ECE 146B
Gibson

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Solutions to Homework No. 2

Problem 4.12

The rectangular pulse given in Fig. P4.12 is defined by

$g(t) = \mathrm{rec}(t/T)$

The Fourier transform of $g(t)$ is given by

$$G(f) = \int_{-T/2}^{T/2} \exp(-j2\pi ft)dt$$

$$= T \,\mathrm{sinc}\,(fT)$$

We thus have the Fourier-transform pair

$$\mathrm{rec}(t/T) \rightleftharpoons T \,\mathrm{sinc}\,(fT)$$

The magnitude spectrum $|G(f)|/T$ is plotted as the solid line in Fig. 1, shown on the next page.

Consider next a Nyquist pulse (raised cosine pulse with a rolloff factor of zero). The magnitude spectrum of this second pulse is a rectangular function of frequency, as shown by the dashed curve in Fig. 1.

Comparing the two spectral characteristics of Fig. 1, we may say that the rectangular pulse of Fig. P4.12 provides a crude approximation to the Nyquist pulse.

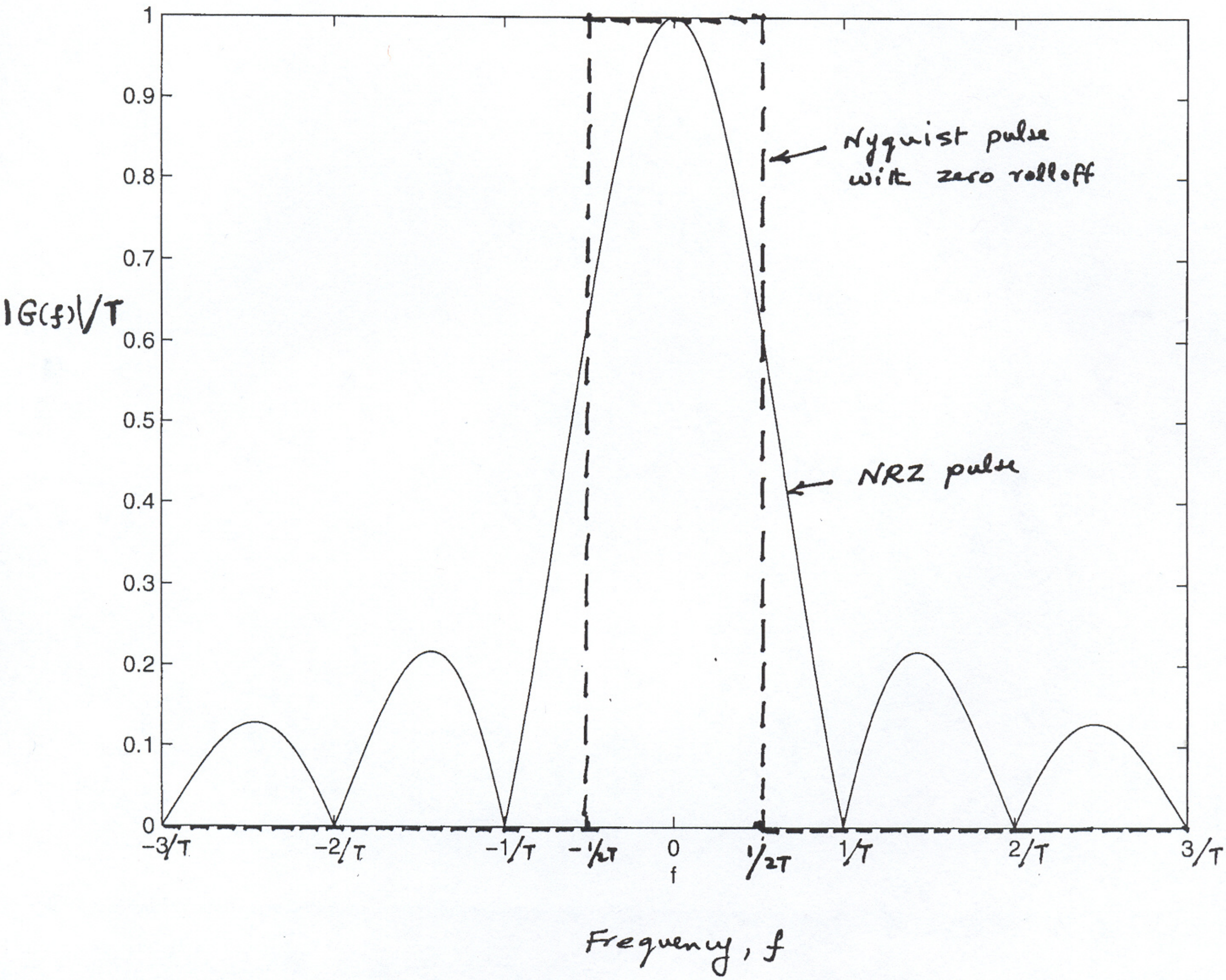


Figure 1 Spectral characteristics

Problem 4.13

Since P(f) is an even real function, its inverse Fourier transform equals

$$p(t) = 2 \int_0^{\infty} P(f) \cos(2\pi f t)\, df \tag{1}$$

The P(f) is itself defined by Eq. (7.60) which is reproduced here in the form

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 < |f| < f_1 \\ \dfrac{1}{4W}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2W - 2f_1}\right]\right\}, & f_1 < f < 2W - f_1 \\ 0, & |f| > 2W - f_1 \end{cases} \tag{2}$$

