ECE 146B
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Spring 2008
04/11/2008

Solutions to Homework No. 1

Problem 3.5

The spectrum of the flat-top pulses is given by

$$H(f) = T\,\text{sinc}(fT)\exp(-j\pi fT)$$

$$= 10^{-4}\,\text{sinc}(10^{-4}f)\exp(-j\pi f 10^{-4})$$

Let $s(t)$ denote the sequence of flat-top pulses:

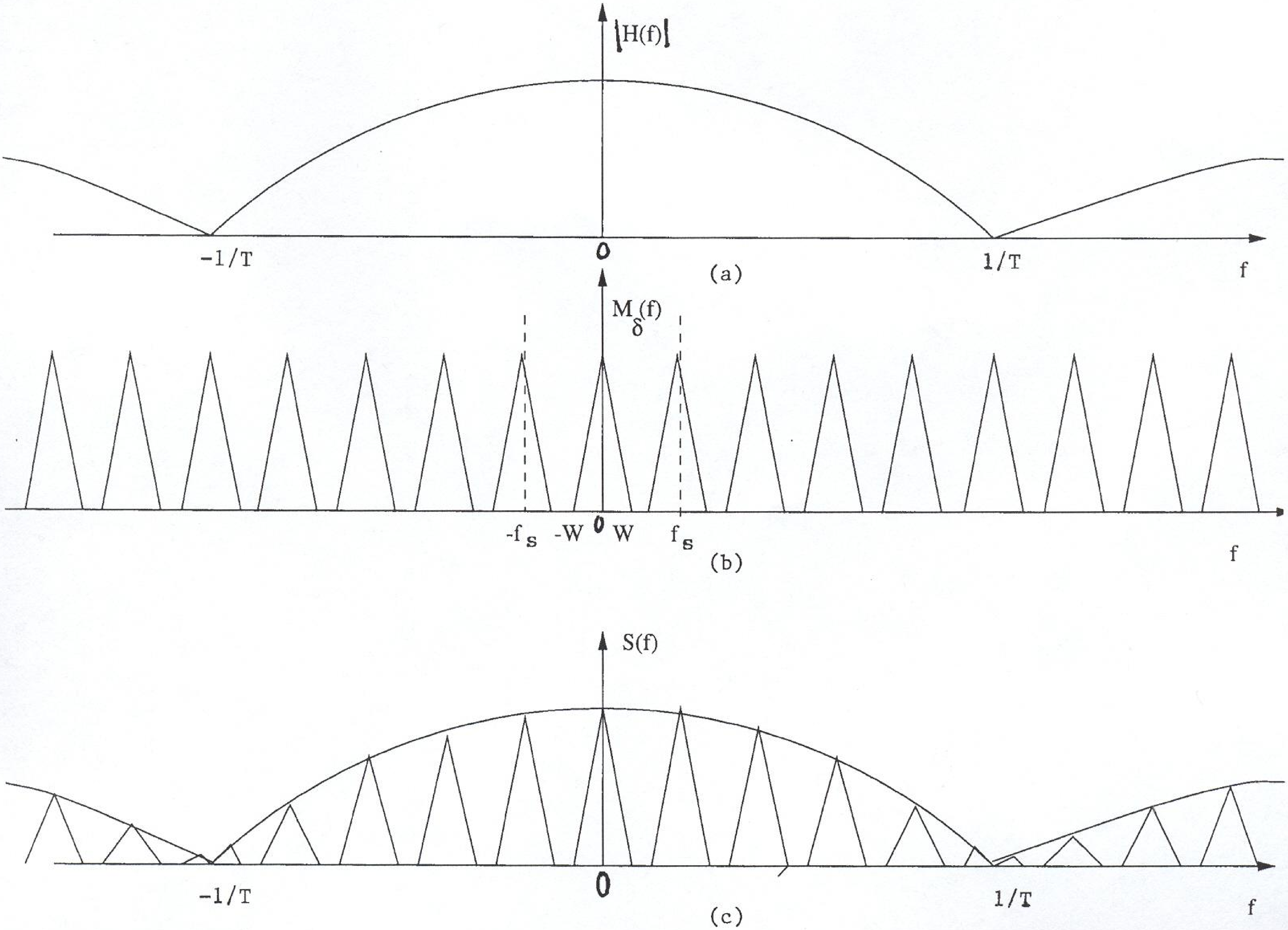
$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t-nT_s)$$

