ECE 146B
Gibson

Spring 2008
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Solutions to Homework No. 3

Problem 4.21

(a)	binary sequence b_k	0	0	1	1	0	1	0	0	1
	polar representation	−1	−1	1	1	−1	1	−1	−1	1
	duobinary coder output c_k		−2	0	2	0	0	0	−2	0
	receiver output $\hat{b}_k$	−1	−1	1	1	−1	1	−1	−1	1
	output binary sequence	0	0	1	1	0	1	0	0	1
(b)	receiver input		0	0	2	0	0	0	−2	0
	receiver output $\hat{b}_k$	−1	1	−1	1	−1	1	−1	−1	1
	output binary sequence	0	1	0	1	0	1	0	0	1

We see that not only is the second digit in error, but also the error propagates.

Problem 4.22

(a)	binary sequence b_k		0	0	1	1	0	1	0	0	1
	coded sequence d_k	1	1	1	0	1	1	0	0	0	1
	polar representation	1	1	1	−1	1	1	−1	−1	−1	1
	duobinary coder output c_k		2	2	0	0	2	0	−2	−2	0
	receiver output		0	0	1	1	0	1	0	0	1
(b)	receiver input		2	0	0	0	2	0	−2	−2	0
	receiver output		0	1	1	1	0	1	0	0	1

In this case we see that only the second digit is in error, and there is no error propagation.

Problem 4.30

(a) The power spectral density of the signal generated by the NRZ transmitter is given by

$$S(f) = \frac{\sigma^2}{T}|G(f)|^2 \tag{1}$$

where σ^2 is the symbol variance, T is the symbol duration, and

$$G(f) = \int_{-T/2}^{T/2} 1 \cdot e^{-j2\pi ft}\,dt = T\,\mathrm{sinc}\,(fT) = \frac{1}{R}\mathrm{sinc}\left(\frac{f}{R}\right) \tag{2}$$

is the Fourier transform of the generating function for NRZ symbols. Here, we have used the fact that the symbol rate $R = 1/T$. A 2B1Q code is a multi-level block code where each block has 2 bits and the bit rate $R = 2/T$ (i.e., m/T, where m is the number of bits in a block). Since the 2B1Q pulse has the shape of an NRZ pulse, the power spectral density of 2B1Q signals is given by

$$S_{2B1Q} = \frac{\sigma^2}{T}|G_{2B1Q}(f)|^2$$

where

$$G_{2B1Q}(f) = \frac{\sin(2\pi(f/R))}{\sqrt{2}\pi f}$$

The factor $\sqrt{2}$ in the denominator is introduced to make the average power of the 2B1Q signal equal to the average power of the corresponding NRZ signal. Hence,

$$S_{2B1Q}(f) = \frac{\sigma^2}{T}\left(\frac{\sin(2\pi(f/R))}{\sqrt{2}\pi f}\right)^2$$

$$= \frac{2\sigma^2}{R}\mathrm{sinc}^2(2(f/R)) \tag{3}$$

(b) The transfer functions of pulse-shaping filters for the Manchester code, modified duobinary code, and bipolar return-to-zero code are as follows:

(i) Manchester code:

$$G(f) = \frac{j}{\pi f}\left[1 - \cos\left(\pi\frac{f}{R}\right)\right] \tag{4}$$

(ii)Modified duobinary code:

