[
G_{M1}R_1V_{in} = V_o \left( \frac{(R_1(C_1 + C)s + 1)(R_2(C_2 + C)s + 1)}{R_2Cs - G_{M2}R_2} - R_1Cs \right).
\]

Solving for the transfer function

\[
\frac{V_o}{V_{in}}(s) = \frac{G_{M1}R_1(R_2Cs - G_{M2}R_2)}{(R_1(C_1 + C)s + 1)(R_2(C_2 + C)s + 1) - (R_2Cs - G_{M2}R_2)R_1Cs
\]

\[
\frac{V_o}{V_{in}}(s) = \frac{G_{M1}R_1G_{M2}R_2(Cs/G_{M2} - 1)}{R_1R_2(C_1C_2 + C_1C + C_2) + (R_1C_1 + R_1C + R_2C_2 + R_2C + G_{M2}R_2R_1Cs + 1}.
\]

This second-order transfer function is the expected result from the topology in Figure 10. All of this math can be avoided using the feedback techniques in Section IV and Appendix V.
Appendix IV
Closed-Loop Transfer Function Trick

The work for finding an approximate transfer function of a feedback system, such as shown in Figure 46, can be simplified by taking advantage of a useful trick involving Black’s formula

\[
\frac{C}{R}(s) = \frac{G(s)}{1 + G(s)H(s)}.
\]

Black’s formula can be simplified when

\[ |G(s)H(s)| \gg 1 \]

or equivalently, when

\[ |G(s)| \gg \frac{1}{|H(s)|}. \]

When this inequality is true, the \(GH\) term dominates in the denominator, and the magnitude of the closed-loop transfer function can be rewritten as

\[
|C \frac{R}{R}(s)| = \left| \frac{G(s)}{1 + G(s)H(s)} \right| \approx \frac{1}{|H(s)|}.
\]

Similarly, when

\[ |G(s)H(s)| \ll 1 \]

or equivalently, when

\[ |G(s)| \ll \frac{1}{|H(s)|} \]

the unity term dominates in the denominator. The magnitude of the closed-loop transfer function is then approximately

\[
|C \frac{R}{R}(s)| = \left| \frac{G(s)}{1 + G(s)H(s)} \right| \approx |G(s)|.
\]

Therefore, the asymptotic frequency response of a closed-loop feedback system, can be plotted by graphing \(|G(s)|\) and \(|H(s)|^{-1}\) and then tracing the lower curve for all frequencies.

Appendix V
Right Half-Plane Zero

In the analysis in Section IV, the effect of the voltage \(V_1\) on the current through the compensation capacitor was ignored. Unfortunately, the output voltage of the first stage, while indeed small, is not zero. Both currents that flow through the compensation capacitor are shown in Figure 47, where

\[ I_c = sCV_o \]

and

\[ I_r = sCV_1. \]

The feedback current \(I_r\) is the compensation current discussed in Section IV. The feedforward current \(I_c\) causes a right half-plane zero in the op-amp transfer function. By superposition, the sum of these two currents is the total current flowing in the compensation capacitor.

A close-up of the effect of the feedforward current \(I_c\) on the second stage is shown in Figure 48. The output voltage is zero when the total current flowing into the output stage is zero, that is

\[ sCV_1 - G_{M2}V_1 = 0. \]

This zero occurs at a frequency

\[ \omega_z = \frac{G_{M2}}{C}. \]

Therefore equation (7) must now be written as

\[ V_o = (-G_{M2}V_1 + sCV_1) \frac{R_2}{R_2C_4s + 1}. \]

The complete block diagram of the op-amp equivalent circuit, including the feedforward current through the compensation capacitor, is shown in Figure 49. The parallel blocks in the forward path can be collapsed into a single block

\[ -G_{M2} + sC = -G_{M2} \left( 1 - \frac{sC}{G_{M2}} \right). \]

Thus the feedforward current causes a right half-plane zero. The negative phase shift from this right half-plane zero can place considerable limits on op-amp performance.

This problem can be fixed. A compensation network to cancel the right half-plane zero is shown in Figure 50 with admittance

\[ Y_c(s) = \frac{I_c}{V_o} = \frac{Cs}{RCs + 1}. \]
Fig. 49. Block diagram of the op-amp equivalent circuit, including the feedforward current through the compensation capacitor. The feedforward term causes a right half-plane zero in the op-amp transfer function.

\[ \frac{C}{R} \]

Fig. 50. Compensation network to cancel the right half-plane zero due to the feedforward current through the compensation capacitor.

\[ -G_{M2} + \frac{Cs}{RCs + 1} = \frac{-G_{M2}RCs - G_{M2} + Cs}{RCs + 1} = -G_{M2}\frac{(R - 1/G_{M2})Cs + 1}{RCs + 1}. \]

Thus, for a choice of \( R = 1/G_{M2} \), the zero moves out of the right half-plane to infinity.

The right half-plane zero is usually not a problem in bipolar op amps. For example, in the \( \mu A741 \)

\[ f_z = \frac{G_{M2}}{2\pi C} = \frac{6.8 \text{ m}\Omega}{2\pi 30 \text{ pF}} = 36 \text{ MHz}. \]

In a CMOS op amp, where transistor transconductances can be much lower, the right-half plane zero frequency can be quite close to the unity-gain frequency of the op amp.

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