

①

Homework-1 solutions.

$$1. P(X > 5) = P\left(\frac{X-1}{2} > 2\right) = P(Y > 2)$$

$$Y = N(0, 1)$$

$$= Q(2)$$

2. (a) For no aliasing to occur we must sample at the Nyquist rate

$$f_s = 2 \times 6000 \text{ samples/sec} = 12000 \text{ samples/sec.}$$

(b) With guard band of 2000,

$$f_s - 2W = 2000 \Rightarrow f_s = \underline{14000}$$

The reconstruction filter should not pick-up frequencies of images of spectrum $X(f)$. The nearest image spectrum is centered at f_s and occupies the frequency band $[f_s - W, f_s + W]$. Thus the highest frequency of the reconstruction filter ($= 10000$) should satisfy,

$$10000 < f_s - W \Rightarrow f_s \geq 16000$$

For value $f_s = 16000$, K should be such that

$$K \cdot f_s = 1 \Rightarrow K = \frac{1}{16000}$$

$$\begin{aligned}
 3. \quad x_p(t) &= \sum_{n=-\infty}^{\infty} x(nT_s) p(t - nT_s) \\
 &= p(t) * \sum_{n=-\infty}^{\infty} x(nT_s) \delta(t - nT_s) \\
 &= p(t) * x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)
 \end{aligned}$$

$$\begin{aligned}
 X_p(f) &= P(f) X(f) * \mathcal{F} \left[\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \right] \\
 &= P(f) \cdot X(f) * \left(\frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_s}\right) \right) \\
 &= \frac{1}{T_s} P(f) \sum_{n=-\infty}^{\infty} X\left(f - \frac{n}{T_s}\right)
 \end{aligned}$$

$$\textcircled{2} \quad \frac{1}{T_s} > 2W$$

$\textcircled{3}$ $X(f)$ can be recovered using reconstruction filter $\Pi\left(\frac{f}{2W_r}\right)$ with $W < W_r < \frac{1}{T_s} - W$. In this case

$$X(f) = \frac{X_p(f) T_s \Pi\left(\frac{f}{2W_r}\right)}{P(f)}.$$

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$$4. \quad v(t) = \sum_{n=-\infty}^{\infty} I_n g(t-nT)$$

$R =$ bit rate.

$$\frac{1}{T} = R/k \Rightarrow \text{symbol rate.}$$

$\{I_n\} =$ sequence of symbols.

$$\phi_{vv}(t+\tau, t) = E[v(t)^* v(t+\tau)]$$

$$= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} E[I_n^* I_m] g^*(t-nT) g(t+\tau-mT)$$

$\{I_n\}$ is WSS mean μ_i and autocorrelation $\phi_{ii}(m)$

$$\phi_{ii}(m) = E[I_n^* I_{n+m}]$$

$$\therefore \phi_{vv}(t+\tau, t) = \sum_{m=-\infty}^{\infty} \phi_{ii}(m) \sum_{n=-\infty}^{\infty} g^*(t-nT) g(t+\tau-nT-mT)$$

Now

$$\sum_{n=-\infty}^{\infty} g^*(t-nT) g(t+\tau-nT-mT)$$

is periodic in t with period T ,

$\therefore \phi_{vv}(t+\tau, t)$ is also periodic in t with

period T

$$\phi_{vv}(t+T+\tau; t+T) = \phi_{vv}(t+\tau; t)$$

(4)

also,

$$E\{v(t)\} = \mu_i \sum_{n=-\infty}^{\infty} g(t-nT)$$

$\therefore v(t)$ is a stochastic process having periodic mean and autocorrelation function. Such a process is called cyclostationary process or periodically stationary process in the wide sense.

In order to compute PSD of cyclostationary process dependence of $\phi_{vv}(t+\tau; t)$ on variable t must be eliminated.

This is accomplished by averaging $\phi_{vv}(t+\tau; t)$ over a single period. Thus,

$$\begin{aligned} \bar{\phi}_{vv}(\tau) &= \frac{1}{T} \int_{-T/2}^{T/2} \phi_{vv}(t+\tau; t) dt \\ &= \sum_{m=-\infty}^{\infty} \phi_{ii}(m) \sum_{n=-\infty}^{\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^*(t-nT) g(t+\tau-nT-mT) dt \end{aligned}$$

$$= \sum_{m=-\infty}^{\infty} \phi_{ii}(m) \sum_{n=-\infty}^{\infty} \frac{1}{T} \int_{-T/2-nT}^{T/2-nT} g^*(t) g(t+\tau-mT) dt$$

Let's define $\phi_{gg}(\tau) = \int_{-\infty}^{\infty} g(t) g^*(t+\tau) dt$

$$\bar{\phi}_{vv}(t) = \frac{1}{T} \sum_{m=-\infty}^{\infty} \phi_{ii}(m) \phi_{gg}(t-mT) \quad (5)$$

$$\phi_w(f) = \frac{1}{T} |G(f)|^2 \phi_{ii}(f)$$

$$\phi_{ii}(f) = \sum_{m=-\infty}^{\infty} \phi_{ii}(m) e^{-j2\pi f m T}$$

Now

$$\phi_{ii}(m) = \frac{1}{T} \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \phi_{ii}(f) e^{j2\pi f m T} df$$

because $\phi_{ii}(f)$ is periodic with period $\frac{1}{T}$ as $\phi_{ii}(t)$ is periodic with period T .

Now for special case where $\{I_n\}$ are real and mutually uncorrelated,

$$\phi_{ii}(m) = \begin{cases} \sigma_i^2 + \mu_i^2 & (m=0) \\ \mu_i^2 & (m \neq 0) \end{cases}$$

$$\phi_{ii}(f) = \sigma_i^2 + \mu_i^2 \sum_{m=-\infty}^{\infty} e^{-j2\pi f m T}$$

$$\begin{matrix} \infty & \downarrow \\ \frac{1}{T} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T}\right) \end{matrix}$$

$$\therefore \phi_{ii}(f) = \sigma_i^2 + \frac{\mu_i^2}{T} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T}\right) \quad (6)$$

$$\therefore \phi_{vv}(f) = \frac{\sigma_i^2}{T} |G(f)|^2 + \frac{\mu_i^2}{T^2} \sum_{m=-\infty}^{\infty} \left| G\left(\frac{m}{T}\right) \right|^2 \delta\left(f - \frac{m}{T}\right)$$