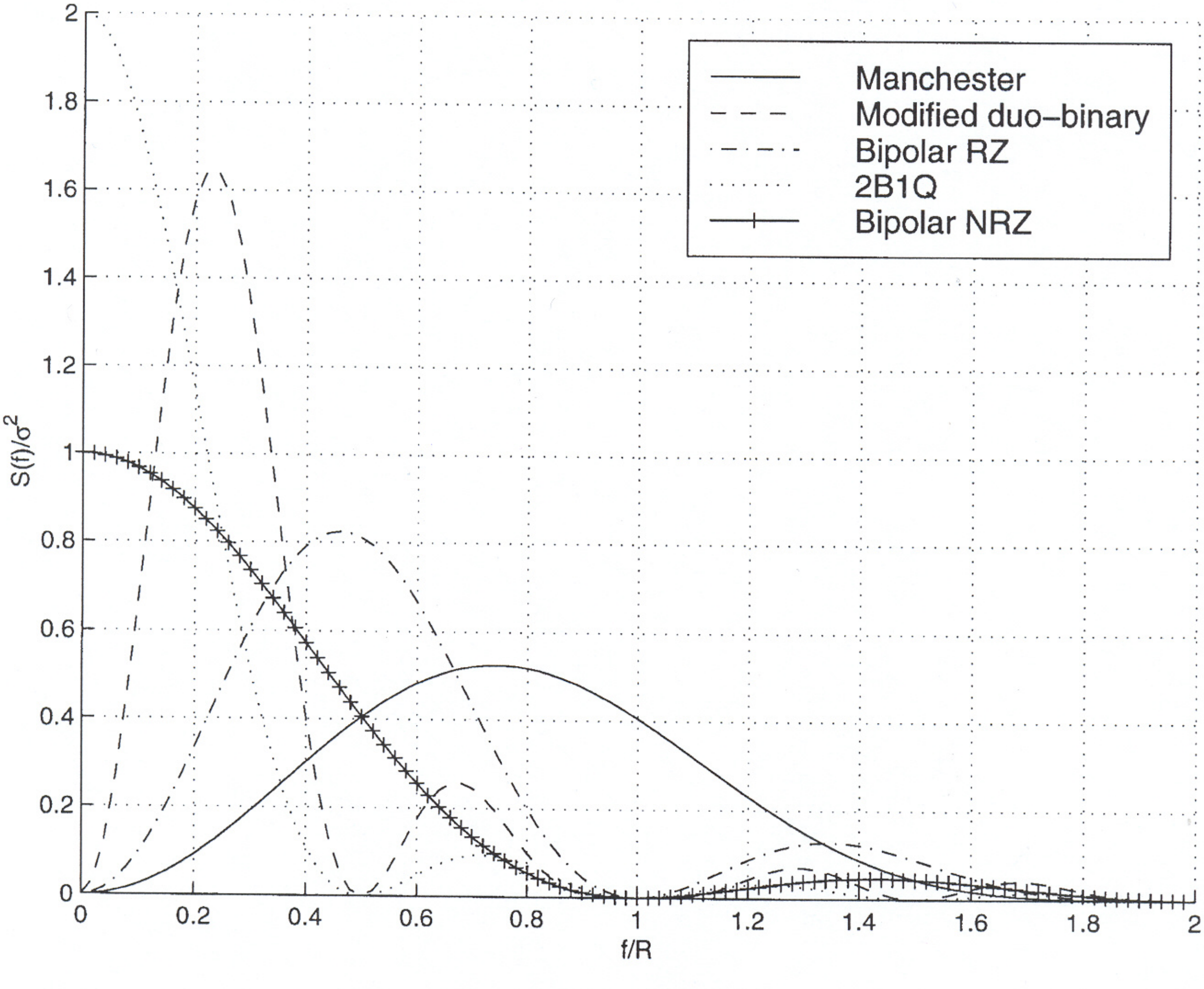
$$G(f) = \frac{1}{j\sqrt{2}\pi f}\left[\cos\left(3\pi\frac{f}{R}\right) - \cos\left(\pi\frac{f}{R}\right)\right] \tag{5}$$

(iii)Bipolar return-to-zero code:

$$G(f) = \frac{2}{\pi f}\left[\sin\left(\pi\frac{f}{2R}\right) \times \sin\left(\pi\frac{f}{R}\right)\right] \tag{6}$$

Hence, using Eqs. (4), (5), and (6) in the formula of Eq. (1) for the power spectral density of PAM line codes, we get the normalized spectral plots shown in Fig. 1. In this figure, the spectral density is normalized with respect to the symbol variance σ^2 and the frequency is normalized with respect to the data rate R.

From Fig. 1, we may make the following observations: Among the four line codes displayed here, the 2B1Q code has much of its power concentrated inside the frequency band $-R/2 \le f \le R/2$, which is much more compact than all the other three codes: Manchester code, modified duobinary code, and bipolar return-to-zero code.



