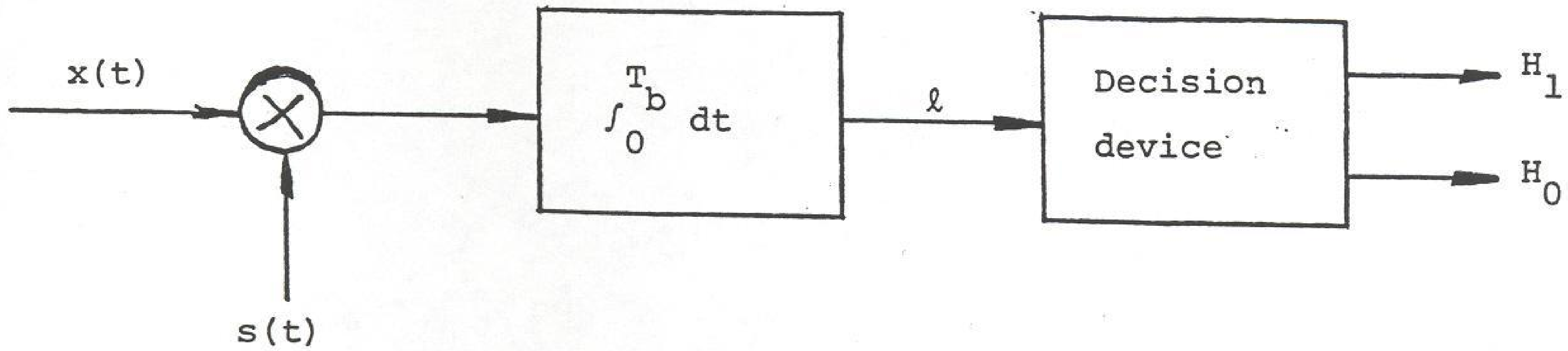
Solutions to Homework No. 5

Problem 6.1

(a) ASK with coherent reception



Denoting the presence of symbol 1 or symbol 0 by hypothesis H_1 or H_0, respectively, we may write

$$H_1: \quad x(t) = s(t) + w(t)$$

$$H_0: \quad x(t) = w(t)$$

where $s(t) = A_c \cos(2\pi f_c t)$, with $A_c = \sqrt{2E_b/T_b}$. Therefore,

$$\ell = \int_0^{T_b} x(t)\, s(t)\, dt$$

If $\ell > E_b/2$, the receiver decides in favor of symbol 1. If $\ell < E_b/2$, it decides in favor of symbol 0.

The conditional probability density functions of the random variable L, whose value is denoted by ℓ, are defined by

$$f_{L|0}(\ell|0) = \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left(-\frac{\ell^2}{N_0 E_b}\right)$$

$$f_{L|1}(\ell|1) = \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left[-\frac{(\ell - E_b)^2}{N_0 E_b}\right]$$

The average probability of error is therefore,

$$P_e = p_o \int_{E_b/2}^{\infty} f_{L|0}(\ell|0)d\ell + p_1 \int_{-\infty}^{E_b/2} f_{L|1}(\ell|1)d\ell$$

$$= \frac{1}{2} \int_{E_b/2}^{\infty} \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left(-\frac{\ell^2}{N_0 E_b}\right)d\ell + \frac{1}{2} \int_{-\infty}^{E_b/2} \frac{1}{\sqrt{\pi N_0 E_b}} \exp\left[-\frac{(\ell - E_b)^2}{N_0 E_b}\right]d\ell$$

$$= \frac{1}{\sqrt{\pi N_0 E_b}} \int_{E_b/2}^{\infty} \exp\left(-\frac{\ell^2}{N_0 E_b}\right)d\ell$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2} \sqrt{E_b/N_0}\right)$$

Problem 6.2

The transmitted binary PSK signal is defined by

$$s(t) = \begin{cases} \sqrt{E_b}\phi(t), & 0 \le t \le T_b, \quad \text{symbol 1} \\ -\sqrt{E_b}\phi(t), & 0 \le t \le T_b, \quad \text{symbol 0} \end{cases}$$

where the basis function $\phi(t)$ is defined by

$$\phi(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$$

The locally generated basis function in the receiver is

$$\phi_{\text{rec}}(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t + \varphi)$$

$$= \sqrt{\frac{2}{T_b}}[\cos(2\pi f_c t)\cos\varphi - \sin(2\pi f_c t)\sin\varphi]$$

