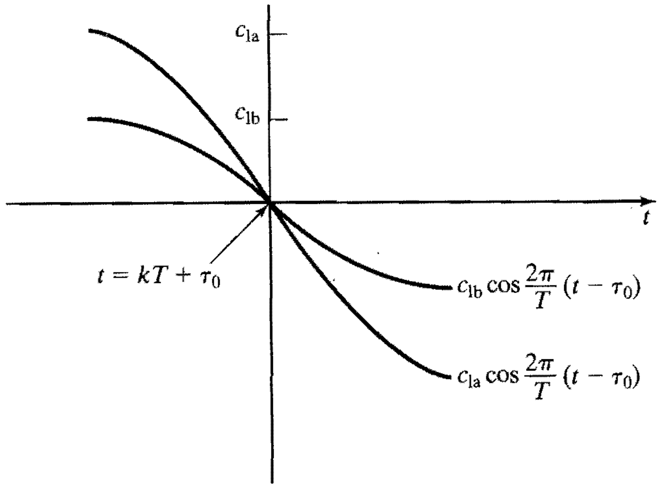

Figure 8.55 Illustration of the slope of the sinusoid at the zero crossing as a function of the amplitude.

as the band of frequencies of the signal $X(f)$ beyond $f = 1/2T$. If the excess bandwidth is zero, i.e., $X(f) = 0$ for $|f| > 1/2T$, then $c_1 = 0$; thus, this method fails to provide a timing signal. If the excess bandwidth is large, such as $\alpha/2T$ where $\alpha = 1/2$ or 1, the timing signal amplitude will be sufficiently large to yield relatively accurate symbol timing estimates.

8.7 FURTHER READING

The geometrical representation of digital signals as vectors was first used by Kotelnikov (1947) and by Shannon (1948a, 1948b) (in his classic papers). This approach was popularized by Wozencraft and Jacobs (1965). Today, this approach to signal analysis and design is widely used. Treatments similar to those given in the text may be found in most books on digital communications.

The matched filter was introduced by North (1943), who showed that it maximized the SNR. Analysis of various binary and M-ary modulation signals in AWGN were performed in the two decades following Shannon's work. Treatments similar to those given in this chapter may be found in most books on digital communications.

A number of books and tutorial papers have been published on the topic of time synchronization. Books that cover both carrier-phase recovery and time synchronization have been written by Stiffler (1971), Lindsey (1972), Lindsey and Simon (1973), Meyr and Ascheid (1990), and Mengali and D'Andrea (1997). The tutorial paper by Franks (1980) presents a very readable introduction to this topic.

PROBLEMS

8.1 Consider the three waveforms $\psi_n(t)$ shown in Figure P-8.1.

1. Show that these waveforms are orthonormal.

2. Express the waveform $x(t)$ as a weighted linear combination of $\psi_n(t)$, $n = 1, 2, 3$ if

$$x(t) = \begin{cases} -1, & 0 \le t \le 1 \\ 1, & 1 \le t \le 3 \\ -1, & 3 \le t \le 4 \end{cases}$$

and determine the weighting coefficients.