Manchester
Modified duo-binary
Bipolar RZ
2B1Q
Bipolar NRZ
S(f)/σ2
f/R
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Figure 1

Problem 4.34

(a) The channel output is

$$x(t) = a_1\, s(t-t_{01}) + a_2\, s(t-t_{02})$$

Taking the Fourier transform of both sides:

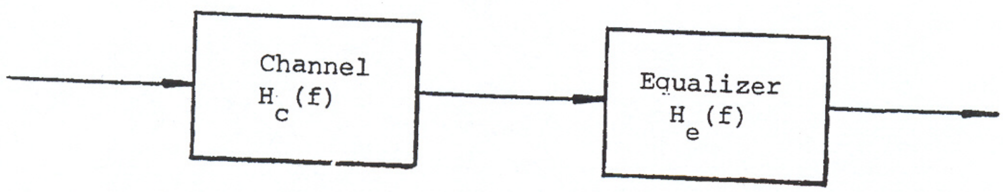
$$X(f) = [a_1 \exp(-j2\pi f t_{01}) + a_2 \exp(-j2\pi f t_{02})]\, S(f)$$

The transfer function of the channel is

$$H_c(f) = \frac{X(f)}{S(f)}$$

$$= a_1 \exp(-j2\pi f t_{01}) + a_2 \exp(-j2\pi f t_{02})$$

(b)



Ideally, the equalizer should be designed so that

$$H_c(f)\, H_e(f) = K_0 \exp(-j2\pi f t_0)$$

where K_0 is a constant gain and t_0 is the transmission delay. The transfer function of the equalizer is

$$H_e(f) = w_0 + w_1 \exp(-j2\pi fT) + w_2 \exp(-j4\pi fT)$$

$$= w_0 \left[1 + \frac{w_1}{w_0} \exp(-j2\pi fT) + \frac{w_2}{w_0} \exp(-j4\pi fT)\right] \qquad (1)$$

Therefore

$$H_e(f) = \frac{K_0 \exp(-j2\pi f t_0)}{H_c(f)}$$

$$= \frac{K_0 \exp(-j2\pi f t_0)}{a_1 \exp(-j2\pi f t_{01}) + a_2 \exp(-j2\pi f t_{02})}$$

$$= \frac{(K_0/a_1)\exp[-j2\pi f(t_0 - t_{01})]}{1 + \frac{a_2}{a_1}\exp[-j2\pi f(t_{02} - t_{01})]}$$

Since $a_2 \ll a_1$, we may approximate $H_e(f)$ as follows

$$H_e(f) = \frac{K_0}{a_1}\exp[-j2\pi f(t_0 - t_{01})]\,\{1 - \frac{a_2}{a_1}\exp[-j2\pi f(t_{02} - t_{01})]$$

$$+ (\frac{a_2}{a_1})^2 \exp[-j4\pi f(t_{02} - t_{01})]\} \qquad (2)$$

Comparing Eqs. (1) and (2), we deduce that

$$\frac{K_0}{a_1} \simeq w_0$$

$$t_0 - t_{01} \simeq 0$$

$$-\frac{a_2}{a_1} \simeq \frac{w_1}{w_0}$$

$$(\frac{a_2}{a_1})^2 \simeq \frac{w_2}{w_0}$$

$$T \simeq t_{02} - t_{01}$$

Choosing $K_0 = a_1$, we find that the tap weights of the equalizer are as follows

$$w_0 = 1$$

$$w_1 = -\frac{a_2}{a_1}$$

$$w_2 = (\frac{a_2}{a_1})^2$$

5. $H(\omega) = e^{-j(\omega t_d - \varepsilon \sin \omega t_o)} = e^{-j\omega t_d} e^{j\varepsilon \sin \omega t_o}$

$$\simeq e^{-j\omega t_d} (1 + j\varepsilon \sin \omega t_o)$$

$$= e^{-j\omega t_d} + j\frac{\varepsilon}{2j} e^{-j\omega(t_d - t_o)} - j\frac{\varepsilon}{2j} e^{-j\omega(t_d + t_o)}$$

$$= e^{-j\omega t_d} + \frac{\varepsilon}{2} e^{-j\omega(t_d - t_o)} - \frac{\varepsilon}{2} e^{-j\omega(t_d + t_o)}$$

$$Y(\omega) = X(\omega) H(\omega)$$

$$Y(\omega) \simeq X(\omega) e^{-j\omega t_d} + \frac{\varepsilon}{2} X(\omega) e^{-j\omega(t_d - t_o)} - \frac{\varepsilon}{2} X(\omega) e^{-j\omega(t_d + t_o)}$$

$$x(t - \tau) \overset{F}{\longleftrightarrow} X(\omega) e^{-j\omega \tau}$$

$$y(t) \simeq x(t - t_d) + \frac{\varepsilon}{2} x(t - (t_d - t_o)) - \frac{\varepsilon}{2} x(t - (t_d + t_o))$$

$$= x(t - t_d) + \frac{\varepsilon}{2} x(t - t_d + t_o) - \frac{\varepsilon}{2} x(t - t_d - t_o)$$

6. Figures 1 and 2 shows the plots for equations 8.4.3 and 8.4.8 respectively. $x(t)$ is a raised cosine pulse with 100% roll-off. $y(t)$ is sketched in the range $\frac{-1}{2W} \le t \le \frac{1}{2W}$, with $W = 1001$ Hz, and $\varepsilon = 0.1$.