where φ is the phase error. The correlator output is given by

$$y = \int_0^{T_b} x(t)\varphi_{\text{rec}}(t)dt$$

where

$$x(t) = s_k(t) + w(t), \qquad k = 1, 2$$

Assuming that f_c is an integer multiple of $1/T_b$, and recognizing that $\sin(2\pi f_c t)$ is orthogonal to $\cos(2\pi f_c t)$ over the interval $0 \le t \le T_b$, we get

$$y = \pm\sqrt{E_b}\cos\varphi + W$$

when the plus sign corresponds to symbol 1 and the minus sign corresponds to symbol 0, and W is a zero-mean Gaussian variable of variance $N_0/2$. Accordingly, the average probability of error of the binary PSK system with phase error φ is given by

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b \cos^2\varphi}{N_0}}\right)$$

When $\varphi = 0$, this formula reduces to that for the standard PSK system equipped with perfect phase recovery. At the other extreme, when $\varphi = \pm 90°$, P_e attains its worst value of unity.

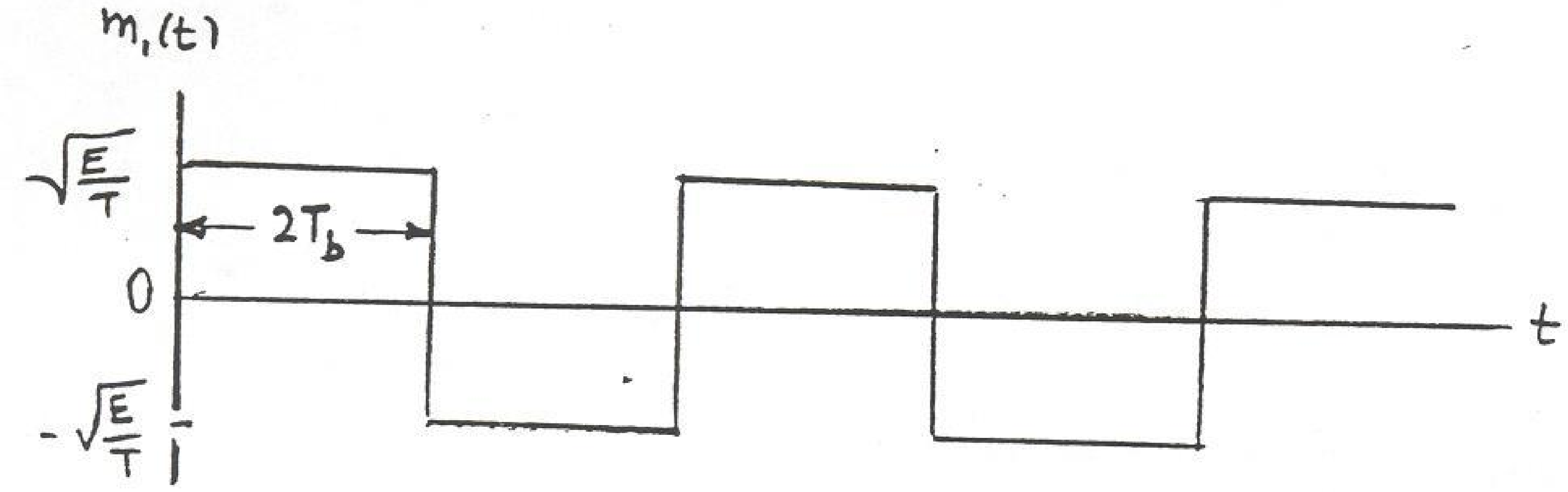
Problem 6.5

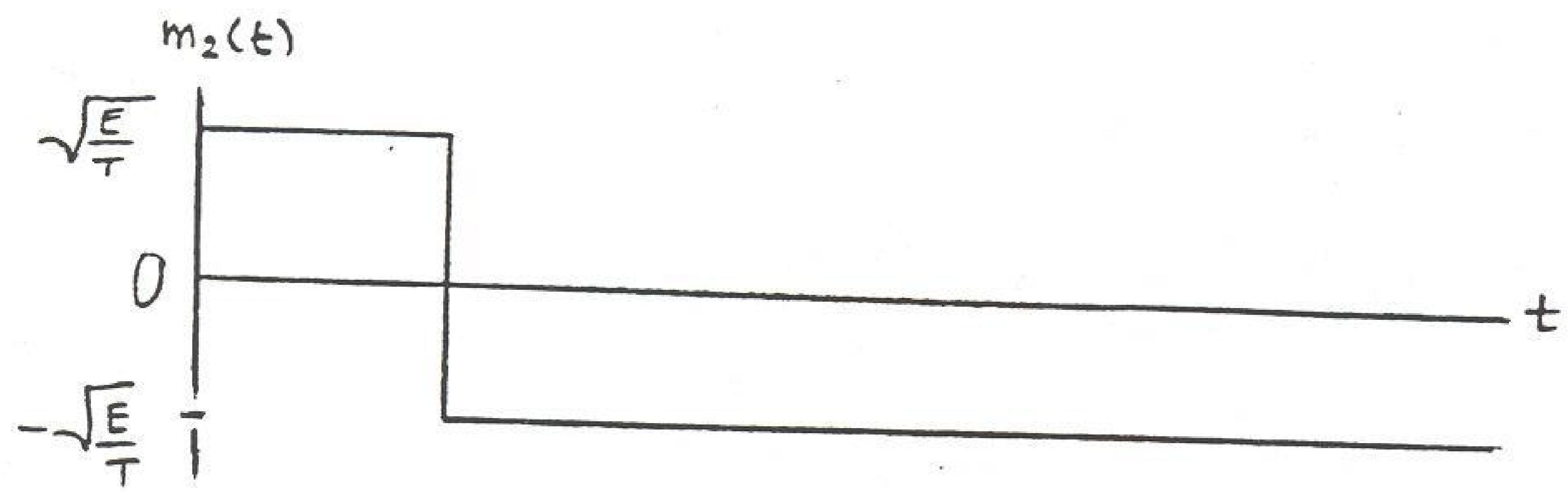
a) The QPSK wave can be expressed as

$$s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t).$$

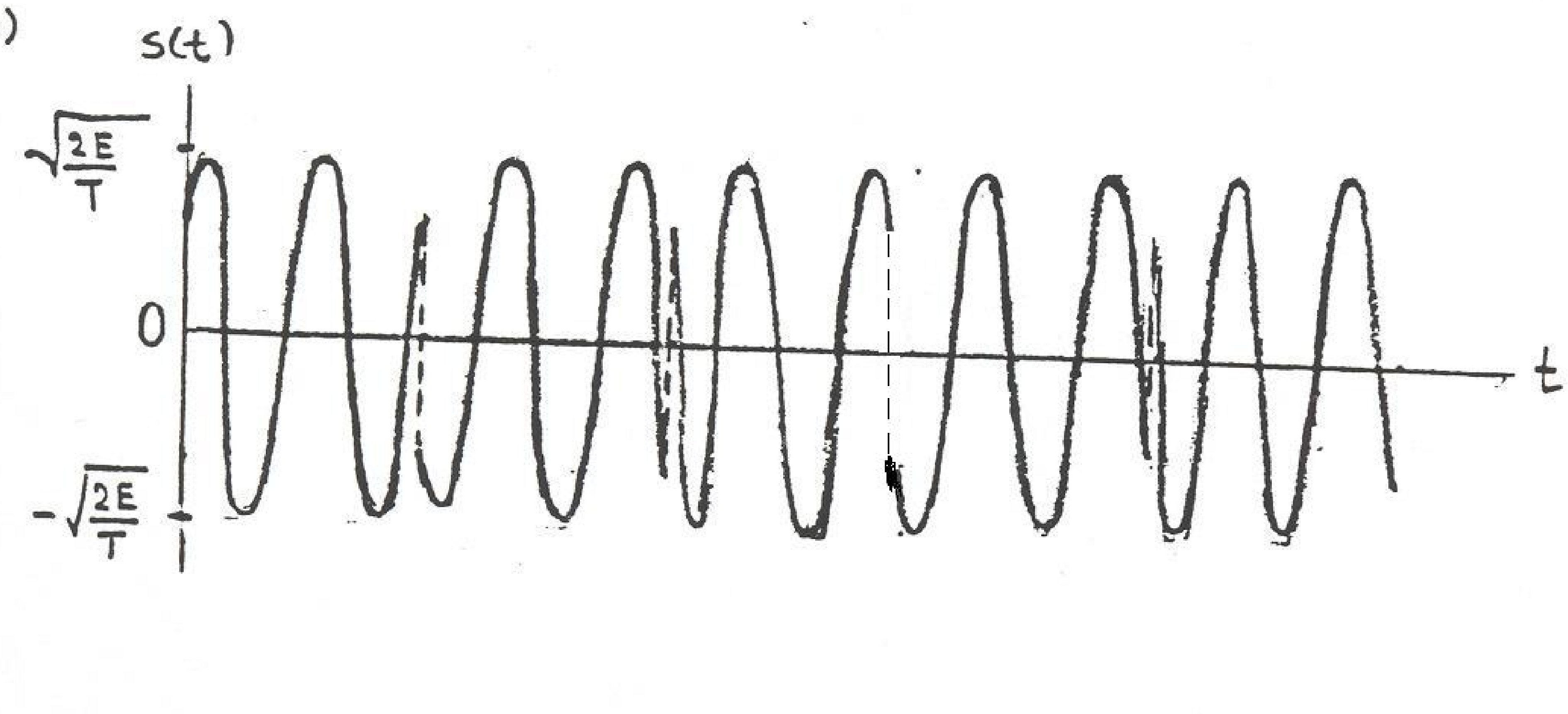
Dividing the binary wave into dibits and finding $m_1(t)$ and $m_2(t)$ for each dibit:

