Sections 8.4 and 8.5

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8.4 Deterministic Distortion

Impairments that can affect data transmission over telephone channels can be classified as being one of two types: deterministic impairments or random impairments. *Deterministic impairments* include amplitude distortion, delay distortion, nonlinearities, and frequency offset. These impairments are deterministic in that they can be measured for a particular transmission path and they do not change perceptibly over a relatively short period of time. On the other hand, *random impairments* include Gaussian noise, impulsive noise, and phase jitter. Clearly, these random disturbances require a probabilistic description, and hence are treated in a later chapter.

Amplitude distortion and delay distortion can cause intersymbol interference, which decreases the system margin against noise. The magnitude response of a "typical" telephone channel is sketched in Fig. 8.4.1. Note that what is shown is not "gain versus frequency," but "attenuation versus frequency." It is quite evident that the amplitude response as a function of frequency between 200 and 3200 Hz is far from flat, and hence certainly not distortionless.

A demonstration of the detrimental effects of a nonflat frequency response is obtainable by considering the classic example of a filter with ripples in its amplitude response. In particular, we wish to examine the time-domain response of a system with the transfer function

$$H(\omega) = [1 + 2\varepsilon \cos \omega t_0]e^{-j\omega t_d}, \tag{8.4.1}$$

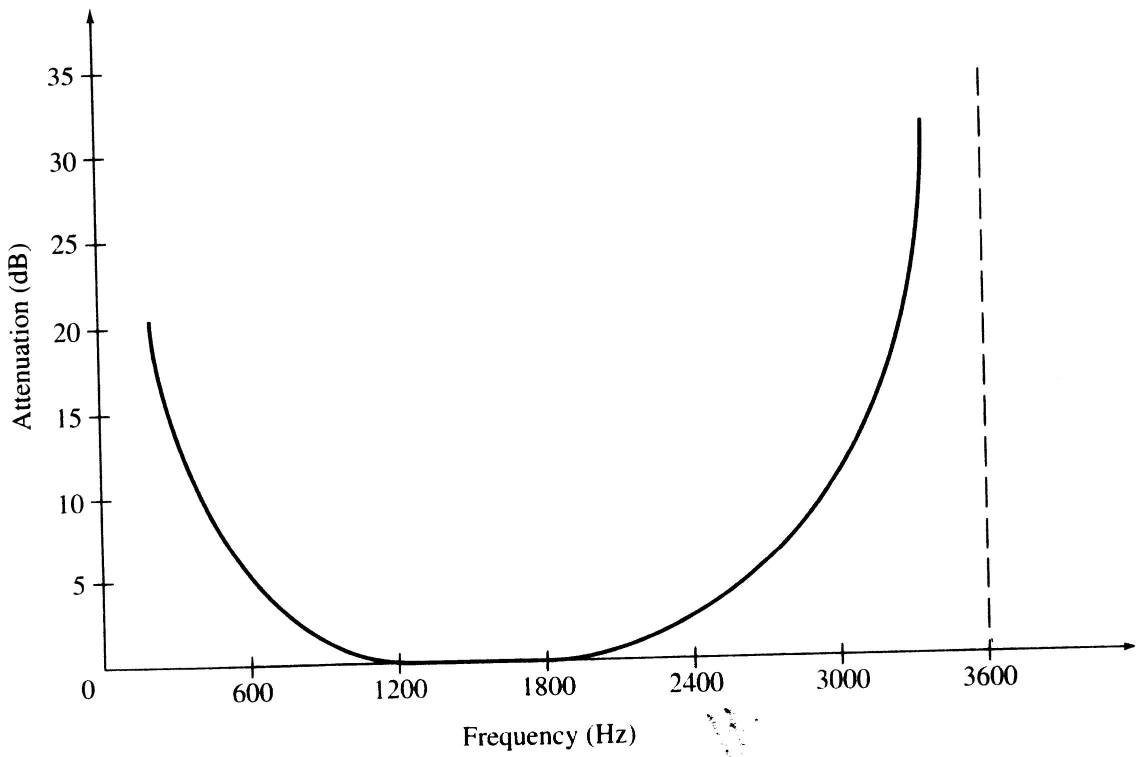


FIGURE 8.4.1 Magnitude response of a typical telephone channel.