The spectrum $S(f) = F[s(t)]$ is as follows:

$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f-kf_s)H(f)$$

$$= f_s H(f) \sum_{k=-\infty}^{\infty} M(f-kf_s)$$

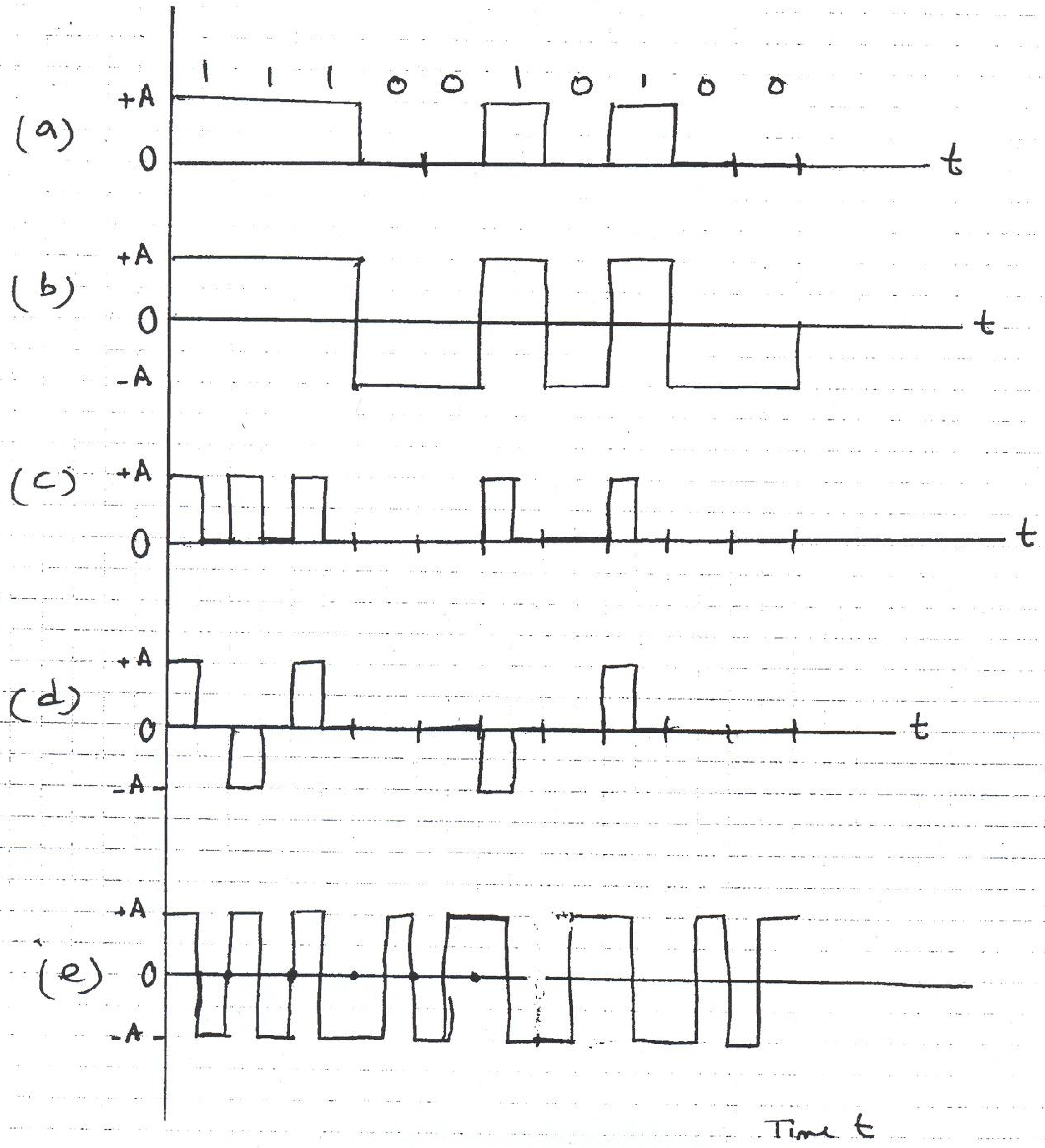
The magnitude spectrum $|S(f)|$ is thus as shown in Fig. 1c.