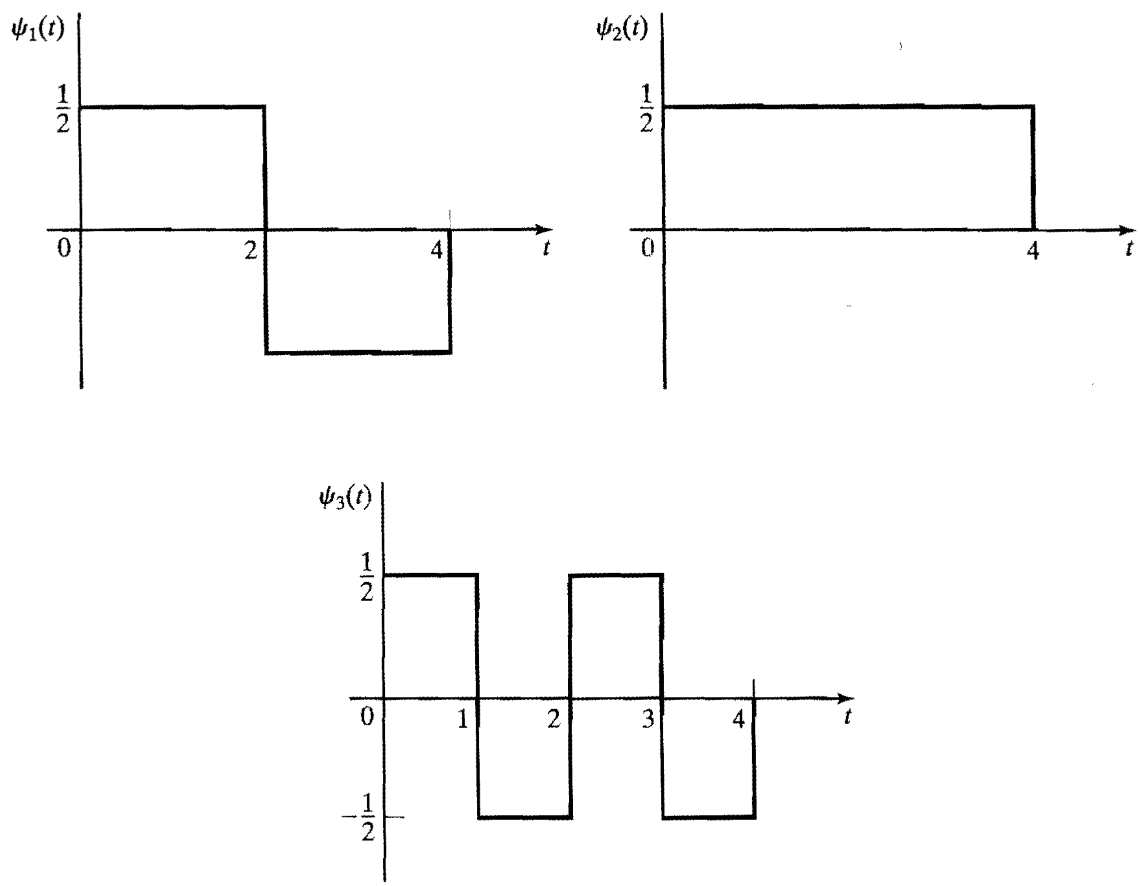

Figure P-8.1

8.2 Use the orthonormal waveforms in Figure P-8.1 to approximate the function

$$x(t) = \sin(\pi t/4)$$

over the interval $0 \le t \le 4$ by the linear combination

$$\hat{x}(t) = \sum_{n=1}^{3} c_n \psi_n(t).$$

1. Determine the expansion coefficients $\{c_n\}$ that minimize the mean square approximation error

$$E = \int_0^4 \left[x(t) - \hat{x}(t)\right]^2 dt.$$

2. Determine the residual mean square error $E_{\min}$.

8.3 Consider the four waveforms shown in Figure P-8.3.

1. Determine the dimensionality of the waveforms and a set of basis functions.
2. Use the basis functions to represent the four waveforms by vectors s_1, s_2, s_3, s_4.
3. Determine the minimum distance between any pair of vectors.

