Midterm Exam: Thursday, February 14, 5:00 - 6:15 p.m. (open book, open notes)

Reading: Chapter 2 (2.7)

Problems:

1. Consider the SSB signal

\[ s(t) = m(t) \cos(2\pi f_c t) - \hat{m}(t) \sin(2\pi f_c t) \]

where \( f_c \) is the carrier frequency, \( m(t) \) is the message signal, and \( \hat{m}(t) \) is its Hilbert transform. This modulated wave is applied to a square-law device characterized by

\[ y(t) = s^2(t). \]

Show that the output \( y(t) \) contains a frequency component twice the carrier frequency and has a time-varying phase. Determine if it is possible to extract \( m(t) \) after \( y(t) \) is low-pass filtered.

2. (a) Let \( s_u(t) \) denote the SSB signal obtained by transmitting the upper sideband, and let \( \hat{s}_u(t) \) be its Hilbert transform. Show that

\[ m(t) = \frac{2}{A_c} [s_u(t) \cos(2\pi f_c t) + \hat{s}_u(t) \sin(2\pi f_c t)] \]

\[ \hat{m}(t) = \frac{2}{A_c} [\hat{s}_u(t) \cos(2\pi f_c t) - s_u(t) \sin(2\pi f_c t)] \]

where \( m(t) \) is the message signal, \( \hat{m}(t) \) is its Hilbert transform, \( f_c \) is the carrier frequency, and \( A_c \) is the carrier amplitude.

(b) Specify the corresponding equations for the SSB signal \( s_l(t) \) obtained by transmitting the lower sideband.
Using these results, sketch a block diagram of a coherent receiver for demodulating an SSB signal.

Consider a frequency-division multiplexed (FDM) system in which four message signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$ are, respectively, multiplied by the carrier signals

$$c_1(t) = \cos(2\pi f_a t) + \cos(2\pi f_b t),$$
$$c_2(t) = \cos(2\pi f_a t + \alpha_1) + \cos(2\pi f_b t + \beta_1),$$
$$c_3(t) = \cos(2\pi f_a t + \alpha_2) + \cos(2\pi f_b t + \beta_2),$$
$$c_4(t) = \cos(2\pi f_a t + \alpha_3) + \cos(2\pi f_b t + \beta_3),$$

and the resulting DSB-SC signals are summed and transmitted over a common channel. In the receiver, demodulation is achieved by multiplying the sum of the DSB-SC signals by the four carrier signals separately and then filtering to remove the unwanted components.

(a) Determine the conditions that the phase angles $\{\alpha_i\} (i = 1, 2, 3)$ and $\{\beta_j\} (j = 1, 2, 3)$ must satisfy so that the output of the $k$th demodulator is $m_k(t)$ for $k = 1, 2, 3, 4$.

(b) Determine the minimum separation of the carrier frequencies $f_a$ and $f_b$ relative to the bandwidth of the input signals in order to ensure satisfactory operation of the system.

4. Problem 2.15 (you can use Matlab)

5. Problem 2.17

6. Problem 2.21