Hence, using Eq. (2) in (1):

$$p(t) = \frac{1}{W}\int_0^{f_1} \cos(2\pi f t)\, df + \frac{1}{2B}\int_{f_1}^{2W-f_1}\left[1 + \cos\left(\frac{\pi(f-f_1)}{2W\alpha}\right)\right]\cos(2\pi f t)\, df$$

$$= \left[\frac{(\sin(2\pi f t)}{2\pi W t}\right] + \left[\frac{\sin(2\pi f t)}{4\pi W t}\right]_{f_1}^{2W-f_1}$$

$$+ \frac{1}{4}W\left[\frac{\sin\left(2\pi f t + \dfrac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t + \pi/2W\alpha}\right]_{f_1}^{2W-f_1} + \frac{1}{4W}\left[\frac{\sin\left(2\pi f t - \dfrac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t - \pi/2W\alpha}\right]_{f_1}^{2W-f_1}$$

$$= \frac{\sin(2\pi f_1 t)}{4\pi W t} + \frac{\sin[2\pi t(2W-f_1)]}{4\pi W t}$$

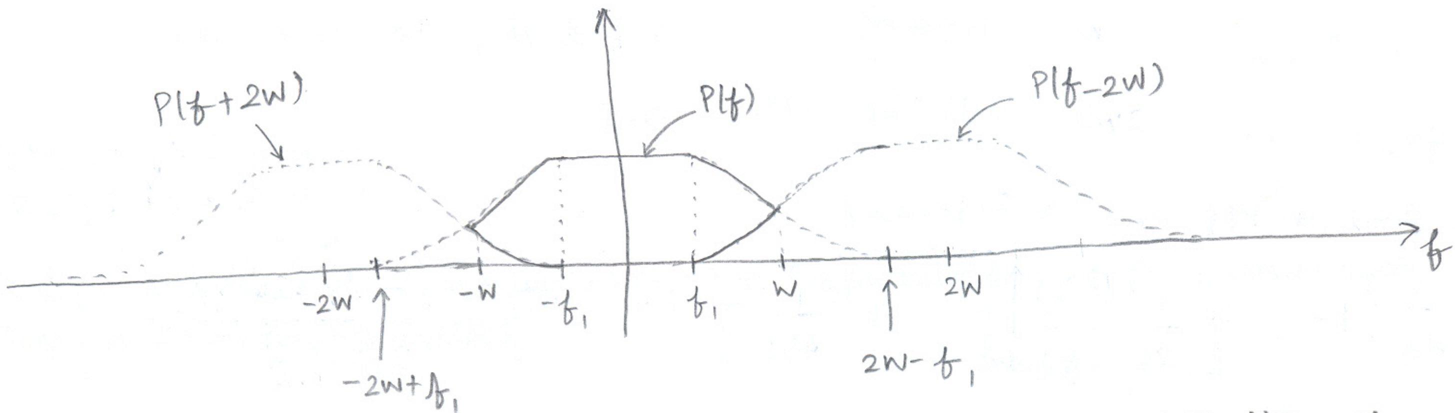
$$- \frac{1}{4W}\,\frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]}{2\pi t - \pi/2W\alpha} + \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1 t)]}{2\pi t - \pi/2W\alpha}$$

$$= \frac{1}{W}\left[\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]\right]\left[\frac{1}{4\pi t} - \frac{\pi t}{(2\pi t)^2 - (\pi/2W\alpha)^2}\right]$$

$$= \frac{1}{W}[\sin(2\pi Wt)\cos(2\pi\alpha W)] \left[\frac{-(\pi/2W\alpha)^2}{4\pi t\,[(2\pi t)^2 - (\pi/2W\alpha)^2]}\right]$$

$$= \mathrm{sinc}(2Wt)\cos(2\pi\alpha Wt)\left[\frac{1}{1-16\,\alpha^2W^2\,t^2}\right]$$

3.



Note that in the range $-f_1 < f < f_1$, the contribution of $P(f+2W)$ and $P(f-2W)$ is zero.

$\therefore$ $P(f) + P(f+2W) + P(f-2W) = \frac{1}{2W}$ for $-f_1 < f < f_1$.