where $|\varepsilon| \leq \frac{1}{2}$. It should be evident to the reader that $|H(\omega)|$ has ripples of maximum value $\pm 2\varepsilon$ about 1 and that the phase of $H(\omega)$ is linear. We thus have amplitude distortion only. Rewriting Eq. (8.4.1), we obtain

$$\begin{aligned} H(\omega) &= [1 + \varepsilon(e^{j\omega t_0} + e^{-j\omega t_0})]e^{-j\omega t_d} \\ &= e^{-j\omega t_d} + \varepsilon e^{j\omega(t_0 - t_d)} + \varepsilon e^{-j\omega(t_0 + t_d)}. \end{aligned} \tag{8.4.2}$$

For a general input pulse shape, say $x(t)$, the time-domain response of the filter in Eq. (8.4.2) is

$$y(t) = x(t - t_d) + \varepsilon x(t - t_d + t_0) + \varepsilon x(t - t_d - t_0). \tag{8.4.3}$$

The ripples in the magnitude response of $H(\omega)$ have thus generated scaled replicas of $x(t)$ that both precede and follow an undistorted (but delayed) version of the input. These replicas of $x(t)$ have been called "echoes" in the literature. Depending on the value of t_0, we find that the echoes can interfere with adjacent transmitted pulses, yielding intersymbol interference, and/or the echoes can overlap the main output pulse $x(t - t_d)$, also causing distortion. Sketches of these various cases are left as a problem.

A filter (or channel) with ripples in the phase but a flat magnitude response will also cause echoes. To demonstrate this claim, we consider the transfer function

$$H(\omega) = \exp\{-j[\omega t_d - \varepsilon \sin \omega t_0]\} \tag{8.4.4}$$

with $|\varepsilon| \ll \pi$. To proceed, we note that $e^{-j\omega t_d}$ represents a pure time delay and that

$$e^{j\varepsilon \sin \omega t_0} = \cos[\varepsilon \sin \omega t_0] + j \sin[\varepsilon \sin \omega t_0]. \tag{8.4.5}$$

Since $|\varepsilon| \ll \pi$, we can approximate Eq. (8.4.5) as

$$e^{j\varepsilon \sin \omega t_0} \cong 1 + j\varepsilon \sin \omega t_0, \tag{8.4.6}$$

so

$$H(\omega) \cong e^{-j\omega t_d}\left[1 + \frac{\varepsilon}{2} e^{j\omega t_0} - \frac{\varepsilon}{2} e^{-j\omega t_0}\right]. \tag{8.4.7}$$

For a general input pulse shape $x(t)$, then, the filter output is approximately

$$y(t) \cong x(t - t_d) + \frac{\varepsilon}{2} x(t - t_d + t_0) - \frac{\varepsilon}{2} x(t - t_d - t_0) \tag{8.4.8}$$

and again we have echoes.

We have not established as yet that phase distortion is actually significant over telephone channels. Since the telephone network was originally designed with only voice signals in mind, there is considerable phase nonlinearity over telephone channels. Voice communication is relatively unaffected by the phase nonlinearities present, but as we have seen, data transmission can be impaired significantly. Phase information for a telephone channel is not usually expressed as a system phase response, due to the difficulty of establishing an absolute phase reference and the necessity to count modulo 2π or 360°. To obtain phase information, a related quantity called *envelope delay* is measured instead.

The envelope delay is defined as the rate of change of the phase versus frequency response, and hence can be expressed as

$$t_R = \frac{-d}{d\omega}\,\theta(\omega), \tag{8.4.9}$$

where $\theta(\omega)$ is the channel phase in radians and t_R denotes the envelope delay in seconds. A related quantity is the *phase delay* or *carrier delay*, which is defined as the change in phase versus frequency, and it is expressible as

$$t_c = \frac{-\theta(\omega)}{\omega}, \tag{8.4.10}$$

where t_c denotes the phase or carrier delay. These definitions are instrumental to the measurement of phase distortion over a channel, and the following derivation illustrates the reasoning behind their names as well as the technique used to measure the envelope delay.