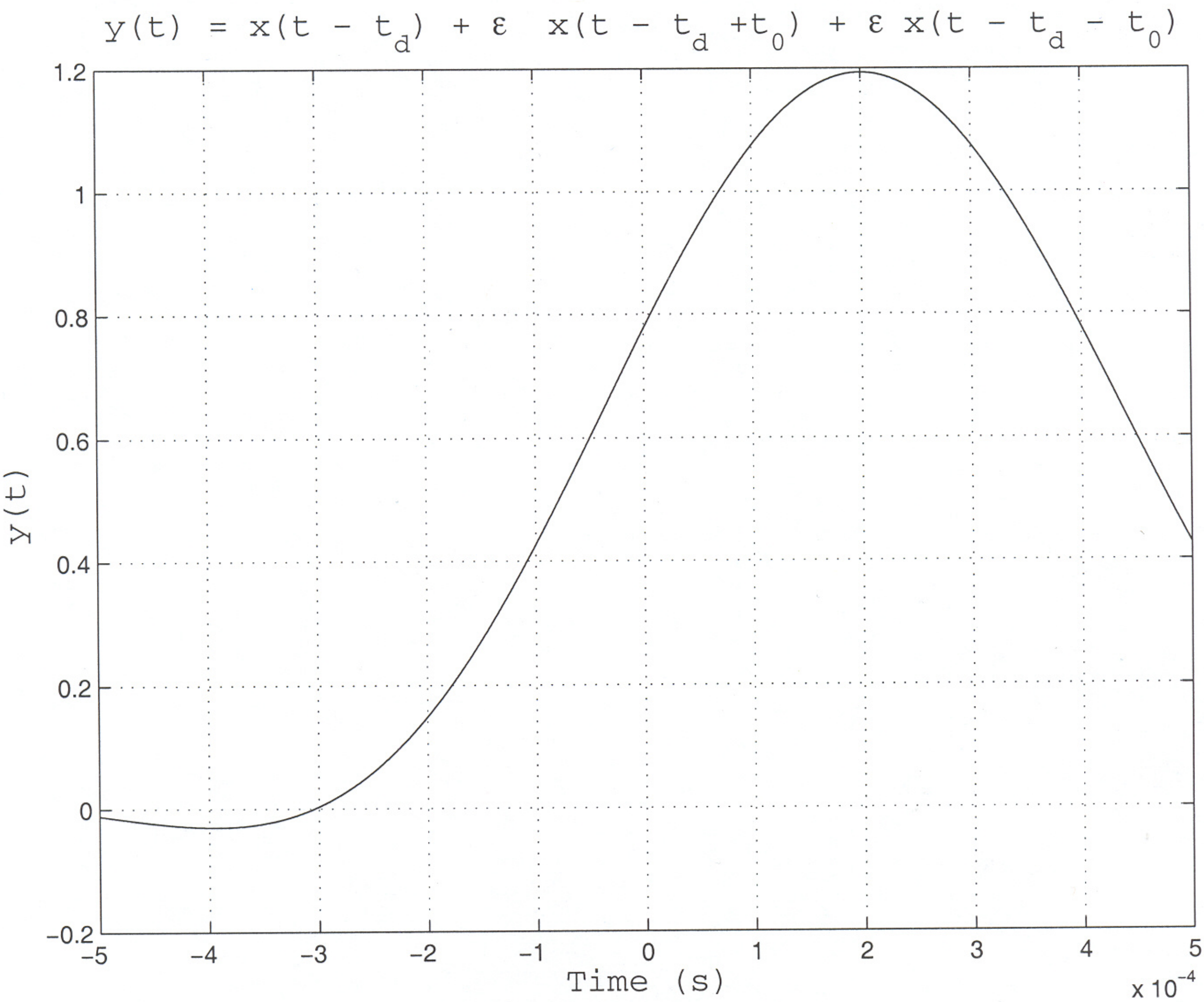

y(t) = x(t − t_d) + ε x(t − t_d + t_0) + ε x(t − t_d − t_0)
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
y(t)
−5
−4
−3
−2
−1
0
1
2
3
4
5
Time (s)
x 10^-4