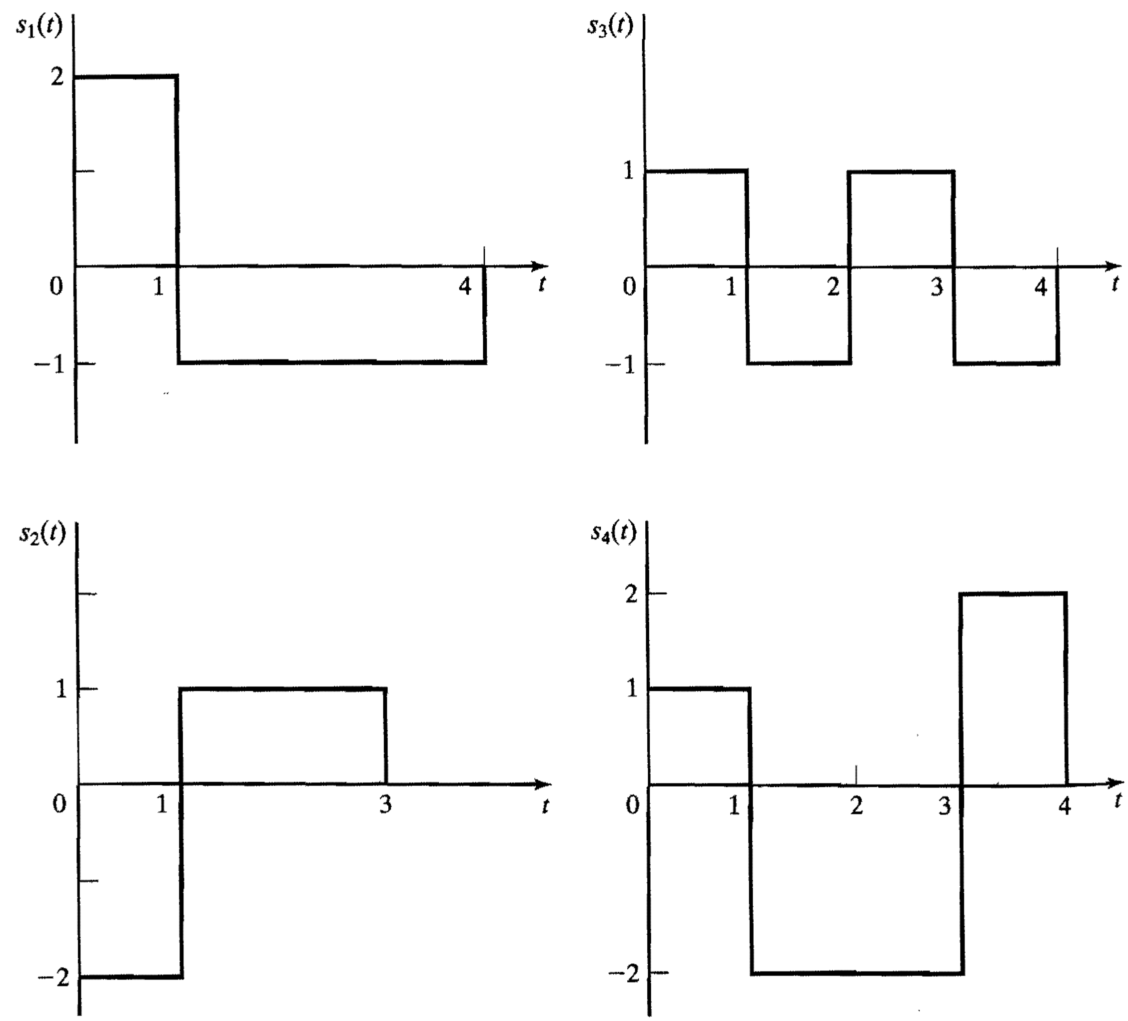

Figure P-8.3

8.4 Determine a set of orthonormal functions for the four signals shown in Figure P-8.4.

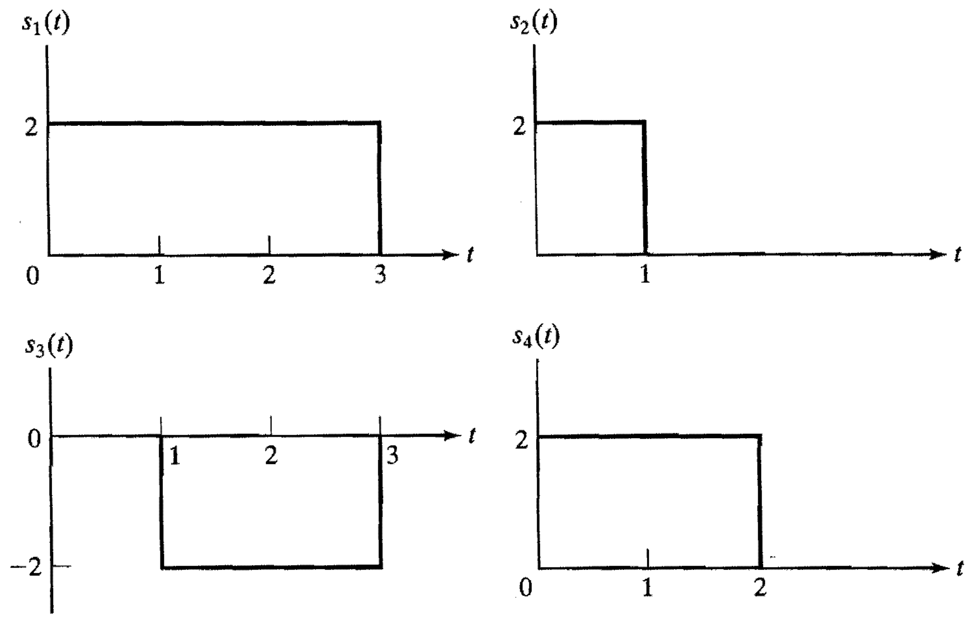

Figure P-8.4

8.5 The received signal in a binary communication system that employs antipodal signals is

$$r(t) = s(t) + n(t),$$

where $s(t)$ is shown in Figure P-8.5 and $n(t)$ is AWGN with power spectral density $N_0/2$ W/Hz.

1. Sketch the impulse responses of the filter matched to $s(t)$.
2. Sketch the output of the matched filter to the input $s(t)$.
3. Determine the variance at the noise of the output of the matched filter at $t = 3$.
4. Determine the probability of error as a function of A and N_0.

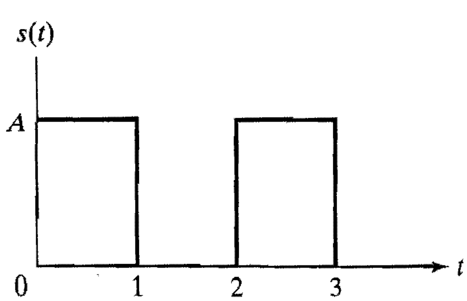

Figure P-8.5

8.16 Suppose that binary PAM is used for transmitting information over an AWGN with power spectral density of $N_0/2 = 10^{-10}$ W/Hz. The transmitted signal energy is $\mathcal{E}_b = A^2T/2$, where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6} if the data rate is (a)10 kbps, (b)100 kbps, and (c) 1 Mbps.