We consider a bandpass channel with a constant magnitude response, $|H(\omega)| = K$, but with a nonlinear phase response, $\underline{/H(\omega)} = \theta(\omega)$. If the input to this channel is a narrowband signal

$$x(t) = m(t)\cos\omega_c t, \tag{8.4.11}$$

we wish to write an expression for the channel output, denoted $y(t)$, in terms of the envelope delay and the phase delay. To begin, we assume that the phase nonlinearity is not too severe, and thus we can expand the phase in a Taylor's series about the carrier components $\pm\omega_c$. About $\omega = +\omega_c$, we have

$$\theta(\omega) = \theta(\omega_c) + (\omega - \omega_c)\left.\frac{d\theta(\omega)}{d\omega}\right|_{\omega=\omega_c}, \tag{8.4.12}$$

where higher-order terms are assumed negligible. At the frequency of interest, namely $\omega = +\omega_c$, we know that

$$t_R = -\left.\frac{d\theta(\omega)}{d\omega}\right|_{\omega=\omega_c} \tag{8.4.13}$$

and

$$t_c = -\left.\frac{\theta(\omega)}{\omega}\right|_{\omega=\omega_c}, \tag{8.4.14}$$

so

$$\theta(\omega) = -\omega_c t_c + (\omega - \omega_c)(-t_R) = -\omega_c t_c - (\omega - \omega_c)t_R. \tag{8.4.15}$$

Similarly, about $\omega = -\omega_c$, we find that

$$\theta(\omega) = \omega_c t_c - (\omega + \omega_c)t_R. \tag{8.4.16}$$

Letting $M(\omega) = \mathscr{F}\{m(t)\}$ and $Y(\omega) = \mathscr{F}\{y(t)\}$, we can write the channel response as

$$Y(\omega) = \frac{K}{2}\,[M(\omega - \omega_c) + M(\omega + \omega_c)]e^{j\theta(\omega)}$$

$$= \frac{K}{2}\,[M(\omega - \omega_c)e^{j\theta(\omega)} + M(\omega + \omega_c)e^{j\theta(\omega)}]. \tag{8.4.17}$$

Since $M(\omega - \omega_c)$ is located about $\omega = +\omega_c$ and since $M(\omega + \omega_c)$ is located about $\omega = -\omega_c$, we make the appropriate substitutions into Eq. (8.4.17) using Eqs. (8.4.15) and (8.4.16), respectively. Hence

$$Y(\omega) = \frac{K}{2}\,[M(\omega - \omega_c)e^{j[-\omega_c t_c - (\omega - \omega_c)t_R]} + M(\omega + \omega_c)e^{j[\omega_c t_c - (\omega + \omega_c)t_R]}]$$

$$= \frac{K}{2}\,e^{-j\omega t_R}[M(\omega - \omega_c)e^{-j\omega_c[t_c - t_R]} + M(\omega + \omega_c)e^{j\omega_c[t_c - t_R]}]. \tag{8.4.18}$$

Therefore, using Fourier transform properties we obtain

$$y(t) = \frac{K}{2}\,[m(t)e^{j\omega_c[t - t_c + t_R]} + m(t)e^{-j\omega_c[t - t_c + t_R]}]\Big|_{t = t - t_R}$$

$$= Km(t - t_R)\cos\,[\omega_c(t - t_c)]. \tag{8.4.19}$$

Only the envelope $m(t)$ is delayed by t_R, hence t_R is called the *envelope delay*, while only the carrier is delayed by t_c, hence the name *carrier delay*.

The method used to measure the envelope delay over real channels satisfies the assumptions used in the derivation. For example, $m(t)$ is selected to be a low-frequency sinusoid, say $\cos\omega_m t$, so that we have $\omega_c \pm \omega_m \cong \omega_c$ and hence the phase nonlinearity will not be too bad over this region. We can also neglect higher-order terms in the Taylor's series since our range of frequencies of interest is $\omega_c - \omega_m \le \omega \le \omega_c + \omega_m$ and ω_m is small. To measure t_R, the derivative is approximated by $\Delta\theta(\omega)/\Delta\omega$, where $\Delta\omega = \omega_m$, and the phase of the received envelope $\cos(\omega_m t - \omega_m t_R)$ is compared to the transmitted envelope to yield $\Delta\theta(\omega) = \omega_m t_R$. Taking the ratio yields the envelope delay.

Results published by Sunde [1961] clearly demonstrate the effects of phase distortion on data transmission. For zero phase distortion, the phase response should be linear, and hence the envelope delay should be constant. Figure 8.4.2 shows plots of 100% roll-off raised cosine pulses subjected to quadratic envelope delay distortion. To obtain these plots, Sunde [1961] inserted delay distortion, which increased quadratically from 0 at $\omega = 0$ to some final value at $\omega = \omega_{max}$. As the envelope delay increases, the pulse amplitude is reduced and the pulse peak no longer occurs at the desired sampling instant. Furthermore, the zero crossings become shifted and the trailing pulse becomes large enough in amplitude to interfere with adjacent pulses. Obviously, delay distortion or phase distortion can be exceedingly detrimental in data transmission.