1/T = 10,000Hz
f_s = 1,000Hz
W = 400Hz

Figure 1

Problem 3.14



Problem 3.18

(a) Let the message bandwidth be W. Then, sampling the message signal at its Nyquist rate, and using an R-bit code to represent each sample of the message signal, we find that the bit duration is

$$T_b = \frac{T_s}{R} = \frac{1}{2WR}$$

The bit rate is

$$\frac{1}{T_b} = 2WR$$

The maximum value of message bandwidth is therefore

$$W_{max} = \frac{50 \times 10^6}{2 \times 7}$$

$$= 3.57 \times 10^6 \text{ Hz}$$

(b) The output signal-to-quantizing noise ratio is given by (see Example 2):

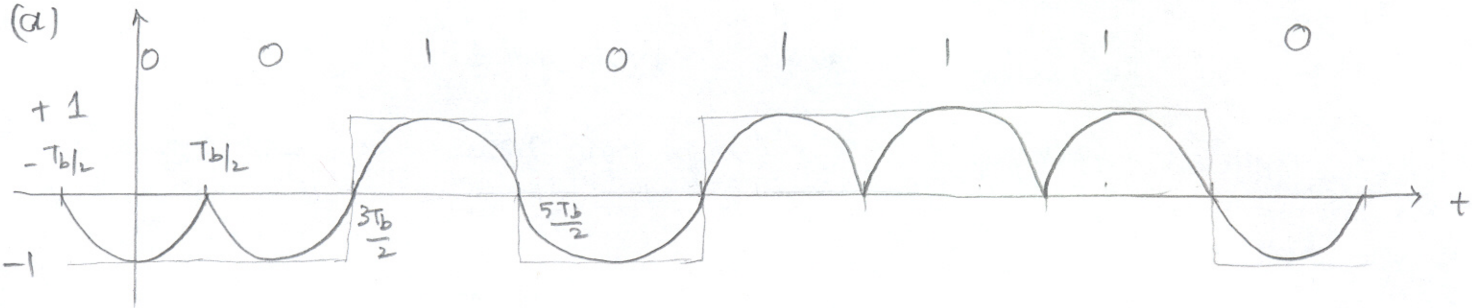
$$10 \log_{10}(SNR)_0 = 1.8 + 6R$$

$$= 1.8 + 6 \times 7$$

$$= 43.8 \text{ dB}$$

4. Problem 3.13

(a)



(b)

$$s(t) = \cos\left(\frac{\pi t}{T_b}\right) I_{[-\frac{T_b}{2}, \frac{T_b}{2}]} \quad \text{where } I_{[-\frac{T_b}{2}, \frac{T_b}{2}]} = \begin{cases} 1 & -\frac{T_b}{2} \le t \le \frac{T_b}{2} \\ 0 & \text{elsewhere} \end{cases}$$

