

8.1.  $\psi_n(t)$ ,  $n = 1, 2, 3$  are orthogonal.  $\Rightarrow$  we have to prove.

$$\int_{-\infty}^{\infty} \psi_m(t) \psi_n(t) dt = 0 \quad m \neq n$$

$$\textcircled{a} C_{12} = \int_{-\infty}^{\infty} \psi_1(t) \psi_2(t) dt = \int_0^4 \psi_1(t) \psi_2(t) dt.$$

$$= \int_0^2 \psi_1(t) \psi_2(t) dt + \int_2^4 \psi_1(t) \psi_2(t) dt.$$

$$= \frac{1}{4} \times 2 - \frac{1}{4} \times (4-2) = \underline{0}$$

$$C_{13} = \int_{-\infty}^{\infty} \psi_1(t) \psi_3(t) dt = \int_0^4 \psi_1(t) \psi_3(t) dt$$

$$= \frac{1}{4} \int_0^1 dt - \frac{1}{4} \int_1^2 dt - \frac{1}{4} \int_2^3 dt + \frac{1}{4} \int_3^4 dt$$

$$= \underline{0}.$$

$$C_{23} = \int_{-\infty}^{\infty} \psi_2(t) \psi_3(t) dt = \int_0^4 \psi_2(t) \psi_3(t) dt.$$

$$= \frac{1}{4} \int_0^1 dt - \frac{1}{4} \int_1^2 dt + \frac{1}{4} \int_2^3 dt - \frac{1}{4} \int_3^4 dt$$

$$= \underline{0}.$$

$\therefore$  Signals  $\psi_n(t)$  are orthogonal.

2) we find the weighting coefficients

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$$x_n = \int_{-\infty}^{\infty} x(t) \psi_n(t) dt, \quad n=1,2,3.$$

$$x_1 = \int_0^4 x(t) \psi_1(t) dt = \frac{1}{2} \int_0^1 dt + \frac{1}{2} \int_1^2 dt - \frac{1}{2} \int_2^3 dt + \frac{1}{2} \int_3^4 dt \\ = \underline{\underline{0}}$$

$$x_2 = \int_0^4 x(t) \psi_2(t) dt = \frac{1}{2} \int_0^4 x(t) dt = 0.$$

$$x_3 = \int_0^4 x(t) \psi_3(t) dt = -\frac{1}{2} \int_0^1 dt - \frac{1}{2} \int_1^2 dt + \frac{1}{2} \int_2^3 dt + \frac{1}{2} \int_3^4 dt \\ = 0.$$

~~$x_3$~~

$\therefore x(t)$  is orthogonal to  $\psi_i(t)$ ,  $i=1,2,3$ , and thus it cannot be represented as a linear combination of these functions.

8.2 The expansion coefficients  $\{c_n\}$  that minimize the mean square error satisfy

$$c_n = \int_{-\infty}^{\infty} x(t) \psi_n(t) dt = \int_0^4 \sin \frac{\pi t}{4} \psi_n(t) dt$$

②

p.3

$$\begin{aligned}
 C_1 &= \int_0^4 \sin \frac{\pi t}{4} \psi_1(t) dt = \frac{1}{2} \int_0^2 \sin \frac{\pi t}{4} dt - \frac{1}{2} \int_2^4 \sin \frac{\pi t}{4} dt \\
 &= \left. -\frac{2}{\pi} \cos \left( \frac{\pi t}{4} \right) \right|_0^2 + \left. \frac{2}{\pi} \cos \frac{\pi t}{4} \right|_2^4 \\
 &= -\frac{2}{\pi} (0 - 1) + \frac{2}{\pi} (-1 - 0) = \underline{\underline{0}}
 \end{aligned}$$

$$\begin{aligned}
 C_2 &= \int_0^4 \sin \frac{\pi t}{4} \psi_2(t) dt = \frac{1}{2} \int_0^4 \sin \frac{\pi t}{4} dt \\
 &= \left. -\frac{2}{\pi} \cos \frac{\pi t}{4} \right|_0^4 = -\frac{2}{\pi} (-1 - 1) = \underline{\underline{\frac{4}{\pi}}}
 \end{aligned}$$

$$\begin{aligned}
 C_3 &= \int_0^4 \sin \frac{\pi t}{4} \psi_3(t) dt \\
 &= \frac{1}{2} \int_0^1 \sin \frac{\pi t}{4} dt - \frac{1}{2} \int_1^2 \sin \frac{\pi t}{4} dt + \frac{1}{2} \int_2^3 \sin \frac{\pi t}{4} dt \\
 &\quad - \frac{1}{2} \int_3^4 \sin \frac{\pi t}{4} dt \\
 &= \underline{\underline{0}}
 \end{aligned}$$