Figure 1

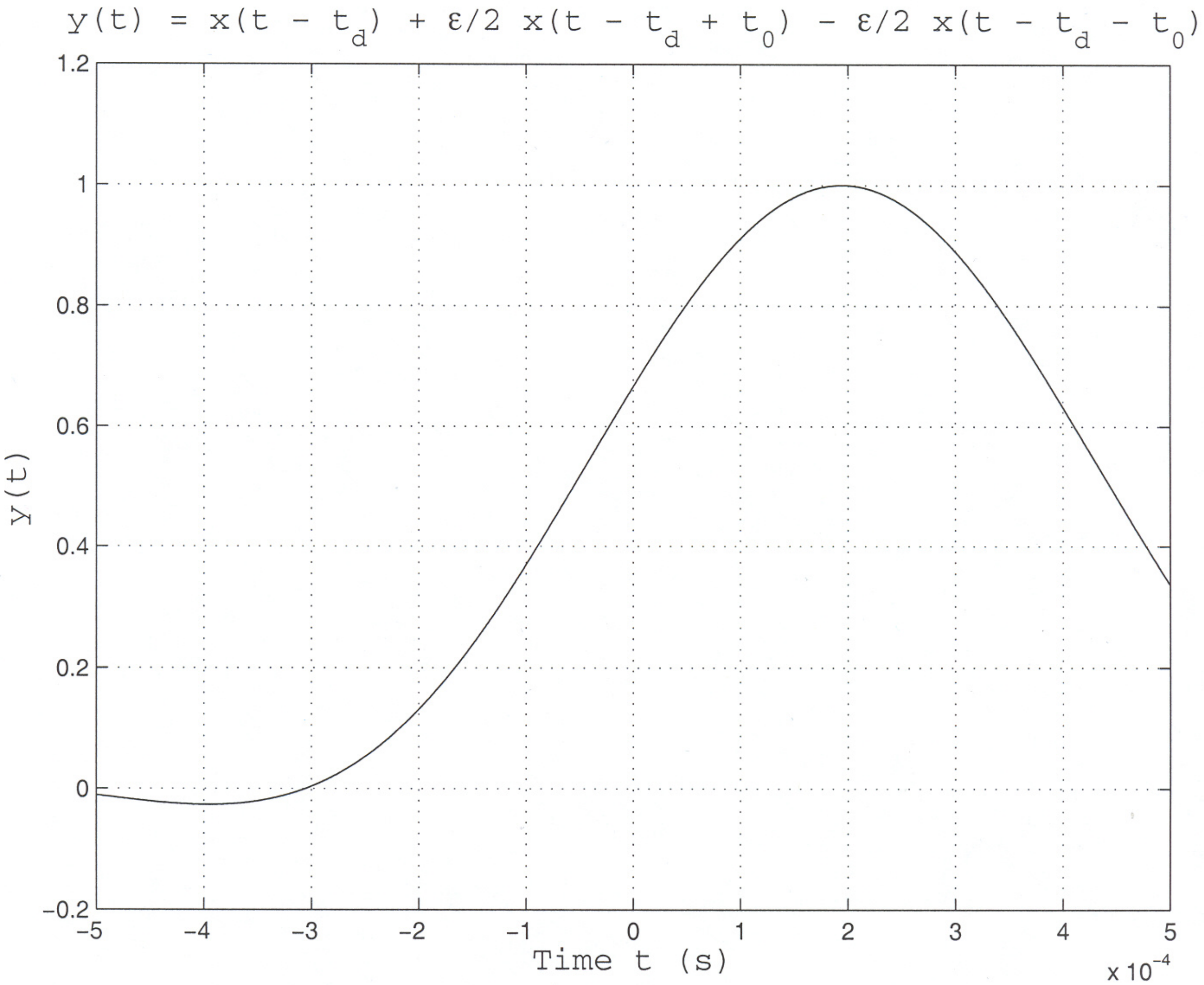

y(t) = x(t - t_d) + ε/2 x(t - t_d + t_0) - ε/2 x(t - t_d - t_0)
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
y(t)
−5
−4
−3
−2
−1
0
1
2
3
4
5
Time t (s)
x 10^−4

Figure 2