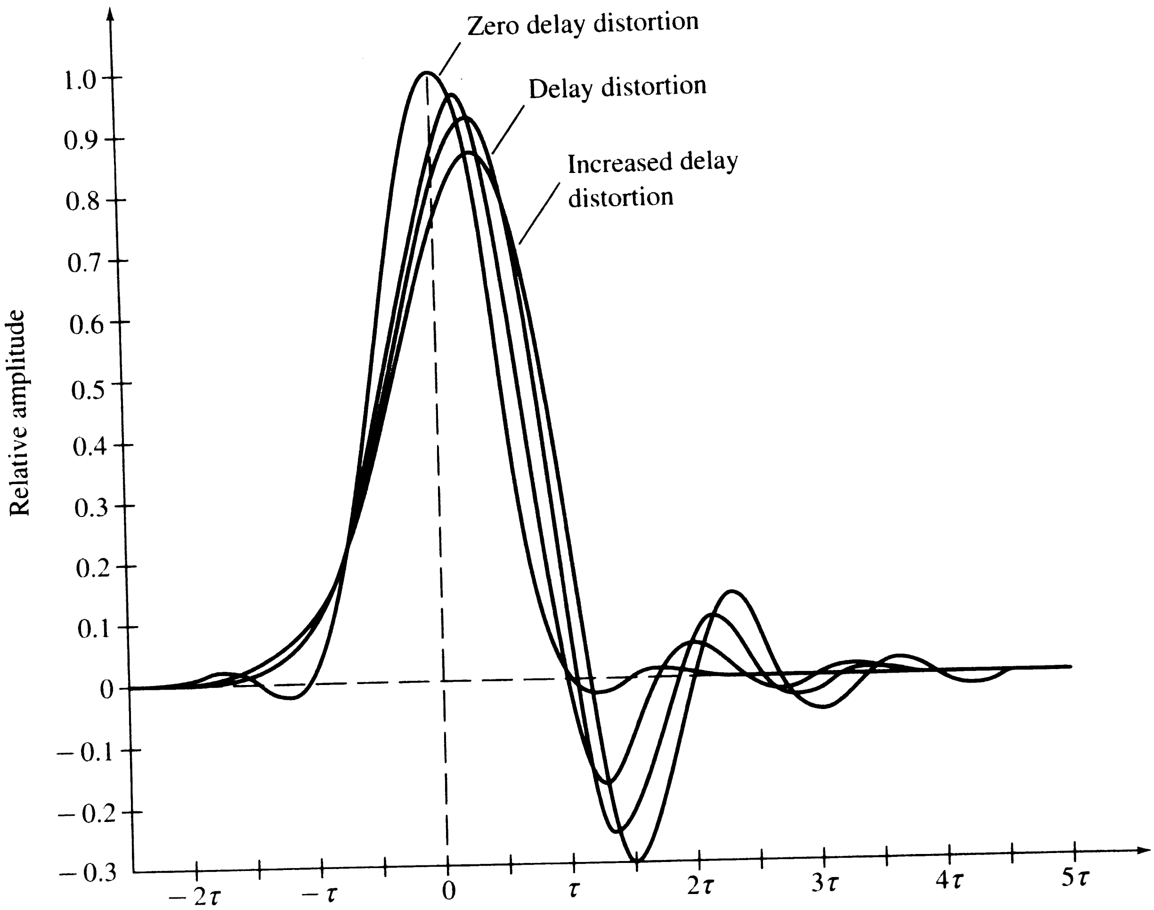
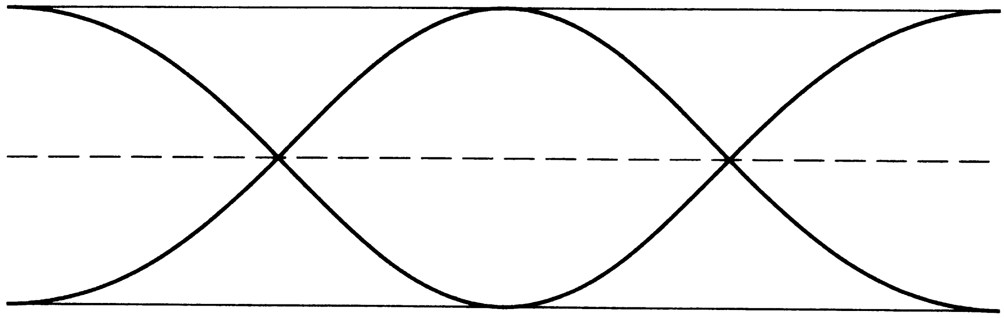


