Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 146A
Winter 2008
Shynk
H.O. #10

HOMEWORK #4

Due Friday, February 8, 2008 (5:00 p.m.)

Midterm Exam: Thursday, February 14, 5:00 - 6:15 p.m. (open book, open notes)

Reading: Chapter 2 (2.7)

Problems:

1. Consider the SSB signal

$$s(t) = m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)$$

where f_c is the carrier frequency, $m(t)$ is the message signal, and $\hat{m}(t)$ is its Hilbert transform. This modulated wave is applied to a square-law device characterized by

$$y(t) = s^2(t).$$

Show that the output $y(t)$ contains a frequency component twice the carrier frequency and has a time-varying phase. Determine if it is possible to extract $m(t)$ after $y(t)$ is low-pass filtered.

2. **(a)** Let $s_u(t)$ denote the SSB signal obtained by transmitting the upper sideband, and let $\hat{s}_u(t)$ be its Hilbert transform. Show that

$$\begin{aligned} m(t) &= \frac{2}{A_c}[s_u(t)\cos(2\pi f_c t) + \hat{s}_u(t)\sin(2\pi f_c t)] \\ \hat{m}(t) &= \frac{2}{A_c}[\hat{s}_u(t)\cos(2\pi f_c t) - s_u(t)\sin(2\pi f_c t)] \end{aligned}$$

where $m(t)$ is the message signal, $\hat{m}(t)$ is its Hilbert transform, f_c is the carrier frequency, and A_c is the carrier amplitude.

(b) Specify the corresponding equations for the SSB signal $s_l(t)$ obtained by transmitting the lower sideband.

(c) Using these results, sketch a block diagram of a coherent receiver for demodulating an SSB signal.

3. Consider a frequency-division multiplexed (FDM) system in which four message signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$ are, respectively, multiplied by the carrier signals

$$\begin{aligned}
c_1(t) &= \cos(2\pi f_a t) + \cos(2\pi f_b t), \\
c_2(t) &= \cos(2\pi f_a t + \alpha_1) + \cos(2\pi f_b t + \beta_1), \\
c_3(t) &= \cos(2\pi f_a t + \alpha_2) + \cos(2\pi f_b t + \beta_2), \\
c_4(t) &= \cos(2\pi f_a t + \alpha_3) + \cos(2\pi f_b t + \beta_3),
\end{aligned}$$

and the resulting DSB-SC signals are summed and transmitted over a common channel. In the receiver, demodulation is achieved by multiplying the sum of the DSB-SC signals by the four carrier signals separately and then filtering to remove the unwanted components.

(a) Determine the conditions that the phase angles $\{\alpha_i\}$ $(i = 1, 2, 3)$ and $\{\beta_j\}$ $(j = 1, 2, 3)$ must satisfy so that the output of the kth demodulator is $m_k(t)$ for $k = 1, 2, 3, 4$.

(b) Determine the minimum separation of the carrier frequencies f_a and f_b relative to the bandwidth of the input signals in order to ensure satisfactory operation of the system.

4. Problem 2.15 (you can use Matlab)

5. Problem 2.17

6. Problem 2.21