Consider the range $-W \le f \le -f_1$. The contribution from $P(f-2W)$ will be zero, and

positive half of $P(f)$ left-shifted by $2W$

$P(f) + P(f+2W) + P(f-2W)$

$$= \frac{1}{4W}\left\{1 - \sin\left[\frac{\pi(-f-W)}{2W-2f_1}\right]\right\} + \frac{1}{4W}\left\{1 - \sin\left[\frac{\pi(f+2W-W)}{2W-2f_1}\right]\right\}$$

$$= \frac{1}{2W} + \frac{1}{4W}\left\{\sin\left[\frac{\pi(f+W)}{2W-2f_1}\right]\right\} - \frac{1}{4W}\left\{\sin\left[\frac{\pi(f+W)}{2W-2f_1}\right]\right\}$$

$$= \frac{1}{2W}.$$

Similarly, in the interval $f_1 \le f \le W$, the contribution from $P(f+2W)$ will be zero, and

$$P(f) + P(f-2W) + P(f+2W)$$

$$= \frac{1}{4W}\left\{1 - \sin\left[\frac{\pi(f-W)}{2W-2f_1}\right]\right\} + \frac{1}{4W}\left\{1 - \sin\left[\frac{\pi\,(\overbrace{-(f-2W)-W}^{\text{negative half of } P(f) \text{ right shifted by } 2W})}{2W-2f_1}\right]\right\}$$

$$= \frac{1}{2W} - \frac{1}{4W}\sin\left[\frac{\pi(f-W)}{2W-2f_1}\right] - \frac{1}{4W}\sin\left[\frac{\pi(-f+W)}{2W-2f_1}\right]$$

$$= \frac{1}{2W}.$$

Hence, we have shown that $P(f) + P(f-2W) + P(f+2W) = \frac{1}{2W}$ throughout the interval $-W \le f \le W$.

4. (a) 100% roll off $\Rightarrow \alpha = 1$

Transmission bandwidth, $B_T = W(1+\alpha) = 4000$

$\Rightarrow 2W = 4000$

Symbol rate $= 2W = 4000$ bits/s $= \underline{4 \text{ kbps}}$.

(b) 50% roll off $\Rightarrow \alpha = 0.5$

$B_T = W(1+\alpha) = 4000$

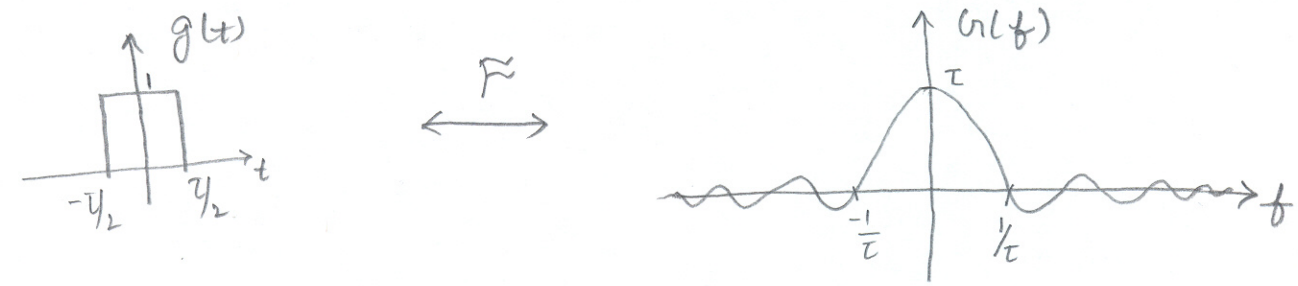
$\Rightarrow 1.5W = 4000$

Symbol rate $= 2W = 5333.33$ bits/s $= \underline{5.33 \text{ kbps}}$.

5. 100% roll off $\Rightarrow \alpha = 1 \Rightarrow f_1 = 0$

$$P(f) = \frac{1}{4W}\left\{1 - \sin\left[\frac{\pi(|f|-W)}{2W}\right]\right\} \qquad 0 \le |f| < 2W$$

Let $g(t)$ be the rectangular pulse of width τ.



It has the Fourier transform $G(f) = \tau \operatorname{Sinc}(f\tau)$

$$g(t) \longrightarrow \underset{\text{Shaping filter}}{\boxed{h(t)}} \longrightarrow \underset{\text{Raised cosine pulse}}{p(t)}$$

Now, we need to find the transfer function $H(f)$.

$$G(f)\, H(f) = P(f)$$

$$\therefore \quad H(f) = \frac{P(f)}{G(f)} = \frac{1}{4W\tau} \; \frac{1 - \sin\left[\dfrac{\pi(|f| - W)}{2W}\right]}{\operatorname{Sinc}(f\tau)} \qquad 0 \le |f| < 2W$$

a raised cosine divided by a sinc in the range $-2W < f < 2W$.