8.17 A Manchester encoder maps an information 1 into 10 and a 0 into 01. The signal waveforms corresponding to the Manchester code are shown in Figure P-8.17. Determine the probability of error if the two signals are equally probable.

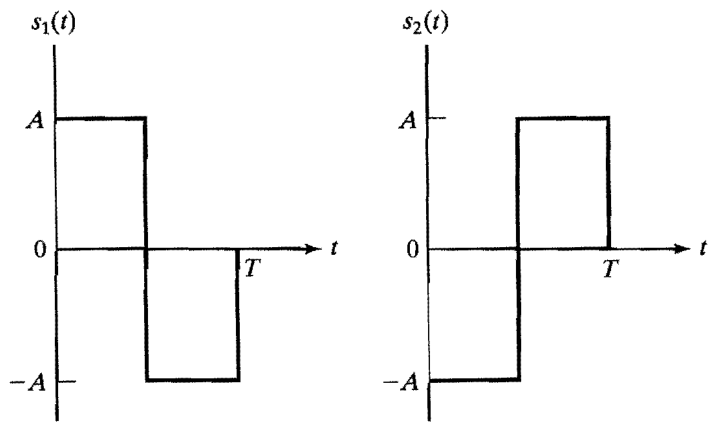

Figure P-8.17

8.18 The signal waveform

$$s(t) = \begin{cases} e^{-t} & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

is passed through its matched filter, $h(t) = s(T - t)$. Determine the output of the matched filter.

8.19 The data rate in a binary PAM communication system with AWGN is 2 Mbps. If the desired average error probability is 10^{-6}, determine the SNR/bit, $\mathcal{E}_b/N_0$, and the power-to-noise ratio P_{av}/N_0.

8.20 A binary digital communication system employs the signals

$$s_0(t) = 0, \quad 0 \le t \le T$$

and

$$s_1(t) = A, \quad 0 \le t \le T$$

for transmitting the information. This is called *on–off signaling*. The demodulator cross-correlates the received signal $r(t)$ with $s(t)$ and samples the output of the correlator at $t = T$.

1. Determine the optimum detector for an AWGN channel and the optimum threshold, assuming that the signals are equally probable.