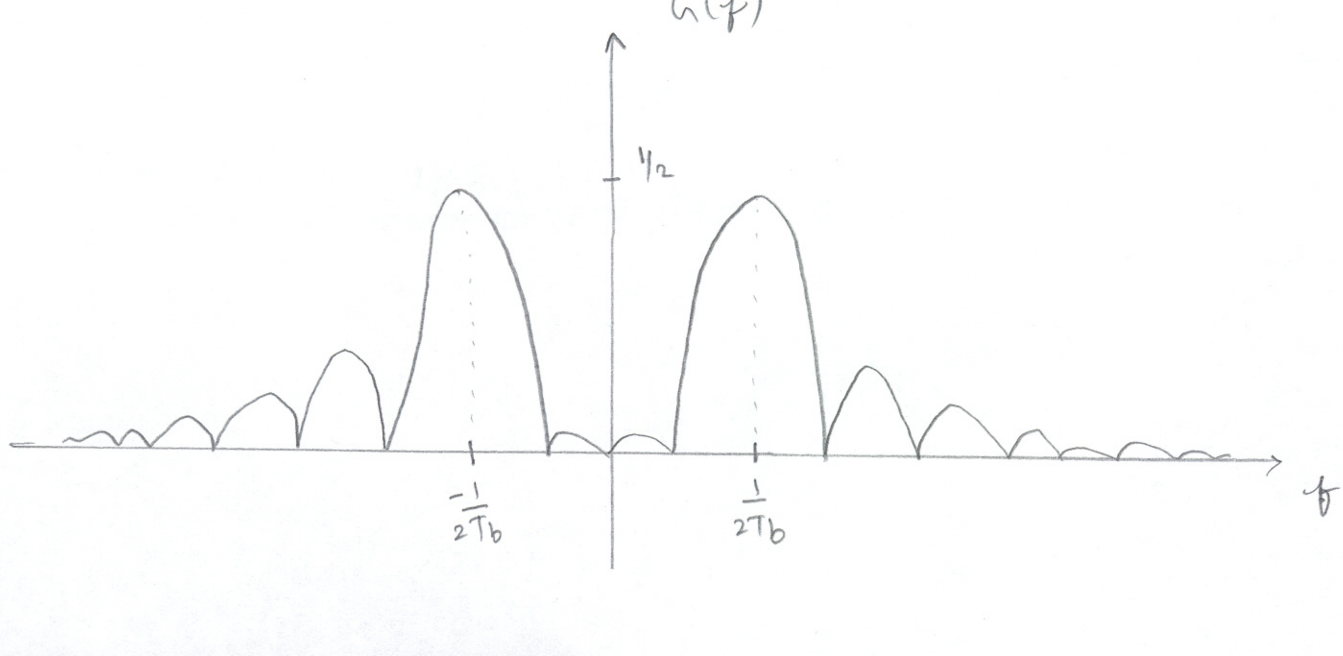
$$I_{[-\frac{T_b}{2}, \frac{T_b}{2}]} \overset{\mathcal{F}}{\longleftrightarrow} T_b \operatorname{sinc}(f T_b)$$

$$S(f) = \frac{1}{2}\left(\delta\left(f - \frac{1}{2T_b}\right) + \delta\left(f + \frac{1}{2T_b}\right)\right) * T_b \operatorname{sinc}(f T_b)$$

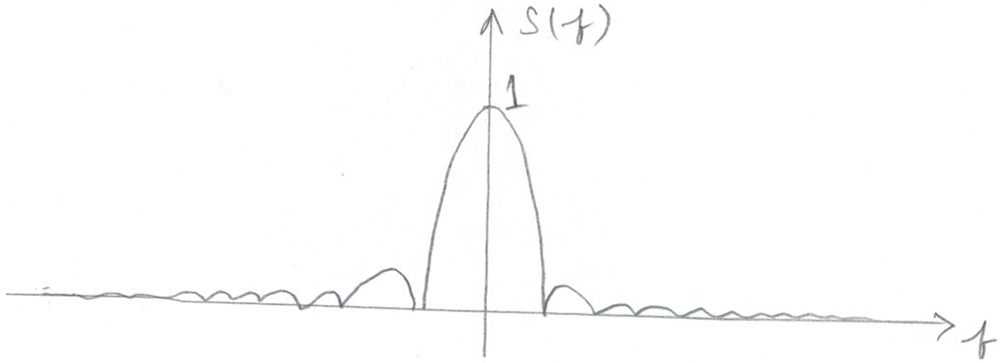
$$= \frac{T_b}{2}\left(\operatorname{sinc}\left(T_b\left(f - \frac{1}{2T_b}\right)\right) + \operatorname{sinc}\left(T_b\left(f + \frac{1}{2T_b}\right)\right)\right)$$

Power spectral density: $G(f) = \dfrac{|S(f)|^2}{T_b}$

$$= \frac{1}{T_b}\frac{T_b^2}{4}\left[\operatorname{sinc}^2\left(T_b\left(f - \frac{1}{2T_b}\right)\right) + \operatorname{sinc}^2\left(T_b\left(f + \frac{1}{2T_b}\right)\right) + 2\operatorname{sinc}\left(T_b\left(f - \frac{1}{2T_b}\right)\right)\operatorname{sinc}\left(T_b\left(f + \frac{1}{2T_b}\right)\right)\right]$$



(c) $S(f) = A^2 T_b \operatorname{sinc}^2(f T_b)$



When using rectangular pulse, there is a significant DC component. When using cosine pulse, the energy at DC has been shifted to the frequencies $\pm \frac{1}{2T_b}$.