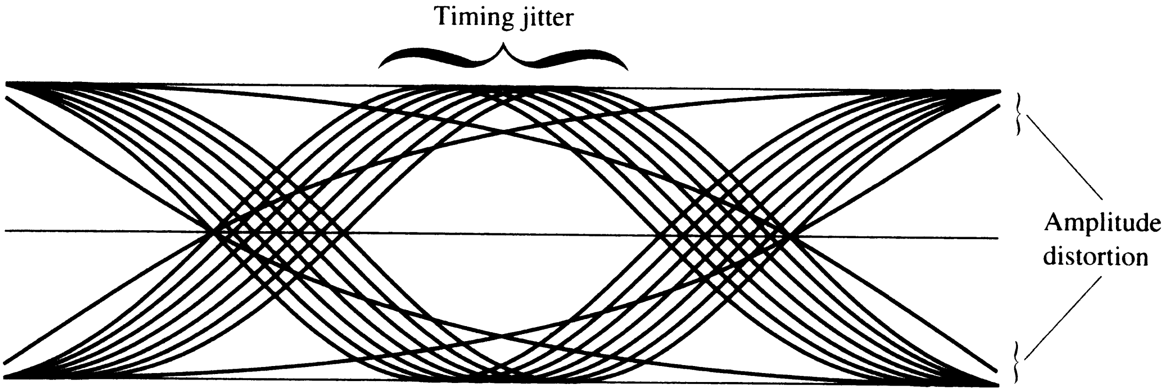
FIGURE 8.4.2 Raised cosine pulses with quadratic delay distortion. From E. D. Sunde, "Pulse Transmission by AM, FM, and PM in the Presence of Phase Distortion," *Bell Syst. Tech. J.*, © 1961 AT&T Bell Laboratories.

8.5 Eye Patterns

A convenient way to see the distortion present on a channel is to display what is called the system *eye pattern* or *eye diagram*. The eye pattern is obtained by displaying the data pulse stream on an oscilloscope, with the pulse stream applied to the vertical input and the sampling clock applied to the external trigger. A drawing of a two-level eye pattern is shown in Fig. 8.5.1, and the source of its name is clearly evident. Typically, one to three pulse (symbol) intervals are displayed and several kinds of distortion are easily observed. For minimum error probability, sampling should occur at the point where the eye is open widest. If all of the traces go through allowable (transmitted) pulse amplitudes only at the sampling instants, the eye is said to be *100% open* or *fully open*. A fully open eye pattern for three-level pulse transmission is sketched in Fig. 8.5.2. The eye pattern is said to be 80% open if, at the sampling instant, the traces deviate by 20% from the fully open eye diagram. This degradation is some-



(a)



(b)

FIGURE 8.5.1 Two-level eye diagrams: (a) Two-level pattern (distortionless); (b) two-level eye pattern with timing jitter.

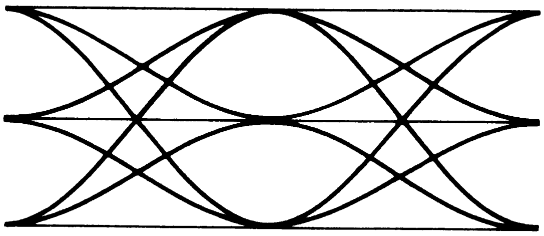


FIGURE 8.5.2 Three-level eye pattern.

times expressed as a loss in signal-to-noise ratio (S/N) by $S/N = 20 \log_{10} 0.8 = -1.9$ dB, where the minus sign indicates a degradation in S/N.

The distance between the decision thresholds and adjacent received pulse traces at the sampling time is the margin of the system against additional noise. As the sampling time is varied about the time instant of maximum eye opening, the eye begins to close. The rate that the eye closes as the sampling instant is varied is an indication of the system's sensitivity to timing error. Jitter in received zero crossings (or threshold crossings) can be particularly insidious, since many receivers extract timing information by averaging zero crossings. The various kinds of distortion are labeled in Fig. 8.5.1.