1.

(a)

$$Z_i = Z_0 \cdot \frac{Z_L + Z_0 \cdot \tanh(\gamma l)}{Z_0 + Z_L \cdot \tanh(\gamma l)}$$

$$= \frac{Z_0}{\tanh(\gamma l)} \qquad (Z_L = +\infty)$$

$$\tanh(\gamma l) = \tanh((\alpha + j\beta)l) = \frac{\sinh(\alpha l + j \cdot \beta l)}{\cosh(\alpha l + j \cdot \beta l)}$$

$$= \frac{\sinh(\alpha l) \cdot \cos(\beta l) + j \cdot \cosh(\alpha l) \cdot \sin(\beta l)}{\cosh(\alpha l) \cdot \cos(\beta l) + j \cdot \sinh(\alpha l) \cdot \sin(\beta l)}$$

Since $\alpha l \ll 1$,

$$\sinh(\alpha l) = \frac{e^{\alpha l} - e^{-\alpha l}}{2} = \frac{(1+\alpha l) - (1-\alpha l)}{2} = \alpha l.$$

$$\cosh(\alpha l) = \frac{e^{\alpha l} + e^{-\alpha l}}{2} = \frac{(1+\alpha l) + (1-\alpha l)}{2} = 1.$$

$$\therefore \tanh(\gamma l) = \frac{\alpha l \cdot \cos(\beta l) + j \cdot \sin(\beta l)}{\cos(\beta l) + j \cdot (\alpha l) \cdot \sin(\beta l)}$$

When $\beta l = n\pi$, i.e. $l = \frac{\lambda}{2} \cdot n$,

$\tanh(\gamma l)$ attains its minima αl.

Thus $Z_i = \frac{Z_0}{\alpha l}$ at $l = \frac{\lambda}{2} n$.

Note $\tanh(\gamma l)$ doesn't vanish as in lossless case.

Hence $(Z_i)_{max} = \frac{Z_0}{\alpha l}$, where $l = \frac{\lambda}{2}$, $n=1$.

(b).

$$\alpha = 0.01 \text{ dB/m}.$$

$$\Rightarrow \quad -20 \cdot \log_{10} e^{-\alpha} = 0.01$$

$$\alpha = \frac{0.01}{20} \cdot \frac{1}{\log_{10} e} = 1.15 \times 10^{-3} \text{ Np/m}.$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{c}{\sqrt{\varepsilon_r} \cdot f}} = \frac{2\pi \cdot 2 \cdot 3 \times 10^8}{3 \times 10^8} = 4\pi.$$

$$Q = \frac{f_0}{\Delta f} = \frac{\beta}{2\alpha} = \frac{4\pi}{2 \times 1.15 \times 10^{-3}} = 5463.6$$

The half-power bandwidth is

$$\Delta f = \frac{f_0}{Q} = \frac{3 \times 10^8}{5463.6} \doteq 54.9 \text{ kHz}.$$

2.

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2.$$

Thus, $|\Gamma| = \frac{1}{3}$.

The distance between successive voltage maxima is $\frac{\lambda}{2} = 18$ (cm)

$$\lambda = 36 \text{ (cm)}.$$

(a). $\Gamma = |\Gamma| \cdot e^{j\theta_\Gamma}$.

voltage maxima $\Leftrightarrow$ $\theta_\Gamma - 2\beta \cdot l = 2n\pi$.

$\therefore \quad \theta_\Gamma - 2 \cdot \frac{2\pi}{\lambda} \cdot 6 = \theta_\Gamma - 2 \cdot \frac{2\pi}{36} \cdot 6 = 0$.

$\theta_\Gamma = \frac{4\pi}{3\times 2} = \frac{2\pi}{3}$.

$\Gamma = \frac{1}{3} \cdot e^{j\frac{2\pi}{3}}$.

(b). $Z_L = Z_0 \cdot \frac{1+\Gamma}{1-\Gamma}$

$= 50 \cdot \frac{1+\frac{1}{3} \cdot e^{j\frac{2\pi}{3}}}{1-\frac{1}{3} \cdot e^{j\frac{2\pi}{3}}}$

$= 30.77 + j \cdot 20$.

(c). at $z' = 15$ (cm).

$\Gamma' = \frac{1}{3} \cdot e^{j(\theta_\Gamma - 2\beta \cdot z')} = -\frac{1}{3}$.

$Z_{in} = Z_0 \cdot \frac{1+\Gamma'}{1-\Gamma'} = Z_0 \cdot \frac{\frac{2}{3}}{\frac{4}{3}} = 25$ (Ω).

3. (a). $P_i = \frac{1}{2} \cdot \frac{|V_0|^2}{R_0} = 10^2$ (W).

(b). At the load,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j\cdot 50 - 50}{50 + j\cdot 50 + 50}$$

$$= 0.2 + j\cdot 0.4.$$

(c). The reflected power flow is

$$P_r = \frac{1}{2}\cdot\frac{|V_r|^2}{R_0} = \frac{1}{2}\cdot\frac{|V_0|^2\cdot|\Gamma|^2}{R_0} = 10^2 \times |\Gamma|^2$$

$$= 10^2 \times 0.2 = 20 \text{ (W)}$$

(d). The power dissipation is

$$P_i - P_r = 80 \text{ (W)}.$$