2) The residual mean square error  $E_{\min}$  can be found from

$$E_{\min} = \int_{-\infty}^{\infty} |x(t)|^2 dt - \sum_{i=1}^3 |C_i|^2$$

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$$= \int_0^4 \left( \frac{\sin \frac{\pi t}{4}}{4} \right)^2 dt - \left( \frac{4}{\pi} \right)^2 = \frac{1}{2} \int_0^4 \left( 1 - \cos \frac{\pi t}{2} \right) dt - \frac{16}{\pi^2}$$

$$= 2 - \frac{1}{\pi} \sin \frac{\pi t}{2} \Big|_0^4 - \frac{16}{\pi^2} = \underline{\underline{2 - \frac{16}{\pi^2}}}$$

8.5.

1) Impulse response of matched filter to  $s(t)$  is

$$h(t) = s(T-t) = s(3-t) = s(t) \dots$$

because  $s(t)$  is even wrt  $t = T/2 = 3/2$  axis.

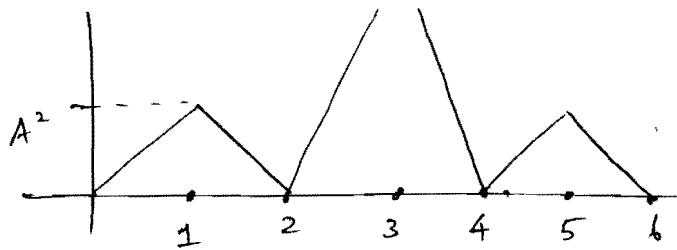
2) The output of the matched filter is

$$y(t) = s(t) * s(t) = \int_a^t s(\tau) s(t-\tau) d\tau$$

$$= \begin{cases} 0, & t < 0 \\ A^2 t, & 0 \leq t < 1 \\ A^2 (2-t), & 1 \leq t < 2 \\ 2A^2 (t-2), & 2 \leq t < 3 \\ 2A^2 (4-t), & 3 \leq t < 4 \\ A^2 (t-4), & 4 \leq t < 5 \\ A^2 (6-t), & 5 \leq t < 6 \\ 0, & 6 \leq t \end{cases}$$

(3)

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3) At the output of the matched filter for  $t = T = 3$ , the noise is

$$\begin{aligned} n_T &= \int_0^T n(\tau) h(T-\tau) d\tau \\ &= \int_0^T n(\tau) h(T - (T-\tau)) d\tau = \int_0^T n(\tau) s(\tau) d\tau \end{aligned}$$

The variance of the noise is

$$\begin{aligned} \sigma_{n_T}^2 &= E \left[ \int_0^T \int_0^T n(\tau) n(\nu) s(\tau) s(\nu) d\tau d\nu \right] \\ &= \int_0^T \int_0^T s(\tau) s(\nu) E[n(\tau) n(\nu)] d\tau d\nu \\ &= \frac{N_0}{2} \int_0^T s^2(\tau) d\tau = \underline{\underline{N_0 A^2}} \end{aligned}$$

4) For antipodal equiprobable signals prob. of error is

$$P(e) = Q \left( \sqrt{\frac{2E_b}{N_0}} \right) \quad E_b = \underline{\underline{2A^2}}$$

$$P(e) = Q \left( \underline{\underline{\frac{2A}{\sqrt{N_0}}}} \right)$$

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The energy of the two signals  $s_1(t)$  and  $s_2(t)$  is

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$$E_b = A^2 T$$

Basis function

$$\phi(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \leq t \leq T/2 \\ -\frac{1}{\sqrt{T}} & T/2 \leq t \leq T \end{cases}$$

The vector representation of signals is

$$\pm A\sqrt{T}$$

$$P(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$\Rightarrow$  equiprobable antipodal signals.

$$= Q\left(\sqrt{\frac{2A^2 T}{N_0}}\right)$$