dibit	11	00	10	00	10
$m_1(t)$	$\sqrt{E/T}$	$-\sqrt{E/T}$	$\sqrt{E/T}$	$-\sqrt{E/T}$	$\sqrt{E/T}$
$m_2(t)$	$\sqrt{E/T}$	$-\sqrt{E/T}$	$-\sqrt{E/T}$	$-\sqrt{E/T}$	$-\sqrt{E/T}$





(b)



Problem 6.15

The transmission bandwidth of 256-QAM signal is

$$B = \frac{2R_b}{\log_2 M}$$

where R_b is the bit rate given by $1/T_b$ and $M = 256$. Thus

$$B_{256} = \frac{2(1/T_b)}{\log_2 256} = \frac{2}{8T_b} = \frac{1}{4T_b}$$

The transmission bandwidth of 64-QAM is

$$B_{64} = \frac{2(1/T_b)}{\log_2 64} = \frac{2}{6T_b} = \frac{1}{3T_b}$$

Hence, the bandwidth advantage of 256-QAM over 64-QAM is

$$-\frac{1}{4T_b} + \frac{1}{3T_b} = \frac{1}{12T_b}$$

The average energy of 256-QAM signal is

$$E_{256} = \frac{2(M-1)E_0}{3} = \frac{2(256-1)E_0}{3}$$

$$= 170E_0$$

where E_0 is the energy of the signal with the lowest amplitude. For the 64-QAM signal, we have

$$E_{64} = \frac{2(63)}{3}E_0 = 42E_0$$

Therefore, the increase in average signal energy resulting from the use of 256-QAM over 64-QAM, expressed in dBs, is

$$10\log_{10}\left(\frac{170E_0}{42E_0}\right) \approx 10\log_{10}(4)$$

$$= 6 \text{ dB}$$

Problem 6.16

The probability of symbol error for 16-QAM is given by

$$P_e = 2\left(1 - \frac{1}{\sqrt{M}}\right)\text{erfc}\left(\sqrt{\frac{3E_{av}}{2(M-1)N_0}}\right)$$

Setting $P_e = 10^{-3}$, we get

$$10^{-3} = 2\left(1 - \frac{1}{4}\right)\text{erfc}\left(\sqrt{\frac{3E_{av}}{30N_0}}\right)$$

Solving this equation for E_{av}/N_0,

$$\frac{E_{av}}{N_0} = 58 = 17.6\text{dB}$$

The probability of symbol error for 16-PSK is given by

$$P_e = \text{erfc}\left(\sqrt{\frac{E}{N_0}}\sin(\pi/M)\right)$$

Setting $P_e = 10^{-3}$, we get

$$10^{-3} = \text{erfc}\left(\sqrt{\frac{E}{N_0}}\sin(\pi/16)\right)$$

Solving this equation for E/N_0, we get

$$\frac{E}{N_0} = 142 = 21.5\text{dB}$$

Hence, on the average, the 16-PSK demands 21.5 - 17.6 = 3.9 dB more symbol energy than the 16-QAM for $P_e = 10^{-3}$.

Thus the 16-QAM requires about 4 dB less in signal energy than the 16-PSK for a fixed N_0 and $P_e = 10^{-3}$, However, for this advantage of the 16-QAM over the 16-PSK to be realized, the channel must be linear.