Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 146A
Winter 2008
Shynk
H.O. #4

HOMEWORK #1

Due Friday, January 18, 2008 (5:00 p.m.)

Reading: Background and Preview (overview), Appendix 2 (review), and Chapter 1 (review)

Problems:

1. Any function $g(t)$ can be written in terms of an even part and an odd part as follows: $g(t) = g_e(t) + g_o(t)$ where

$$\begin{aligned} g_e(t) &= 1/2[g(t) + g(-t)] \\ g_o(t) &= 1/2[g(t) - g(-t)]. \end{aligned}$$

Evaluate the even and odd parts of

$$g(t) = A \ \text{rect}(t/T - 1).$$

2. The following expression can be viewed as an approximate representation of a pulse:

$$g(t) = \frac{1}{\tau} \int_{t-T}^{t+T} e^{-\pi x^2/\tau^2} dx$$

where $T >> \tau$. Find an expression for the Fourier transform $G(f)$, and determine what happens to your result when $\tau \rightarrow 0$.

3. Evaluate the inverse Fourier transform of

$$G(f) = \begin{cases} e^f, & f < 0 \\ 1/2, & f = 0 \\ 0, & f > 0. \end{cases}$$

Show that $g(t)$ is complex and that its real and imaginary parts constitute a Hilbert transform pair.

4. Prove the following properties of the Fourier transform:

(i) If the real signal $g(t)$ is an even function of t, then $G(f)$ is real. If the real signal $g(t)$ is an odd function of t, then $G(f)$ is imaginary.

(ii)

$$\int_{-\infty}^{\infty} g_1(t)g_2^*(t)dt = \int_{-\infty}^{\infty} G_1(f)G_2^*(f)df$$

5. The real finite-energy signal $x(t)$ is applied to a square-law device whose output is

$$y(t) = x^2(t).$$

If the spectrum of $X(f)$ is limited to $|f| < W$, then show that $Y(f)$ is limited to $|f| < 2W$.

6. Determine the pre-envelope $g_+(t)$ for each of the following signals:

(i) $g(t) = \text{sinc}(2t)$

(ii) $g(t) = \sin(2\pi f_m t)\cos(2\pi f_c t)$.